DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :10

Topic :-INVERSE TRIGONOMETRICE FUNCTIONS

2 **(b)**

Given, $(\sqrt{3}-i)=(a+ib)(c+id)$

$=(ac-bd)+i(ad+bc)$

On comparing the real and imaginary part on both sides, we get

$ac-bd=\sqrt{3}$

And $ad+bc=1$

Now, $\tan^{-1}\left(\frac{b}{a}\right)+\tan^{-1}\left(\frac{d}{c}\right)$

$=\tan^{-1}\left(\frac{bc+ad}{ac-bd}\right)$

$=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$=n\pi+\frac{\pi}{6}, n\in I$

3 **(b)**

Given, $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x$

Let $x=\tan\theta$

$\therefore \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)=\frac{1}{2}\tan^{-1}(\tan\theta)$

$\Rightarrow \tan^{-1}\left\{\tan\left(\frac{\pi}{4}-\theta\right)\right\}=\frac{1}{2}\tan^{-1}(\tan\theta)$

$\Rightarrow \frac{\pi}{4}-\theta=\frac{\theta}{2}\Rightarrow\theta=\frac{\pi}{6}$

$\therefore x=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$

4 **(c)**

Let $S_\infty=\cot^{-1}2+\cot^{-1}8+\cot^{-1}18+\cot^{-1}32+\ldots$

$\therefore T_n\cot^{-1}2n^2$

$=\tan^{-1}\frac{1}{2n^2}$

$$= \tan^{-1}\left(\frac{2}{4n^2}\right) = \tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right)$$

$$\therefore S_n = \sum_{n=1}^{\infty}\{\tan^{-1}(2n+1) - \tan^{-1}(2n-1)\}$$

$$= \tan^{-1}\infty - \tan^{-1}1$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

6 **(d)**

$$\sin^{-1}\left\{\tan\left(\frac{-5\pi}{4}\right)\right\} = \sin^{-1}\left\{-\tan\left(\pi+\frac{\pi}{4}\right)\right\}$$

$$= \sin^{-1}\left(-\tan\frac{\pi}{4}\right)$$

$$= \sin^{-1}\left(-\sin\frac{\pi}{2}\right)$$

$$= -\frac{\pi}{2}$$

7 **(a)**

$$\because \tan^{-1}\left(\frac{1}{1+r+r^2}\right) = \tan^{-1}\left(\frac{r+1-r}{1+r(r+1)}\right)$$

$$= \tan^{-1}(r+1) - \tan^{-1}(r)$$

$$\therefore \sum_{r=0}^{n}[\tan^{-1}(r+1) - \tan^{-1}(r)]$$

$$= \tan^{-1}(n+1) - \tan^{-1}(0)$$

$$= \tan^{-1}(n+1)$$

$$\Rightarrow \sum_{r=0}^{\infty}\tan^{-1}\left(\frac{1}{1+r+r^2}\right) = \tan^{-1}(\infty) = \frac{\pi}{2}$$

8 **(c)**

Let $\cos^{-1}x = \theta$. Then, $x = \cos\theta$

Also,

$$-1 \le x \le -\frac{1}{2} \Rightarrow -1 \le \cos\theta \le -\frac{1}{2} \Rightarrow \frac{2\pi}{3} \le \theta \le \pi$$

Now,

$\cos^{-1}(4x^3 - 3x)$

$= \cos^{-1}(\cos 3\theta)$

$= \cos^{-1}(\cos(2\pi - 3\theta))$

$= \cos^{-1}(\cos(3\theta - 2\pi))$

$= 3\theta - 2\pi \left[\because \frac{2\pi}{3} \le \theta \le \pi \Rightarrow 0 \le 3\theta - 2\pi \le \pi\right]$

$= 3\cos^{-1}x - 2\pi$

9 **(c)**

Given that, $\tan^{-1}x - \tan^{-1}y = \tan^{-1}A$

$$\Rightarrow \tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1} A$$

Hence, $A = \frac{x-y}{1+xy}$

10 **(b)**

We have, $2\tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$

$$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2 \operatorname{cosec} x)$$

$$\Rightarrow \frac{2\cos x}{\sin^2 x} = 2 \operatorname{cosec} x$$

$$\Rightarrow \sin x = \cos x \Rightarrow x = \frac{\pi}{4}$$

11 **(c)**

Clearly, $x(x+1) \geq 0$ and $x^2 + x + 1 \leq 1$

$$\Rightarrow x(x+1) = 0$$

$$\Rightarrow x = 0, -1$$

When $x = 0$,

LHS=$\tan^{-1} 0 + \sin^{-1} 1 = \frac{\pi}{2}$

When $x = -1$,

LHS=$\tan^{-1} 0 + \sin^{-1}\sqrt{1-1+1}$

$$= 0 + \sin^{-1}(1) = \frac{\pi}{2}$$

Thus, the number of solution is 2

12 **(b)**

We have,

$$\cos\left\{\cos^{-1}\left(-\frac{1}{7}\right) + \sin^{-1}\left(-\frac{1}{7}\right)\right\} = \cos\frac{\pi}{2} = 0$$

13 **(c)**

The given equation is satisfied only when $x = 1$,

$$y = -1, z = 1$$

14 **(c)**

Given, $\sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1} x$

$\Rightarrow \quad 1 - x = \sin\left(\frac{\pi}{2} + 2\sin^{-1} x\right)$

$\Rightarrow \quad 1 - x = \cos(2\sin^{-1} x)$

$\Rightarrow \quad 1 - x = \cos(2\cos^{-1}\sqrt{1-x^2})$

$\Rightarrow \quad 1 - x = \cos\{\cos^{-1}(1-2x^2)\}$

$\Rightarrow \quad 1 - x = 1 - 2x^2$

$\Rightarrow \quad x = 0, \frac{1}{2}$

$\Rightarrow \quad x = 0 \quad \left[\because x = \frac{1}{2} \text{ does not satisfy the given equation}\right]$

15 **(d)**

We have,

$$\cos^{-1}\left(\frac{15}{17}\right) + 2\tan^{-1}\left(\frac{1}{5}\right)$$

$$= \cos^{-1}\left(\frac{15}{17}\right) + \cos^{-1}\left(\frac{1-1/25}{1+1/25}\right)$$

$$= \cos^{-1}\left(\frac{15}{17}\right) + \cos^{-1}\left(\frac{12}{13}\right)$$

$$= \cos^{-1}\left\{\frac{15}{17}\times\frac{12}{13} - \sqrt{1-\left(\frac{15}{17}\right)^2}\sqrt{1-\left(\frac{12}{13}\right)^2}\right\} = \cos^{-1}\left(\frac{140}{221}\right)$$

16 **(c)**

Let $\cot^{-1} x = \theta$. Then, $x = \cot\theta$

Also, $x < 0 \Rightarrow \cot\theta < 0 \Rightarrow \frac{\pi}{2} < \theta < \pi$

Now,

$$\tan^{-1}\left(\frac{1}{x}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}(\tan\theta)$$

$$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}(-\tan(\pi-\theta))$$

$$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}(\tan(\theta-\pi))$$

$$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right) = \theta - \pi\left[\frac{\pi}{2} < \theta < \pi \Rightarrow -\frac{\pi}{2} < \theta - \pi < 0\right]$$

$$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1} x - \pi$$

17 **(d)**

Let $\alpha = \cos^{-1}\sqrt{P}, \beta = \cos^{-1}\sqrt{1-P}$

And $\gamma = \cos^{-1}\sqrt{1-q}$

$$\Rightarrow \cos\alpha = \sqrt{p}, \cos\beta = \sqrt{1-p}$$

And $\cos\gamma = \sqrt{1-q}$

Therefore, $\sin\alpha = \sqrt{1-p}, \sin\beta = \sqrt{p}$ and $\sin\gamma = \sqrt{q}$

The given equation may be written as

$$\alpha + \beta + \gamma = \frac{3\pi}{4}$$

$$\Rightarrow \alpha + \beta = \frac{3\pi}{4} - \gamma$$

$$\Rightarrow \cos(\alpha+\beta) = \cos\left(\frac{3\pi}{4} - \gamma\right)$$

$$\Rightarrow \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$= \cos\left\{\pi - \left(\frac{\pi}{4}+\gamma\right)\right\} = -\cos\left(\frac{\pi}{4}+\gamma\right)$$

$$\Rightarrow\sqrt{p}\sqrt{1-p}-\sqrt{1-p}\sqrt{p}=-\left(\frac{1}{\sqrt{2}}\sqrt{1-q}-\frac{1}{\sqrt{2}}\sqrt{q}\right)$$

$$\Rightarrow 0=\sqrt{1-q}-\sqrt{q}\Rightarrow 1-q=q$$

$$\Rightarrow q=\frac{1}{2}$$

18 **(c)**

$$\tan^{-1}=\frac{m}{n}-\tan^{-1}\frac{m-n}{m+n}$$

$$=\tan^{-1}\frac{m}{n}-\tan^{-1}\frac{\frac{m}{n}-1}{1+\frac{m}{n}}$$

$$=\tan^{-1}\frac{m}{n}-\tan^{-1}\frac{m}{n}+\tan^{-1}(1)=\frac{\pi}{4}$$

19 **(c)**

$$\sin\left[\frac{\pi}{2}-\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]=\cos\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$=\cos\cos^{-1}\sqrt{1-\frac{3}{4}}$$

$$=\cos\cos^{-1}\left(\frac{1}{2}\right)=\frac{1}{2}$$

20 **(d)**

$\cos^{-1}x, \sin^{-1}x$are real, if $-1\leq x\leq 1$

But $\cos^{-1}x>\sin^{-1}x$

$$\Rightarrow 2\cos^{-1}x>\frac{\pi}{2}$$

$$\Rightarrow \cos^{-1}x=\frac{\pi}{4}$$

$\therefore \cos(\cos^{-1}x)<\cos\frac{\pi}{4}$

$$\Rightarrow x<\frac{1}{\sqrt{2}}$$

The common value are $-1\leq x<\frac{1}{\sqrt{2}}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	B	C	B	D	A	C	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	C	C	D	C	D	C	C	D