DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :3

Topic :-INVERSE TRIGONOMETRICE FUNCTIONS

1 **(d)**

Given, $5\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)+7\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$-4\tan^{-1}\left(\frac{2x}{1-x^2}\right)-\tan^{-1}x=5\pi$$

$\Rightarrow\ 5(2\tan^{-1}x)+7(2\tan^{-1}x)-4(2\tan^{-1}x)-\tan^{-1}x=5\pi$

$\Rightarrow\ 15\tan^{-1}x=5\pi$

$\Rightarrow \tan^{-1}x=\frac{\pi}{3}$

$\therefore\ x=\sqrt{3}$

2 **(c)**

$$\because -\frac{\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2}, -\frac{\pi}{2}\le\sin^{-1}y\le\frac{\pi}{2}$$

And $-\frac{\pi}{2}\le\sin^{-1}z\le\frac{\pi}{2}$

Given that, $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$

Which is possible only when

$$\sin^{-1}x=\sin^{-1}y=\sin^{-1}z=\frac{\pi}{2}$$

Or $x=y=z=1$

Put $p=q=1$

Then $f(2)=f(1)f(1)=2\cdot 2=4$

And put $p=1, q=2$

Then, $f(3)=f(1)f(2)=2\cdot 2^2=8$

$$\therefore x^{f(1)}+y^{f(2)}+z^{f(3)}-\frac{x+y+z}{x^{f(1)}+y^{f(2)}+z^{f(3)}}$$

$$=1+1+1-\frac{3}{1+1+1}$$

$$=3-1=2$$

3 **(a)**

Given, $\tan^{-1}\left(\frac{1}{\sqrt{\cos\alpha}}\right)-\tan^{-1}(\sqrt{\cos\alpha})=x$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{\sqrt{\cos\alpha}} - \sqrt{\cos\alpha}}{1 + \frac{1}{\sqrt{\cos\alpha}}.\sqrt{\cos\alpha}}\right) = x$$

$$\Rightarrow \frac{1 - \cos\alpha}{2\sqrt{\cos\alpha}} = \tan x$$

$$\Rightarrow \frac{2\sqrt{\cos\alpha}}{1 - \cos\alpha} = \cot x$$

$$\Rightarrow \operatorname{cosec} x = \frac{1 + \cos\alpha}{1 - \cos\alpha}$$

$$\Rightarrow \sin x = \frac{1 - \cos\alpha}{1 + \cos\alpha}$$

$$\Rightarrow \sin x = \frac{2\sin^2\left(\frac{\alpha}{2}\right)}{2\cos^2\left(\frac{\alpha}{2}\right)} = \tan^2\left(\frac{\alpha}{2}\right)$$

4 **(b)**

$$\cot^{-1} 9 + \operatorname{cosec}^{-1}\frac{\sqrt{41}}{4} = \tan^{-1}\frac{1}{9} + \tan^{-1}\frac{1}{\sqrt{\frac{41}{16} - 1}}$$

$$\left[\because \operatorname{cosec}^{-1}x = \tan^{-1}\frac{1}{\sqrt{x^2 - 1}}\right]$$

$$= \tan^{-1}\frac{1}{9} + \tan^{-1}\frac{4}{5}$$

$$= \tan^{-1}\left(\frac{\frac{1}{9} + \frac{4}{5}}{1 - \frac{1}{9}.\frac{4}{5}}\right)$$

$$= \tan^{-1}\left(\frac{41}{41}\right) = \frac{\pi}{4}$$

5 **(a)**

We have, $\sum_{m=1}^{n} \tan^{-1}\left(\frac{2m}{m^4 + m^2 + 2}\right)$

$$= \sum_{m=1}^{n} \tan^{-1}\left(\frac{2m}{1 + (m^2 + m + 1)(m^2 - m + 1)}\right)$$

$$= \sum_{m=1}^{n} \tan^{-1}\left(\frac{(m^2 + m + 1) - (m^2 - m + 1)}{1 + (m^2 + m + 1)(m^2 - m + 1)}\right)$$

$$= \sum_{m=1}^{n} [\tan^{-1}(m^2 + m + 1) - \tan^{-1}(m^2 - m + 1)]$$

$= (\tan^{-1} 3 - \tan^{-1} 1) + (\tan^{-1} 7 - \tan^{-1} 3) +$
$(\tan^{-1} 13 - \tan^{-1} 7) + \ldots +$
$[\tan^{-1}(n^2 + n + 1) - \tan^{-1}(n^2 - n + 1)]$

$$= \tan^{-1}\frac{n^2+n+1-1}{1+(n^2+n+1)\cdot 1} = \tan^{-1}\left(\frac{n^2+n}{2+n^2+n}\right)$$

6 **(b)**

$$\therefore \cos^{-1}\frac{3}{5} - \sin^{-1}\frac{4}{5} = \cos^{-1}x$$

$$\Rightarrow \sin^{-1}\frac{4}{5} - \sin^{-1}\frac{4}{5} = \cos^{-1}x$$

$$\Rightarrow \cos^{-1}x = 0 \Rightarrow x = \cos 0 = 1$$

$$\therefore x = 1$$

7 **(a)**

Given, $\cot(\cos^{-1}x) = \sec\left(\tan^{-1}\frac{a}{\sqrt{b^2-a^2}}\right)$

$$\therefore \quad \cot\left(\cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right)$$

$$= \sec\left(\sec^{-1}\frac{b}{\sqrt{b^2-a^2}}\right)$$

$$\Rightarrow \quad \frac{x}{\sqrt{1-x^2}} = \frac{b}{\sqrt{b^2-a^2}}$$

$$\Rightarrow \quad x^2(b^2-a^2) = b^2 - b^2x^2$$

$$\Rightarrow x^2(2b^2-a^2) = b^2$$

$$\Rightarrow \quad x = \frac{b}{\sqrt{2b^2-a^2}}$$

8 **(b)**

Given, $\sin^{-1}x - \cos^{-1}x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\Rightarrow \quad \sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$...(i)

But $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$...(ii)

On solving Eqs. (i) and (ii), we get

$\sin^{-1}x = \frac{\pi}{3}and\cos^{-1}x = \frac{\pi}{6}$

$\Rightarrow \quad x = \frac{\sqrt{3}}{2}$is the unique solution.

9 **(d)**

We have, $\theta = \sin^{-1}x + \cos^{-1}x - \tan^{-1}x$

$$= \frac{\pi}{2} - \tan^{-1}x = \cot^{-1}x$$

Since, $0 \le x \le 1$, therefore $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$

10 **(a)**

$$\therefore \cot^{-1}x + \sin^{-1}\frac{1}{\sqrt{5}} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\frac{1}{x} + \tan^{-1}\frac{1}{2} = \tan^{-1}1$$

$$\Rightarrow \tan^{-1}\frac{1}{x} = \tan^{-1}1 - \tan^{-1}\frac{1}{2}$$

$$\Rightarrow \tan^{-1}\frac{1}{x} = \tan^{-1}\left(\frac{1-\frac{1}{2}}{1+\frac{1}{2}}\right)$$

$$\Rightarrow \tan^{-1}\frac{1}{x} = \tan^{-1}\frac{1}{3}$$

$$\Rightarrow x = 3$$

11 **(b)**

$\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$

Now, put $\frac{4}{5} = \cos 2\theta$

$\therefore \sin\left(\frac{1}{2} \times 2\theta\right)$

$$= \sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}}$$

$$= \sqrt{\frac{1-\frac{4}{5}}{2}}$$

$$= \sqrt{\frac{1}{5 \times 2}}$$

$$= \frac{1}{\sqrt{10}}$$

12 **(b)**

Given, $\sin^{-1}\left(\frac{3}{x}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{4}{x}\right)$

$\Rightarrow \sin^{-1}\left(\frac{3}{x}\right) = \cos^{-1}\left(\frac{4}{x}\right)$

$\Rightarrow \sin^{-1}\left(\frac{3}{x}\right) = \sin^{-1}\left(\frac{\sqrt{x^2-16}}{x}\right)$

$\Rightarrow \frac{3}{x} = \frac{\sqrt{x^2-16}}{x}$

$\Rightarrow x = \pm 5$

$\therefore \quad x = 5$

[$\because -5$ not satisfies the given equation]

13 **(b)**

$$\because 0 \le \cos^{-1}x \le \pi$$

And $0 < \cot^{-1}x < \pi$

Given, $[\cot^{-1}x] + [\cot^{-1}x] = 0$

$$\Rightarrow [\cot^{-1}x] = 0 \text{ and } [\cos^{-1}x] = 0$$

$$\Rightarrow 0 < \cot^{-1}x < 1 \text{ and } 0 \le \cos^{-1}x < 1$$

$$\therefore x \in (\cot 1, \infty) \text{ and } x \in (\cos 1, 1)$$
$$\Rightarrow x \in (\cot 1, 1)$$

14 **(b)**

$$3\sin^{-1}\frac{2x}{1+x^2} - 4\cos^{-1}\frac{1-x^2}{1+x^2} + 2\tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$$

On putting $x = \tan\theta$, we get

$$3\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) - 4\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + 2\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = \frac{\pi}{3}$$

$$\Rightarrow 3\sin^{-1}(\sin 2\theta) - 4\cos^{-1}(\cos 2\theta) + 2\tan^{-1}(\tan 2\theta) = \frac{\pi}{3}$$
$$\Rightarrow 3(2\theta) - 4(2\theta) + 2(2\theta) = \frac{\pi}{3}$$
$$\Rightarrow 6\theta - 8\theta + 4\theta = \frac{\pi}{3}$$
$$\Rightarrow \theta = \frac{\pi}{6} \Rightarrow \tan^{-1} x = \frac{\pi}{6}$$
$$\Rightarrow x = \tan\frac{\pi}{6} \Rightarrow x = \frac{1}{\sqrt{3}}$$

15 **(b)**

Here, $T_n = \cot^{-1}\left(n^2 + \frac{3}{4}\right)$

$$= \tan^{-1}\left(\frac{4}{4n^2+3}\right)$$

$$= \tan^{-1}\left(\frac{1}{1+\left(n+\frac{1}{2}\right)\left(n-\frac{1}{2}\right)}\right)$$

$$= \tan^{-1}\left[\frac{\left(n+\frac{1}{2}\right) - \left(n-\frac{1}{2}\right)}{1+\left(n+\frac{1}{2}\right)\left(n-\frac{1}{2}\right)}\right]$$

$$= \tan^{-1}\left(n+\frac{1}{2}\right) - \tan^{-1}\left(n-\frac{1}{2}\right)$$
$$\therefore S_\infty = T_\infty^{-1} - \tan^{-1}\left(\frac{1}{2}\right)$$
$$= \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}\right)$$
$$\Rightarrow S_\infty = \cot^{-1}\left(\frac{1}{2}\right)$$
$$\Rightarrow S_\infty = \tan^{-1}(2)$$

16 **(d)**

$$2\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left[\frac{2\left(\frac{1}{3}\right)}{1-\frac{1}{9}}\right] + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}\right)$$

$=\tan^{-1}\left(\frac{25}{25}\right) = \frac{\pi}{4}$

17 **(c)**

The given equation is satisfied only when $x = 1$,

$$y = -1, z = 1$$

18 **(d)**

Let $\cot^{-1} x = \theta \Rightarrow x = \cot\theta$

Now, $\operatorname{cosec}\theta = \sqrt{1+\cot^2\theta} = \sqrt{1+x^2}$

$$\Rightarrow \sin\theta = \frac{1}{\operatorname{cosec}\theta} = \frac{1}{\sqrt{1+x^2}}$$

$$\Rightarrow \theta = \sin^{-1}\frac{1}{\sqrt{1+x^2}}$$

$$\therefore \sin(\cot^{-1} x) = \sin\left(\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)$$

$$= \frac{1}{\sqrt{1+x^2}} = (1+x^2)^{-1/2}$$

19 **(c)**

$$\therefore \tan^{-1} x - \tan^{-1} y = 0 \Rightarrow x = y$$

Also, $\cos^{-1} x + \cos^{-1} y = \frac{\pi}{2} \Rightarrow 2\cos^{-1} x = \frac{\pi}{2}$

$$\Rightarrow \cos^{-1} x = \frac{\pi}{4} \Rightarrow x = \frac{1}{\sqrt{2}} \Rightarrow x^2 = \frac{1}{2}$$

Hence, $x^2 + xy + y^2 = 3x^2 = \frac{3}{2}$

20 **(a)**

Let $\tan^{-1} x = \theta$. Then, $x = \tan\theta$

Also, $-1 < x < 1 \Rightarrow -1 < \tan\theta < 1 \Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

Now,

$\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}(\tan 2\theta)$

$= 2\theta \quad \left[\because -\frac{\pi}{4} < \theta < \frac{\pi}{4} \Rightarrow -\frac{\pi}{2} < 2\theta < \frac{\pi}{2}\right]$

$= 2\tan^{-1} x$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	A	B	A	B	A	B	D	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	B	B	B	B	D	C	D	C	A