DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :2

Topic :-INVERSE TRIGONOMETRICE FUNCTIONS

1 **(b)**

We have, $\Sigma x_1 = \sin 2\beta, \Sigma x_1 x_2 = \cos 2\beta, \Sigma x_1 x_2 x_3 = \cos\beta$ and $x_1 x_2 x_3 x_4 = -\sin\beta$

$$\therefore \tan^{-1} x_1 + \tan^{-1} x_2 + \tan^{-1} x_3 + \tan^{-1} x_4$$
$$= \tan^{-1}\left(\frac{\Sigma x_1 - \Sigma x_1 x_2 x_3}{1 - \Sigma x_1 x_2 + x_1 x_2 x_3 x_4}\right)$$
$$= \tan^{-1}\left(\frac{\sin 2\beta - \cos\beta}{1 - \cos 2\beta - \sin\beta}\right)$$
$$= \tan^{-1}\left(\frac{(2\sin\beta - 1)\cos\beta}{\sin\beta(2\sin\beta - 1)}\right)$$
$$= \tan^{-1}(\cot\beta)$$
$$= \tan^{-1}\left(\tan\left(\frac{\pi}{2} - \beta\right)\right) = \frac{\pi}{2} - \beta$$

2 **(d)**

$$\tan^{-1}\left(\frac{\tan x}{4}\right) + \tan^{-1}\left(\frac{3\sin 2x}{5 + 3\cos 2x}\right)$$
$$= \tan^{-1}\left(\frac{\tan x}{4}\right) + \tan^{-1}\left(\frac{\frac{6\tan x}{1 + \tan^2 x}}{5 + \frac{3(1 - \tan^2 x)}{1 + \tan^2 x}}\right)$$
$$= \tan^{-1}\left(\frac{\tan x}{4}\right) + \tan^{-1}\left(\frac{6\tan x}{8 + 2\tan^2 x}\right)$$
$$= \tan^{-1}\left(\frac{\tan x}{4}\right) + \tan^{-1}\left(\frac{3\tan x}{4 + \tan^2 x}\right)$$
$$= \tan^{-1}\left(\frac{\frac{\tan x}{4} + \frac{3\tan x}{4 + \tan^2 x}}{1 - \frac{3\tan^2 x}{4(4 + \tan^2 x)}}\right)\left(\text{as}\left|\frac{\tan x}{4} \cdot \frac{3\tan x}{4\tan^2 x}\right| < 1\right)$$

$$= \tan^{-1}\left(\frac{16\tan x + \tan^3 x}{16 + \tan^2 x}\right)$$

$$= \tan^{-1}(\tan x) = x$$

4 **(c)**

Let $\cos^{-1} x = \theta$. Then, $x = \cos\theta$

Also,

$-1 \le x \le 0 \Rightarrow -1 \le \cos\theta \le 0 \Rightarrow \frac{\pi}{2} \le \theta \le \pi$

Now,

$\cos^{-1}(2x^2 - 1) = \cos^{-1}(\cos 2\theta) = \cos^{-1}(2\pi - 2\theta)$

$= 2\pi - 2\theta \left[\begin{array}{l} \because \frac{\pi}{2} \le \theta \le \pi \Rightarrow \pi \le 2\theta \le 2\pi \\ \Rightarrow 0 \le 2\pi - 2\theta \le \pi \end{array}\right]$

$= 2\pi - 2\cos^{-1} x$

5 **(b)**

$$\therefore \cos^{-1}\frac{3}{5} - \sin^{-1}\frac{4}{5} = \cos^{-1} x$$

$$\Rightarrow \sin^{-1}\frac{4}{5} - \sin^{-1}\frac{4}{5} = \cos^{-1} x$$

$$\Rightarrow \cos^{-1} x = 0 \Rightarrow x = \cos 0 = 1$$

$$\therefore x = 1$$

6 **(d)**

We have,

$\sec^{-1} x = \text{cosec}^{-1} y \Rightarrow \cos^{-1}\frac{1}{x} = \sin^{-1}\frac{1}{y}$

$\therefore \cos^{-1}\frac{1}{x} + \cos^{-1}\frac{1}{y} = \sin^{-1}\frac{1}{y} + \cos^{-1}\frac{1}{y} = \frac{\pi}{2}$

8 **(c)**

Let $\sin^{-1} x = \theta$. Then, $x = \sin\theta$

Also,

$-1 \le x \le -\frac{1}{2}$

$\Rightarrow -1 \le \sin\theta \le -\frac{1}{2} \Rightarrow -\frac{\pi}{2} \le \theta \le -\frac{\pi}{6} \Rightarrow -\frac{3\pi}{2} \le 3\theta \le -\frac{\pi}{2}$

Now,

$\sin^{-1}(3x - 4x^3)$

$= \sin^{-1}(\sin 3\theta)$

$= \sin^{-1}(-\pi - 3\theta)$

$$= -\pi - 3\theta\left[-\frac{3\pi}{2} \le 3\theta\left(2 \Rightarrow -\frac{\pi}{2} \le -\pi - 3\theta \le \frac{\pi}{2}\right)\right]$$

$= -\pi - 3\sin^{-1} x$

9 **(d)**

We have,

$$\frac{\tan\frac{6\pi}{15} - \tan\frac{\pi}{15}}{1 + \tan\frac{6\pi}{15}\tan\frac{\pi}{15}} = \tan\frac{\pi}{3}$$

$$\Rightarrow \tan\frac{6\pi}{15} - \tan\frac{\pi}{15} = \sqrt{3} + \sqrt{3}\tan\frac{6\pi}{15}\tan\frac{\pi}{15}$$

$$\Rightarrow \tan\frac{2\pi}{5} - \tan\frac{\pi}{15} - \sqrt{3}\tan\frac{2\pi}{5}\tan\frac{\pi}{15} = \sqrt{3}$$

10 **(a)**

$$\tan\left\{\cos^{-1}\left(-\frac{2}{7}\right) - \frac{\pi}{2}\right\} = \tan\left\{\pi - \cos^{-1}\left(\frac{2}{7}\right) - \frac{\pi}{2}\right\}$$

$$= \tan\left\{\frac{\pi}{2} - \cos^{-1}\left(\frac{2}{7}\right)\right\}$$

$$= \tan\left\{\sin^{-1}\frac{2}{7}\right\}$$

$$= \tan\left\{\tan^{-1}\left(\frac{2}{3\sqrt{5}}\right)\right\} = \frac{2}{3\sqrt{5}}$$

11 **(a)**

$$\sin\left[\sin^{-1}\left(\frac{1}{3}\right) + \sec^{-1}(3)\right] + \cos\left[\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}(2)\right]$$

$$= \sin\left[\sin^{-1}\left(\frac{1}{3}\right) + \cos^{-1}\left(\frac{1}{3}\right)\right]$$

$$+\cos\left[\tan^{-1}\left(\tfrac{1}{2}\right) + \cot^{-1}\left(\tfrac{1}{2}\right)\right]$$

$$= \sin\frac{\pi}{2} + \cos\frac{\pi}{2}$$

$$\left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \text{ and } \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\right]$$

$$= 1$$

12 **(a)**

Let $\tan^{-1}x = \theta$. Then, $x = \tan\theta$

Also,

$$-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{1}{\sqrt{3}} < \tan\theta < \frac{1}{\sqrt{3}} \Rightarrow -\frac{\pi}{6} < \theta < \frac{\pi}{6} \Rightarrow -\frac{\pi}{2} < 3\theta < \frac{\pi}{2}$$

Now,

$$\tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right) = \tan^{-1}(\tan 3\theta)$$

$$\Rightarrow \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right) = 3\theta \left[\because -\frac{\pi}{2} < 3\theta < \frac{\pi}{2}\right]$$

$$\Rightarrow \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right) = 3\tan^{-1} x$$

13 **(b)**

Let $\cos^{-1}\left(\frac{4}{5}\right) = \theta$. Then, $\cos\theta = \frac{4}{5}$

$$\therefore \sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right) = \sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\frac{4}{5}}{2}} = \frac{1}{\sqrt{10}}$$

14 **(a)**

$$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$$
$$\Rightarrow \frac{3x + 2x}{1 - 6x^2} = \frac{\pi}{4}$$
$$\Rightarrow 5x = 1 - 6x^2$$
$$\Rightarrow 6x^2 + 5x - 1 = 0$$
$$\Rightarrow x = -1, \frac{1}{6}$$

But when $x = -1$,

$$\tan^{-1} 2x = \tan^{-1}(-2) < 0$$

And $\tan^{-1} 3x = \tan^{-1}(-3) < 0$

This value will not satisfy the given equation

Hence, $x = \frac{1}{6}$

15 **(c)**

$$\sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{3} = \sin^{-1}\frac{4}{5} + \tan^{-1}\frac{2\left(\frac{1}{3}\right)}{1-\left(\frac{1}{2}\right)^2}$$

$$= \sin^{-1}\frac{4}{5} + \tan^{-1}\frac{3}{4} = \sin^{-1}\frac{4}{5} + \cos^{-1}\frac{4}{5} = \frac{\pi}{2}$$

$$\left[\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}\right]$$

16 **(a)**

Given equation is

$$2\cos^{-1} x + \sin^{-1} x = \frac{11\pi}{6}$$
$$\Rightarrow \cos^{-1} x + (\cos^{-1} x + \sin^{-1} x) = \frac{11\pi}{6}$$
$$\Rightarrow \cos^{-1} x + \frac{\pi}{2} = \frac{11\pi}{6}$$
$$\Rightarrow \cos^{-1} x = \frac{4\pi}{3}$$

Which is not possible as $\cos^{-1} x \in [0, \pi]$.

17 **(c)**

$$\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\cos\frac{5\pi}{3}\right)$$

$$= \cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left[\sin\left(\frac{\pi}{2}-\frac{5\pi}{3}\right)\right]$$

$$= \frac{5\pi}{3}+\frac{\pi}{2}-\frac{5\pi}{3}=\frac{\pi}{2}$$

Alternate

Since, $\cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}$

$$\therefore \cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\sin\frac{5\pi}{3}\right) = \frac{\pi}{2}$$

18 **(d)**

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}=30°$$

19 **(c)**

We have,

$\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$

$\Rightarrow \sin\{\sin^{-1}x + \sin^{-1}(1-x)\} = \sin(\cos^{-1}x)$

$\Rightarrow x\sqrt{1-(1-x)^2} + \sqrt{1-x^2}(1-x) = \sqrt{1-x^2}$

$\Rightarrow x\sqrt{1-(1-x)^2} = x\sqrt{1-x^2}$

$\Rightarrow x = 0$ or, $2x - x^2 = 1 - x^2 \Rightarrow x = 0$ or $x = \frac{1}{2}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	D	C	C	B	D	D	C	D	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	B	A	C	A	C	D	C	C