DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :9

Topic :-INTEGRALS

1 **(a)**

We have,

$$I=\int_{1/e}^{e} |\log x|\, dx$$

$$\Rightarrow I=\int_{1/e}^{e} -\log x\, dx+\int_{1}^{e} \log x\, dx$$

$$\Rightarrow I=[x-x\log x]_{1/e}^{1}+[x\log x-x]_{1}^{e}=2\left(\frac{e-1}{e}\right)$$

2 **(d)**

Let $I=\int_0^{\pi/8}\cos^3 4\theta d\theta$

$$=\int_0^{\pi/8}\left(\frac{3\cos 4\theta+\cos 12\theta}{4}\right)d\theta$$

$$=\frac{1}{4}\left[\frac{3\sin 4\theta}{4}+\frac{\sin 12\theta}{12}\right]_0^{\pi/8}$$

$$=\frac{1}{4}\left[\frac{3}{4}\left(\sin\frac{\pi}{2}-0\right)+\frac{1}{12}\left(\sin\frac{3\pi}{2}-0\right)\right]$$

$$=\frac{1}{4}\left[\frac{3}{4}+\frac{1}{12}(-1)\right]=\frac{1}{4}\left(\frac{8}{12}\right)=\frac{1}{6}$$

3 **(c)**

$$I=\int_{-1}^{1}\frac{\cos h\, x}{1+e^{2x}}dx=\int_{-1}^{1}\frac{e^x+e^{-x}}{2(1+e^{2x})}dx\left[\because \cosh x=\frac{e^x+e^{-x}}{2}\right]$$

$$=\frac{1}{2}\int_{-1}^{1}\frac{1+e^{2x}}{(1+e^{2x})e^x}dx=\frac{1}{2}\int_{-1}^{1}e^{-x}dx$$

$$=-\frac{1}{2}[e^{-x}]_{-1}^{1}=-\frac{1}{2}(e^{-1}-e^{1})=\frac{e^2-1}{2e}$$

4 **(c)**

We have,

$$u_{10}=\int_0^{\pi/2} x^{10}\sin x\ dx$$

$$\Rightarrow u_{10}=\left[-x^{10}\cos x\right]_0^{\pi/2}+10\int_0^{\pi/2} x^9\cos x\,dx$$

$$\Rightarrow u_{10}=10\left[x^9\sin x\right]_0^{\pi/2}-90\int_0^{\frac{\pi}{2}} x^8\sin x\,dx=10\left(\frac{\pi}{2}\right)^9-90\,u_8$$

$$\Rightarrow u_{10}+90u_8=10\left(\frac{\pi}{2}\right)^9$$

5 **(c)**

Let $I=\int \frac{x^3\sin\left[\tan^{-1}(x^4)\right]dx}{1+x^8}$

Put $x^4=t \Rightarrow 4x^3\,dx=dt$

$$\therefore\ \ I=\int \frac{1}{4}.\ \frac{\sin[\tan^{-1}(t)]dt}{1+t^2}$$

Again, put $\tan^{-1}t=u\ \Rightarrow \frac{1}{1+t^2}dt=du$

$$\therefore\ I=\int \frac{1}{4}\sin u\,du=-\frac{1}{4}\cos u+c=-\frac{1}{4}\cos\left[\tan^{-1}(x^4)\right]+c$$

6 **(d)**

We have,

$$\sum_{n=1}^{10}\int_{-2n-1}^{-2n}\sin^{27}x\,dx+\sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}x\,dx$$

$$=-\sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}t\,dt+\sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}x\,dx\text{, where } x=-t$$

$$=-\sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}x\ dx+\sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}x\,dx=0$$

7 **(d)**

$$\int \frac{1}{\sin^2 x.\cos^2 x}\ dx=\int \frac{\sin^2 x+\cos^2 x}{\sin^2 x\cos^2 x}\ dx$$

$$=\int\left(\frac{1}{\cos^2 x}+\frac{1}{\sin^2 x}\right)dx$$

$$=\int(\sec^2 x+\mathrm{cosec}^2\,x)\,dx$$

$$=\tan x-\cot x+c$$

8 **(a)**

We have,

$$I=\int_{-1}^{1}(x-[2x])\,dx=\int_{-1}^{1}x\,dx-\int_{-1}^{1}[2x]\,dx$$

$$\Rightarrow I=0-\left[\int_{-1}^{-1/2}[2x]\,dx+\int_{-1/2}^{0}[2x]dx+\int_{0}^{1/2}[2x]dx+\int_{1/2}^{1}[2x]\,dx\right]$$

$$\Rightarrow I=-\left[\int_{-1}^{-1/2}-2dx+\int_{-1/2}^{0}-1dx+\int_{0}^{1/2}0dx+\int_{1/2}^{1}1dx\right]$$

$$\Rightarrow I=-\left[-2\left(-\frac{1}{2}+1\right)-1\left(0+\frac{1}{2}\right)+0+\left(1-\frac{1}{2}\right)\right]$$

$$\Rightarrow I=-\left[-1-\frac{1}{2}+\frac{1}{2}\right]=1$$

9 **(a)**

We know that if $f(t)$ is an odd function. Then, $\int_0^x f(t)dt$ is an even function.

Since the function $f(x)=\log\frac{1-x}{1+x}$ is an odd function. Therefore, $F(x)$ is an even function

11 **(c)**

We have

$$\frac{d}{dx}\left\{\int_{f(x)}^{g(x)}\phi(t)dt\right\}=g'(x).\phi(g(x))-f'(x).\phi(f(x))$$

12 **(d)**

$$\int_1^3(x-1)(x-2)(x-3)dx$$

$$=\int_1^3(x-1)(x^2-5x+6)dx$$

$$=\int_1^3(x^3-6x^2+11x-6)dx$$

$$=\left[\frac{x^4}{4}-\frac{6x^3}{3}+\frac{11x^2}{2}-6x\right]_1^3$$

$$=\left[\frac{81}{4}-\frac{162}{3}+\frac{99}{2}-18-\left(\frac{1}{4}-\frac{6}{3}+\frac{11}{2}-6\right)\right]$$

$$=\left[-\frac{27}{12}+\frac{27}{12}\right]=0$$

13 **(c)**

Given, $f(x)=\begin{vmatrix}\sin x+\sin 2x+\sin 3x & \sin 2x & \sin 3x\\ 3+4\sin x & 3 & 4\sin x\\ 1+\sin x & \sin x & 1\end{vmatrix}$

Applying $C_1\to C_1-(C_2+C_3)$, we get

$$=\begin{vmatrix}\sin x & \sin 2x & \sin 3x\\ 0 & 3 & 4\sin x\\ 0 & \sin x & 1\end{vmatrix}$$

$= \sin x(3-4\sin^2 x) = 3\sin x - 4\sin^3 x$

$f(x) = \sin 3x$

$\therefore \int_0^{\pi/2} f(x)dx = \int_0^{\pi/2} \sin 3x\, dx$

$= \left[-\frac{\cos 3x}{3}\right]_0^{\pi/2} = \left(-\frac{1}{3}\right)[\cos 3x]_0^{\pi/2}$

$= \left(-\frac{1}{3}\right)\left[\cos\left(3\times\frac{\pi}{2}\right) - \cos 0\right]$

$= \left(-\frac{1}{3}\right)(0-1) = \frac{-1}{3}(-1) = \frac{1}{3}$

14 **(c)**

We have,

$I = \int g(x)\{f(x)+f'(x)\}\, dx$

$\Rightarrow I = \int g(x)f(x)dx + \int g(x)f'(x)\, dx$

$\Rightarrow I = f(x)\left(\int g(x)dx\right) - \int \left(f'(x)\int g(x)dx\right) dx + \int g(x)f'(x)\, dx$

$\Rightarrow I = f(x)g(x) - \int g(x)g'(x)dx + \int g(x)g'(x)dx + C$

$\Rightarrow I = f(x)g(x) + C \quad \left[\because \int g(x)dx = g(x)\right]$

15 **(a)**

$\int \left(\frac{x+2}{x+4}\right)^2 e^x\, dx = \int e^x\left[\frac{x^2+4x+4}{(x+4)^2}\right]dx$

$= \int e^x\left[\frac{x(x+4)}{(x+4)^2} + \frac{4}{(x+4)^2}\right]dx$

$= \int e^x\left[\frac{x}{x+4} + \frac{4}{(x+4)^2}\right]dx$

$= \int \frac{e^x\, x}{(x+4)}\, dx + \int \frac{4e^x}{(x+4)^2}\, dx$

$= e^x\left(\frac{x}{x+4}\right) - \int \frac{4e^x}{(x+4)^2}\, dx + \int \frac{4e^x}{(x+4)^2}\, dx$

$= \frac{xe^x}{(x+4)} + c$

16 **(c)**

Given, $P = \int_0^{3\pi} f(\cos^2 x)dx$...(i)

and$Q = \int_0^{\pi} f(\cos^2 x)\, dx$ ...(ii)

From Eq. (i)

$P = 3\int_0^{\pi} f(\cos^2 x)dx$

$\Rightarrow P = 3Q \Rightarrow P - 3Q = 0$

17 **(d)**

Let $g(x) = f(\cos^2 x)$. Then,

$g(x + n\pi) = f\{\cos^2(x + n\pi)\} = f(\cos^2 x) = g(x)$

$$\therefore \int_0^{n\pi} g(x)\,dx = n\int_0^{\pi} g(x)\,dx$$

$\Rightarrow \int_0^{3\pi} g(x)dx = 3\int_0^{\pi} g(x)dx$ [Putting $n = 3$]

$$\Rightarrow \int_0^{3\pi} f(\cos^2 x)\,dx = 3\int_0^{\pi} f(\cos^2 x)\,dx$$

$\Rightarrow I_1 = 3I_2$

18 **(c)**

We know that $x - [x]$ is a periodic function with period 1 unit. Therefore,

$$\int_0^{n[x]} (x - [x])dx = n[x]\int_0^1 (x - [x])\,dx = n[x]\int_0^1 xdx = \frac{n}{2}[x]$$

19 **(d)**

Put $t = x^2 + 1 \Rightarrow dt = 2x\,dx$

$$\int_0^2 \frac{x^3}{(x^2+1)^{3/2}}dx = \frac{1}{2}\int_1^5 \frac{(t-1)}{t^{3/2}}dt$$

$$= \frac{1}{2}\int_1^5 [t^{-\frac{1}{2}} - t^{-3/2}]dt$$

$$= \frac{1}{2}\left[2\sqrt{t} + 2\frac{1}{\sqrt{t}}\right]_1^5$$

$$= \frac{1}{2}\left[2\sqrt{5} + \frac{2}{\sqrt{5}} - 2 - 2\right]$$

$$= \left[\sqrt{5} + \frac{1}{\sqrt{5}} - 2\right] = \frac{6 - 2\sqrt{5}}{\sqrt{5}}$$

20 **(d)**

We have,

$$I = \int_0^{2\pi} |\cos x - \sin x|\,dx$$

$$\Rightarrow I = \sqrt{2}\int_0^{2\pi} \left|\cos\left(x + \frac{\pi}{4}\right)\right| dx$$

$$\Rightarrow I = \sqrt{2}\int_{\pi/4}^{9\pi/4} |\cos t|\,dt, \text{ where } t = x + \frac{\pi}{4}$$

$$\Rightarrow I = \sqrt{2}\left\{\int_{\pi/4}^{\pi/2} \cos t\,dt + \int_{\pi/2}^{3\pi/2} (-\cos t)dt + \int_{3\pi/2}^{9\pi/4} \cos t\,dt\right\}$$

$$\Rightarrow I = \sqrt{2}\left[\left(1-\frac{1}{\sqrt{2}}\right) - (-1-1) + \left(\frac{1}{\sqrt{2}}+1\right)\right] = 4\sqrt{2}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	D	C	C	C	D	D	A	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	D	C	C	A	C	D	C	D	D