DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :1

Topic :-INVERSE TRIGONOMETRICE FUNCTIONS

1 **(a)**

We have,

$$\sin^{-1}\frac{3}{5}+\tan^{-1}\frac{1}{7}$$

$$=\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{1}{7}$$

$$=\tan^{-1}\left\{\frac{3/4+1/7}{1-3/4\times 1/7}\right\}=\tan^{-1}\left(\frac{25}{25}\right)=\tan^{-1}1=\frac{\pi}{4}$$

3 **(b)**

Given that, $x^2+y^2+z^2=r^2$

Now, $\tan^{-1}\left(\frac{xy}{zr}\right)+\tan^{-1}\left(\frac{yz}{xe}\right)+\tan^{-1}\left(\frac{xz}{yr}\right)$

$$=\tan^{-1}\left[\frac{\frac{xy}{zr}+\frac{yz}{xr}+\frac{xz}{yr}-\frac{xyz}{r^3}}{1-\left(\frac{x^2+y^2+z^2}{r^2}\right)}\right]$$

$$=\tan^{-1}\left[\frac{\frac{xy}{zr}+\frac{yz}{xr}+\frac{xz}{yr}-\frac{xyz}{r^3}}{1-\frac{r^2}{r^2}}\right]$$

$$=\tan^{-2}\infty=\frac{\pi}{2}$$

4 **(b)**

Put $x=\sin\theta$, we get

$$f(x)=\sin^{-1}\left\{\sin\left(\theta-\frac{\pi}{6}\right)\right\}$$

For, $-\frac{1}{2}\le x\le 1$

$$\Rightarrow\ -\frac{1}{2}\le\sin\theta\le 1$$

$$\Rightarrow -\frac{\pi}{6}\le\theta\le\frac{\pi}{2}$$

$\Rightarrow\ \theta = \frac{\pi}{6}$
is in the fourth on the first quadrant
$\therefore\quad f(x) = \theta - \frac{\pi}{6} = \sin^{-1} x - \frac{\pi}{6}$
5 **(c)**

$$\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$$
$$= \frac{\pi}{3} + 2 \cdot \frac{\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

6 **(c)**
Given, $\tan^{-1} 2\theta + \tan^{-1} 3\theta = \frac{\pi}{4}$
$\therefore\quad \tan^{-1}\left(\frac{2\theta + 3\theta}{1 - 2\theta \times 3\theta}\right) = \tan^{-1} 1$
$\Rightarrow\ 6\theta^2 + 5\theta - 1 = 0$
$\Rightarrow\ \theta = \frac{-5 \pm \sqrt{25 + 24}}{2 \times 6}$
$= \frac{-5 \pm 7}{12} = -1, \frac{1}{6}$
$\Rightarrow\ \theta = \frac{1}{6}$
7 **(b)**
Let $\theta = \cos^{-1}\left(-\frac{1}{2}\right)$
$\Rightarrow\ \cos\theta = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right)$
$= \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$
$\Rightarrow \theta = 2n\pi \pm \frac{2\pi}{3}$
$\Rightarrow\ \theta = \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{14\pi}{3}, \ldots$
8 **(c)**

$$\tan\theta + \tan\left(\frac{\pi}{3} + \theta\right) + \tan\left(-\frac{\pi}{3} + \theta\right) = a\tan 3\theta$$
$$\Rightarrow \tan\theta + \frac{\sqrt{3} + \tan\theta}{1 - \sqrt{3}\tan\theta} + \frac{\tan\theta - \sqrt{3}}{1 + \sqrt{3}\tan\theta} = a\tan 3\theta$$
$$\Rightarrow \tan\theta + \frac{8\tan\theta}{1 - 3\tan^2\theta} = a\tan 3\theta$$
$$\Rightarrow \frac{3(3\tan\theta - \tan^3\theta)}{1 - 3\tan^2\theta} = a\tan 3\theta$$
$$\Rightarrow 3\tan 3\theta = a\tan 3\theta$$
$$\Rightarrow a = 3$$

9 **(a)**

Let $\cot^{-1}\frac{3}{4}=\theta \Rightarrow \cot\theta=\frac{3}{4}$

And $\sin\theta=\frac{1}{\sqrt{1+\cot^2\theta}}=\frac{1}{\sqrt{1+\left(\frac{9}{16}\right)}}=\frac{4}{5}$

$$\therefore \cot^{-1}\frac{3}{4}+\sin^{-1}\frac{5}{13}=\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}$$

$$=\sin^{-1}\left[\frac{4}{5}\sqrt{1-\frac{25}{269}}+\frac{5}{13}\sqrt{1-\frac{16}{25}}\right]$$

$$=\sin^{-1}\left[\frac{4}{5}\cdot\frac{12}{13}+\frac{5}{13}\cdot\frac{3}{5}\right]$$

$$=\sin^{-1}\left[\frac{48+15}{65}\right]=\sin^{-1}\frac{63}{65}$$

10 **(d)**

Now, $\cos^{-1}(\cos 4)=\cos^{-1}\{\cos(2\pi-4)\}=2\pi-4$

$$\Rightarrow 2\pi-4>3x^2-4x$$

$$\Rightarrow 3x^2-4x-(2\pi-4)<0$$

$$\Rightarrow \frac{2-\sqrt{6\pi-8}}{3}<x<\frac{2+\sqrt{6\pi-8}}{3}$$

11 **(c)**

Let $\tan^{-1}x=\theta$. Then, $x=\tan\theta$

Also,

$x\in(-\infty,-1)$

$\Rightarrow -\infty<x<1 \Rightarrow -\infty<\tan\theta<-1 \Rightarrow -\frac{\pi}{2}<\theta<-\frac{\pi}{4}$

Now,

$\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\tan^{-1}(\tan 2\theta)=\tan^{-1}(\tan(\pi+2\theta))$

$=\pi+2\theta\ \left[\because -\frac{\pi}{2}<\theta<-\frac{\pi}{4}\Rightarrow 0<\pi+2\theta<\frac{\pi}{2}\right]$

$=\pi+2\tan^{-1}x$

12 **(b)**

Let $\sin^{-1}x=\theta$. Then, $x=\sin\theta$ and $\sqrt{1-x^2}=\cos\theta$

Also,

$\frac{1}{\sqrt{2}}\le x\le 1 \Rightarrow \frac{1}{\sqrt{2}}\le \sin\theta\le 1 \Rightarrow \frac{\pi}{4}\le\theta\le\frac{\pi}{2}$

$\therefore \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$=\sin^{-1}(\sin 2\theta)$

$=\sin^{-1}(\sin(\pi-2\theta))$

$=\pi-2\theta\ \left[\because \frac{\pi}{4}\le\theta\le\frac{\pi}{2}\Rightarrow 0\le\pi-2\theta\le\frac{\pi}{2}\right]$

$=\pi-2\sin^{-1}x$

14 **(a)**

Since, $-\frac{\pi}{2}<\sin^{-1}x\le\frac{\pi}{2}$

$$\therefore \sin^{-1} x_i = \frac{\pi}{2}, 1 \le i \le 20$$
$$\Rightarrow x_i = 1, 1 \le i \le 20$$

Thus, $\sum_{i=1}^{20} x_i = 20$

15 **(a)**

1 rad>45°

$$\Rightarrow \tan 1° > \tan 45° \Rightarrow \tan 1 > 1$$

Also, $\tan^{-1} 1 = \frac{\pi}{4} < 1$

Hence, $\tan 1 > \tan^{-1} 1$

16 **(c)**

$\alpha + \beta = \sin^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\frac{\sqrt{3}}{2} + \sin^{-1}\frac{1}{3} + \cos^{-1}\frac{1}{3}$

$= \frac{\pi}{2} + \frac{\pi}{2} = \pi$

Also, $\alpha = \frac{\pi}{3} + \sin^{-1}\frac{1}{3} < \frac{\pi}{3} + \sin^{-1}\frac{1}{2}$

As $\sin\theta$ is increasing in $\left[0, \frac{\pi}{2}\right]$

$\therefore \alpha < \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$

$\Rightarrow \beta > \frac{\pi}{2} > \alpha$

$\Rightarrow \alpha < \beta$

17 **(d)**

$$2\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$
$$= \tan^{-1}\left[\frac{2\left(\frac{1}{3}\right)}{1 - \frac{1}{9}}\right] + \tan^{-1}\left(\frac{1}{7}\right)$$
$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$
$$= \tan^{-1}\left(\frac{\frac{3}{4} + \frac{1}{7}}{1 - \frac{3}{4} \times \frac{1}{7}}\right)$$

$=\tan^{-1}\left(\frac{25}{25}\right) = \frac{\pi}{4}$

18 **(c)**

$$\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$$
$$= \tan\left[\frac{1}{2} \cdot 2\tan^{-1} a + \frac{1}{2} \cdot 2\tan^{-1} a\right]$$
$$= \tan(2\tan^{-1} a)$$
$$= \tan\left[\tan^{-1}\left(\frac{2a}{1-a^2}\right)\right]$$

$$= \frac{2a}{1-a^2}$$

19 **(c)**

Let $S_\infty = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + ...$

$$\therefore T_n \cot^{-1} 2n^2$$

$$= \tan^{-1} \frac{1}{2n^2}$$

$$= \tan^{-1}\left(\frac{2}{4n^2}\right) = \tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right)$$

$$\therefore S_n = \sum_{n=1}^{\infty} \{ \tan^{-1}(2n+1) - \tan^{-1}(2n-1)\}$$

$$= \tan^{-1} \infty - \tan^{-1} 1$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

20 **(a)**

Given, $\tan^{-1}\left(\frac{a}{x}\right) + \tan^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{a}{x}+\frac{b}{x}}{1-\frac{ab}{x^2}}\right) = \frac{\pi}{2}$$

$$\Rightarrow \frac{(a+b)x}{x^2-ab}$$

$$= \tan\frac{\pi}{2}$$

$$\Rightarrow \frac{(a+b)x}{x^2-ab} = \frac{1}{0}$$

$$\Rightarrow x^2 - ab = 0$$

$$\Rightarrow x = \sqrt{ab}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	B	B	C	C	B	C	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	C	A	A	C	D	C	C	A