DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :8

Topic :-INTEGRALS

1 **(c)**

Since, $e^{x-[x]}$is a periodic function with period 1.

$$\therefore \int_0^{1000} e^{x-[x]}dx = 1000\int_0^1 e^x dx = 1000[e^x]_0^1$$

$= 1000(e-1)$

2 **(b)**

Put $\tan^{-1}x = t \Rightarrow \frac{1}{(1+x^2)}dx = dt$

$$\therefore \int \frac{(\tan^{-1}x)^3}{(1+x^2)}dx = \int t^3\,dt = \frac{t^4}{4} + c = \frac{(\tan^{-1}x)^4}{4} + c$$

3 **(c)**

Let $I = \int_0^{\pi} \sin^3\theta d\theta$

$\left[\begin{matrix} \because \sin\theta > 0 \\ \text{for } 0 < \theta < \pi \end{matrix}\right]$

$$= \int_0^{\pi} \sin\theta(1-\cos^2\theta)d\theta$$

Put$\cos\theta = t \Rightarrow -\sin\theta d\theta = dt$

$$\therefore I = \int_{-1}^{1}(1-t^2)dt = \left[t - \frac{t^3}{3}\right]_{-1}^{1}$$

$$= 2 - \frac{2}{3}$$

$$= \frac{4}{3}$$

4 **(a)**

We have, $d[f(x)] = e^{\tan x}\sec^2 x\,dx$

On integrating w.r.tx, we get

$f(x) = e^{\tan x} + c$

5 **(c)**

We have,

$$I = \int_1^2 \{f(g(x))\}^{-1} f'(g(x))g'(x)dx$$

$$\Rightarrow I = \int_1^2 \frac{1}{fog(x)} f'(g(x))g'(x)dx$$

$$\Rightarrow I = \int_1^2 \frac{1}{fog(x)} d(fog(x)) = [\log(fog\ (x))]_1^2$$

$$\Rightarrow I = \log\{f(g(2))\} - \log\{f(g(1))\} = 0\ \ [\because g(1) = g(2)]$$

6 **(d)**

Given, $f'(x) \geq 2$

On integrating both sides w.r.t.x, we get

$f(x) \geq 2x + c$

$f(1) \geq 2 \times 1 + c$

$\Rightarrow c \leq 4 - 2 \Rightarrow c \leq 2$

$\Rightarrow f(x) \geq 2x + 2$

$\therefore\ f(4) \geq 2 \times 4 + 2 = 10$

7 **(c)**

Let $I = \int \frac{f'(x)}{f(x)\log f(x)} dx$

Put $\log f(x) = t \Rightarrow \frac{f'(x)}{f(x)} dx = dt$

$$\therefore\ I = \int \frac{1}{t}\, dt = \log t + c$$

$$= \log\log f(x) + c$$

8 **(a)**

Let $I = \frac{2}{3}\int_0^{\pi/2} \frac{\sqrt{\cos\theta}}{(\sqrt{\sin\theta} + \sqrt{\cos\theta})} d\theta$...(i)

And $I = \frac{2}{3}\int_0^{\pi/2} \frac{\sqrt{\cos(\pi/2 - \theta)}}{\sqrt{\sin\,(\pi/2 - \theta)} + \sqrt{\cos(\pi/2 - \theta)}} d\theta$

$\Rightarrow I = \frac{2}{3}\int_0^{\pi/2} \frac{\sqrt{\sin\theta}}{\sqrt{\sin\theta} + \sqrt{\cos\theta}} d\theta$...(ii)

On addingEqs.(i) and (ii), we get

$$2I = \frac{2}{3}\int_0^{\pi/2} 1 d\theta = \frac{2}{3}[\theta]_0^{\pi/2} \Rightarrow I = \frac{1}{3}\cdot\frac{\pi}{2} = \frac{\pi}{6}$$

9 **(a)**

$$\lim_{n\to\infty}\left[\frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \dots + \frac{n}{n^2 + n^2}\right]$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{n}{n^2 + r^2}$$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{1}{1 + \left(\frac{r}{n}\right)^2}$$

$$= \int_0^1 \frac{dx}{1 + x^2} = [\tan^{-1} x]_0^1 = \frac{\pi}{4}$$

10 **(d)**

Integrand is an odd function. So, the value of the integral is zero

11 **(a)**

Let $I = \int \frac{\sin x}{\cos x(1 + \cos x)} dx$

Put $\cos x = t \Rightarrow -\sin x dx = dt$

$$\therefore\ I = \int \frac{-dt}{t(1+t)} = -\int \left[\frac{1}{t} - \frac{1}{(1+t)}\right] dt$$

$= \log\left(\frac{t+1}{t}\right) + c = f(x) + c$ [given]

$$\therefore\ f(x) = \log\left(\frac{t+1}{t}\right) = \log\left(\frac{1+\cos x}{\cos x}\right)$$

12 **(a)**

Since $f(x)$ satisfies conditions of Rolle's theorem on $[1, 2]$. Therefore, $f(1) = f(2)$

Hence, $\int_1^2 f'(x)dx = f(2) - f(1) = 0$

13 **(a)**

$$\int e^{2x}(2\sin 3x + 3\cos 3x)dx$$

$$= 2\int e^{2x}\sin 3x + dx + 3\int e^{2x}\cos 3x\,dx$$

$$= e^{2x}\sin 3x - 3\int e^{2x}\cos 3x\,dx + 3\int e^{2x}\cos 3x\,dx$$

$= e^{2x}\sin 3x + c$

14 **(d)**

$$\int_{-2}^{2} |[x]|dx$$

$$= \int_{-2}^{-1} |[x]|dx + \int_{-1}^{0} |[x]|dx + \int_{0}^{1} |[x]|dx + \int_{1}^{2} |[x]|dx$$

$$= \int_{-2}^{-1} 2\,dx + \int_{-1}^{0} 1\,dx + \int_{0}^{1} 0\,dx + \int_{1}^{2} 1\,dx$$

$= 2[x]_{-2}^{-1} + [x]_{-1}^{0} + 0 + [x]_1^2$

$= 2(-1+2) + (0+1) + (2-1) = 2 + 1 + 1 = 4$

15 **(c)**

We have,

$$\int_0^{\pi/2} \cos^n x \sin^n x\ dx = \lambda \int_0^{\pi/2} \sin^n x\ dx$$

$$\Rightarrow \int_0^{\pi/2} (\sin 2x)^n\,dx = 2^n \lambda \int_0^{\pi/2} \sin^n x\ dx$$

$$\Rightarrow \int_0^{\pi} (\sin t)^n dt = 2^{n+1}\lambda \int_0^{\pi/2} \sin^n x\ dx$$

$$\Rightarrow 2\int_0^{\pi/2} \sin^n t\,dt = 2^{n+1}\lambda \int_0^{\pi/2} \sin^n x\ dx$$

$$\left[\because \int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx \text{ if } f(2a-x) = f(x)\right]$$

$$\Rightarrow 2 = 2^{n+1}\lambda \Rightarrow \lambda = \frac{1}{2^n}$$

16 **(a)**

Clearly, $\frac{t^2 \sin 2t}{t^2+1}$ is an odd function

$$\therefore \int_{-3}^{3} \frac{t^2 \sin 2t}{t^2+1}\,dt = 0$$

Also,

$$\int_0^1 \frac{1}{t^2 + 2t\cos\alpha + 1}\,dt = \frac{1}{\sin\alpha}\left[\tan^{-1}\left(\frac{t+\cos\alpha}{\sin\alpha}\right)\right]_0^1 = \frac{\alpha}{2\sin\alpha}$$

Thus the given equation reduces to

$$x^2 \frac{\alpha}{2\sin\alpha} - 2 = 0 \Rightarrow x = \pm 2\sqrt{\frac{\sin\alpha}{\alpha}}$$

17 **(a)**

Let $I = \int_{\pi/3}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

$$\int_{\pi/6}^{\pi/3} \left(\frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\right) dx \ \ldots\text{(i)}$$

$$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} \ldots\text{(ii)}$$

$$\left(\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right)$$

On addingEqs. (i) and (ii), we get

$$2I = \int_{\pi/6}^{\pi/3} 1\,dx = [x]_{\pi/6}^{\pi/3} = \frac{\pi}{6}$$

$$\Rightarrow I = \frac{\pi}{12}$$

18 **(d)**

$$\int_0^2 [x^2]dx$$

$$= \int_0^1 [x^2]dx + \int_1^{\sqrt{2}} [x^2]dx = \int_{\sqrt{2}}^{\sqrt{3}} [x^2]dx + \int_{\sqrt{3}}^{2} [x^2]dx$$

$= \int_0^1 0\,dx + \int_1^{\sqrt{2}} 1dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dx + \int_{\sqrt{3}}^{2} 3dx$

$= [x]_1^{\sqrt{2}} + [2x]_{\sqrt{2}}^{\sqrt{3}} + [3x]_{\sqrt{3}}^{2}$

$= 5 - \sqrt{3} - \sqrt{2}$

19 **(a)**

Given, $\int f(x)\sin x\cos x dx = \frac{1}{2(b^2-a^2)}\log\{f(x)\} + c$

On differentiating both sides, we get

$f(x)\sin x\cos x = \frac{1}{2(b^2-a^2)}\cdot\frac{1}{f(x)}f'(x)$

$\Rightarrow\ 2(b^2-a^2)\sin x\cos x = \frac{f'(x)}{[f(x)]^2}$

$\Rightarrow\ \int(2b^2\sin x\cos x - 2a^2\sin x\cos x)dx = \int\frac{f'(x)}{[f(x)]^2}dx$

$I_1 - I_2 = \int\frac{f'(x)}{[f(x)]^2}$...(i)

Where $I_1 = \int 2b^2\sin x\cos x\,dx$ and $I_2 = \int 2a^2\sin x\cos x dx$

Now, $I_1 = 2b^2\sin x\cos x dx$

Put $\cos x = t \Rightarrow -\sin x dx = dt$

$\therefore\ I_1 = -\int 2b^2 t dt = -b^2t^2 = -b^2\cos^2 x$

and $I_2 = \int 2a^2\sin x\cos x dx$

Put $\sin x = t \Rightarrow \cos x dx = t$

$\therefore\ I_2 = \int 2a^2\,t dt = a^2t^2 = a^2\sin^2 x$

$\therefore$ From Eq. (i),

$-b^2\cos^2 x - a^2\sin^2 x = -\frac{1}{f(x)}$

$\Rightarrow\ f(x) = \frac{1}{a^2\sin^2 x + b^2\cos^2 x}$

20 **(c)**

Let $I = \int_0^a xf(x)dx$...(i)

$\Rightarrow I = \int_0^a (a-x)f(a-x)dx$

$I = \int_0^a (a-x)f(x)dx$

$[\because f(x) = f(a-x)$given$]$

$\Rightarrow I = a\int_0^a f(x)dx - \int_0^a xf(x)dx$ ...(ii)

On addingEqs.(i) and (ii), we get

$$2I = a\int_0^a f(x)dx$$

$$\Rightarrow I = \frac{a}{2}\int_0^a f(x)dx$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	C	A	C	D	C	A	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	A	D	C	A	A	D	A	C