DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :4

Topic :-INVERSE TRIGONOMETRICE FUNCTIONS

1 **(c)**

Given that, $\theta = \tan^{-1} a$ and $\phi = \tan^{-1} b$

And $ab = -1$

$$\therefore \tan\theta \tan\phi = ab = -1$$
$$\Rightarrow \tan\theta = -\cot\phi$$
$$\Rightarrow \tan\theta = \tan\left(\frac{\pi}{2} + \phi\right)$$

$\Rightarrow \theta - \phi = \frac{\pi}{2}$

2 **(b)**

Let $\cot^{-1}\frac{1}{2} = \phi \Rightarrow \frac{1}{2} = \cot\phi$

$$\Rightarrow \sin\phi = \frac{1}{\sqrt{1+\cot^2\phi}} = \frac{2}{\sqrt{5}}$$
$$\text{Let } \cos^{-1} x = \theta \Rightarrow \sec\theta = \frac{1}{x}$$
$$\Rightarrow \tan\theta = \sqrt{\sec^2\theta - 1}$$
$$\Rightarrow \tan\theta = \sqrt{\frac{1}{x^2} - 1}$$
$$\Rightarrow \tan\theta = \frac{\sqrt{1-x^2}}{x}$$

Now, $\tan(\cos^{-1} x) = \sin\left(\cot^{-1}\frac{1}{2}\right)$

$$\Rightarrow \tan\left(\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right) = \sin\left(\sin^{-1}\frac{2}{\sqrt{5}}\right)$$
$$\Rightarrow \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}}$$
$$\Rightarrow \sqrt{(1-x^2)5} = 2x$$

On squaring both sides, we get

$$(1-x^2)5 = 4x^2$$
$$\Rightarrow 9x^2 = 5$$

$$\Rightarrow x = \pm \frac{\sqrt{5}}{3}$$

3 **(b)**

We have, $\sin^{-1} x + \sin^{-1} y = \pi - \sin^{-1} z$

or$x\sqrt{1-y^2} + y\sqrt{1-x^2} = z$

or$x^2(1-y^2) = z^2 + y^2(1-x^2) - 2yz\sqrt{(1-x^2)}$

or$(x^2 - z^2 - y^2)^2 = 4y^2z^2(1-x^2)$

or$x^4 + y^4 + z^4 - 2x^2z^2 + 2y^2z^2 - 2x^2y^2 + 4x^2y^2z^2 - 4y^2z^2 = 0$

or$x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$

$$\therefore\ k = 2$$

4 **(c)**

Let $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

$$\therefore\ \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right) - \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$= 3\tan^{-1} x - 2\tan^{-1} x$$

$$= \tan^{-1} x$$

5 **(d)**

Let $\alpha = \cos^{-1}\sqrt{P}, \beta = \cos^{-1}\sqrt{1-P}$

And $\gamma = \cos^{-1}\sqrt{1-q}$

$$\Rightarrow \cos\alpha = \sqrt{p}, \cos\beta = \sqrt{1-p}$$

And $\cos\gamma = \sqrt{1-q}$

Therefore, $\sin\alpha = \sqrt{1-p}, \sin\beta = \sqrt{p}$ and $\sin\gamma = \sqrt{q}$

The given equation may be written as

$$\alpha + \beta + \gamma = \frac{3\pi}{4}$$

$$\Rightarrow \alpha + \beta = \frac{3\pi}{4} - \gamma$$

$$\Rightarrow \cos(\alpha + \beta) = \cos\left(\frac{3\pi}{4} - \gamma\right)$$

$$\Rightarrow \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$= \cos\left\{\pi - \left(\frac{\pi}{4} + \gamma\right)\right\} = -\cos\left(\frac{\pi}{4} + \gamma\right)$$

$$\Rightarrow \sqrt{p}\sqrt{1-p} - \sqrt{1-p}\sqrt{p} = -\left(\frac{1}{\sqrt{2}}\sqrt{1-q} - \frac{1}{\sqrt{2}}\sqrt{q}\right)$$

$$\Rightarrow 0 = \sqrt{1-q} - \sqrt{q} \Rightarrow 1 - q = q$$

$$\Rightarrow q = \frac{1}{2}$$

6 **(b)**

Let α, β are the roots of given equation $6x^2 - 5x + 1 = 0$

$\Rightarrow\ \alpha + \beta = \frac{5}{6}$ and $\alpha\beta = \frac{1}{6}$

$$\therefore \tan^{-1}\alpha + \tan^{-1}\beta = \tan^{-1}\left(\frac{\alpha+\beta}{1-\alpha\beta}\right)$$

$$= \tan^{-1}\left(\frac{\frac{5}{6}}{1-\frac{1}{6}}\right) = \tan^{-1}(1) = \frac{\pi}{4}$$

7 **(a)**

Since, $\alpha = \sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{1}{3}\right)$

$$= \sin^{-1}\left(\frac{4}{5}\sqrt{1-\frac{1}{9}} + \frac{1}{3}\sqrt{1-\frac{16}{25}}\right)$$

$$\Rightarrow \alpha = \sin^{-1}\left(\frac{8\sqrt{2}}{15} + \frac{3}{15}\right) = \sin^{-1}\left(\frac{8\sqrt{2}+3}{15}\right)$$

Since, $\frac{8\sqrt{2}+3}{15} < 1$

$$\therefore \alpha < \frac{\pi}{2}$$

Now, $\beta = \cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{1}{3}\right)$

$$\Rightarrow \beta = \frac{\pi}{2} - \sin^{-1}\left(\frac{4}{5}\right) + \frac{\pi}{2} - \sin^{-1}\left(\frac{1}{3}\right)$$

$$= \pi - \left(\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{1}{3}\right)$$

$$= \pi - \alpha$$

$$\Rightarrow \beta > \alpha \left(\therefore \alpha < \frac{\pi}{2}\right)$$

8 **(c)**

$$\because [\sin^{-1} x] > [\cos^{-1} x]$$

$$\Rightarrow x > 0$$

Here, $[\cos^{-1} x] = \begin{cases} 0, & x \in (\cos 1, 1) \\ 1, & x \in (0, \cos 1) \end{cases}$

and, $[\sin^{-1} x] = \begin{cases} 0, & x \in (0, \sin 1) \\ 1, & x \in (\sin 1, 1) \end{cases}$

$$\therefore x \in [\sin 1, 1)$$

$$\because \left[\frac{x}{2}\right] = 1$$

Or we say that $x \in [\sin 1, 1]$

9 **(c)**

We have,

$\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$

$$= \tan^{-1} 1 + \pi + \tan^{-1}\left(\frac{2+3}{1-2\times 3}\right)$$

$$= \tan^{-1} 1 + \pi + \tan^{-1}(-1) = \pi$$

10 **(d)**

We have, $(\sin^{-1} x)^3 + (\cos^{-1} x)^3$

$$= (\sin^{-1} + \cos^{-1} x)^3 - 3 \sin^{-1} x \cos^{-1} x \,(\sin^{-1} x + \cos^{-1} x)$$

$$= \frac{\pi^3}{8} - 3(\sin^{-1} x \cos^{-1} x) \frac{\pi}{2}$$

$$= \frac{\pi^3}{8} - \frac{3\pi}{2} \sin^{-1} x \left(\frac{\pi}{2} - \sin^{-1} x\right)$$

$$= \frac{\pi^3}{8} - \frac{3\pi}{2} \sin^{-1} x + \frac{3\pi}{2} (\sin^{-1} x)^2$$

$$= \frac{\pi^3}{8} + \frac{3\pi}{2}\left[(\sin^{-1} x)^2 - \frac{\pi}{2} \sin^{-1} x\right]$$

$$= \frac{\pi^3}{8} + \frac{3\pi}{2}\left[(\sin^{-1} x)^2 - \frac{\pi}{2} \sin^{-1} x + \frac{\pi^2}{16} - \frac{\pi^2}{16}\right]$$

$$= \frac{\pi^3}{8} + \frac{3\pi}{2}\left[\left(\sin^{-1} x - \frac{\pi}{4}\right)^2\right] - \frac{3\pi^3}{32}$$

$$= \frac{\pi^3}{32} + \frac{3\pi}{2}\left(\sin^{-1} x - \frac{\pi}{4}\right)^2$$

∴ The least value is $\frac{\pi^3}{32}$

Since, $\left(\sin^{-1} x - \frac{\pi}{4}\right)^2 \leq \left(\frac{3\pi}{4}\right)^2$

∴ The greatest value is $\frac{\pi^3}{32} + \frac{9\pi^2}{16} \times \frac{3\pi}{2} = \frac{7\pi^3}{8}$

11 **(d)**

Given, $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{3}{4}\right) = \tan^{-1}\left(\frac{x}{3}\right)$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{3} + \frac{3}{4}}{1 - \frac{1}{3} \times \frac{3}{4}}\right) = \tan^{-1}\left(\frac{x}{3}\right)$$

$$\Rightarrow \frac{13}{9} = \frac{x}{3} \Rightarrow x = \frac{13}{3}$$

13 **(b)**

$$\tan^{-1}\frac{x}{y} - \tan^{-1}\frac{x-y}{x+y}$$

$$= \tan^{-1}\frac{x}{y} - \tan^{-1}\left[\frac{1 - \frac{y}{x}}{1 + \frac{y}{x}}\right]$$

$$= \tan^{-1}\frac{x}{y} - \tan^{-1} 1 + \tan^{-1}\frac{y}{x}$$

$$= \tan^{-1}\frac{x}{y} + \cot^{-1}\frac{x}{y} - \tan^{-1} 1$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

14 **(c)**

Given, two angles of triangle are $\tan^{-1} 2$ and $\tan^{-1} 3$.

Let third angle be θ. Then,

$\tan^{-1} 2 + \tan^{-1} 3 + \theta = 180°$

$\Rightarrow \quad \tan^{-1}\left(\frac{2+3}{1-2\times 3}\right) = 180° - \theta$

$\Rightarrow \quad \frac{5}{-5} = \tan(180° - \theta) = -\tan\theta$

$\Rightarrow \quad \tan\theta = 1 = \tan\frac{\pi}{4}$

$\Rightarrow \quad \theta = \frac{\pi}{4}$

15 **(c)**

$$8x^2 + 22x + 5 = 0 \Rightarrow x = -\frac{1}{4}, -\frac{5}{2}$$

$$\therefore -1 < -\frac{1}{4} < 1 \text{ and } -\frac{5}{2} < -1$$

$\therefore \sin^{-1}\left(-\frac{1}{4}\right)$exists but $\sin^{-1}\left(-\frac{5}{2}\right)$ does not exist.

$\sec^{-1}\left(-\frac{5}{2}\right)$existsbut$\sec^{-1}\left(-\frac{1}{4}\right)$does not exist.

$\tan^{-1}\left(-\frac{1}{4}\right)and\tan^{-1}\left(-\frac{5}{2}\right)$ both exist.

16 **(b)**

We have, $\Sigma x_1 = \sin 2\beta, \Sigma x_1 x_2 = \cos 2\beta, \Sigma x_1 x_2 x_3 = \cos\beta$ and $x_1 x_2 x_3 x_4 = -\sin\beta$

$$\therefore \tan^{-1} x_1 + \tan^{-1} x_2 + \tan^{-1} x_3 + \tan^{-1} x_4$$

$$= \tan^{-1}\left(\frac{\Sigma x_1 - \Sigma x_1 x_2 x_3}{1 - \Sigma x_1 x_2 + x_1 x_2 x_3 x_4}\right)$$

$$= \tan^{-1}\left(\frac{\sin 2\beta - \cos\beta}{1 - \cos 2\beta - \sin\beta}\right)$$

$$= \tan^{-1}\left(\frac{(2\sin\beta - 1)\cos\beta}{\sin\beta(2\sin\beta - 1)}\right)$$

$$= \tan^{-1}(\cot\beta)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{2} - \beta\right)\right) = \frac{\pi}{2} - \beta$$

17 **(d)**

$$\cos(2\cos^{-1} x + \sin^{-1} x)$$

$$= \cos[2(\cos^{-1} x + \sin^{-1} x) - \sin^{-1} x]$$

$$= \cos(\pi - \sin^{-1} x) = -\cos(\sin^{-1} x)$$

$$= -\cos\left[\sin^{-1}\left(-\frac{1}{5}\right)\right]\left(\therefore x = \frac{1}{5}\right)$$

$$= -\cos\left(\cos^{-1}\frac{2\sqrt{6}}{5}\right)$$

$$= -\frac{2\sqrt{6}}{5}$$

18 **(a)**

$$\therefore \cot^{-1} x + \sin^{-1}\frac{1}{\sqrt{5}} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\frac{1}{x} + \tan^{-1}\frac{1}{2} = \tan^{-1} 1$$

$$\Rightarrow \tan^{-1}\frac{1}{x} = \tan^{-1} 1 - \tan^{-1}\frac{1}{2}$$

$$\Rightarrow \tan^{-1}\frac{1}{x} = \tan^{-1}\left(\frac{1-\frac{1}{2}}{1+\frac{1}{2}}\right)$$

$$\Rightarrow \tan^{-1}\frac{1}{x} = \tan^{-1}\frac{1}{3}$$

$$\Rightarrow x = 3$$

19 **(b)**

We have,

$\cos(2\tan^{-1} x) = \frac{1}{2}$

$\Rightarrow 2\tan^{-1} x = \frac{\pi}{3} \Rightarrow \tan^{-1} x = \frac{\pi}{6} \Rightarrow x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

20 **(c)**

Given that, $\sin^{-1}\frac{1}{3} + \sin^{-1}\frac{2}{3} = \sin^{1} x$

$\Rightarrow \sin^{-1}\left(\frac{1}{3}\sqrt{1-\frac{4}{9}} + \frac{2}{3}\sqrt{1-\frac{1}{9}}\right) = \sin^{-1} x$

$\Rightarrow \sin^{-1}\left(\frac{1}{3}.\frac{\sqrt{5}}{3} + \frac{2}{3}.\frac{\sqrt{8}}{3}\right) = \sin^{-1} x$

$\Rightarrow \sin^{-1}\left(\frac{\sqrt{5}+4\sqrt{2}}{9}\right) = \sin^{-1} x$

$\therefore x = \left(\frac{\sqrt{5}+4\sqrt{2}}{9}\right)$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	B	C	D	B	A	C	C	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	C	B	C	C	B	D	A	B	C