

CLASS : XII<sup>th</sup>  
DATE :

**SOLUTIONS**

SUBJECT : MATHS  
DPP NO. :8

**Topic :-INTEGRALS**

1      **(c)**

Since,  $e^{x-[x]}$  is a periodic function with period 1.

$$\therefore \int_0^{1000} e^{x-[x]} dx = 1000 \int_0^1 e^x dx = 1000 [e^x]_0^1 \\ = 1000(e - 1)$$

2      **(b)**

Put  $\tan^{-1} x = t \Rightarrow \frac{1}{(1+x^2)} dx = dt$

$$\therefore \int \frac{(\tan^{-1} x)^3}{(1+x^2)} dx = \int t^3 dt = \frac{t^4}{4} + c = \frac{(\tan^{-1} x)^4}{4} + c$$

3      **(c)**

Let  $I = \int_0^\pi \sin^3 \theta d\theta$

$[\because \sin \theta > 0]$   
[for  $0 < \theta < \pi$ ]

$$= \int_0^\pi \sin \theta (1 - \cos^2 \theta) d\theta$$

Put  $\cos \theta = t \Rightarrow -\sin \theta d\theta = dt$

$$\therefore I = \int_{-1}^1 (1-t^2) dt = \left[ t - \frac{t^3}{3} \right]_{-1}^1$$

$$= 2 - \frac{2}{3}$$

$$= \frac{4}{3}$$

4      **(a)**

We have,  $d[f(x)] = e^{\tan x} \sec^2 x dx$

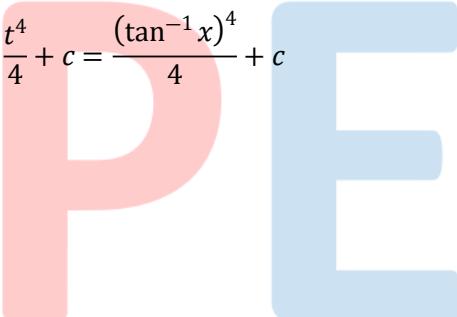
On integrating w.r.t x, we get

$$f(x) = e^{\tan x} + c$$

5      **(c)**

We have,

$$I = \int_1^2 \{f(g(x))\}^{-1} f'(g(x)) g'(x) dx$$



$$\Rightarrow I = \int_1^2 \frac{1}{fog(x)} f'(g(x))g'(x)dx$$

$$\Rightarrow I = \int_1^2 \frac{1}{fog(x)} d(fog(x)) = [\log(fog(x))]_1^2$$

$$\Rightarrow I = \log\{f(g(2))\} - \log\{f(g(1))\} = 0 \quad [\because g(1) = g(2)]$$

6      **(d)**

Given,  $f'(x) \geq 2$

On integrating both sides w.r.t.x, we get

$$f(x) \geq 2x + c$$

$$f(1) \geq 2 \times 1 + c$$

$$\Rightarrow c \leq 4 - 2 \Rightarrow c \leq 2$$

$$\Rightarrow f(x) \geq 2x + 2$$

$$\therefore f(4) \geq 2 \times 4 + 2 = 10$$

7      **(c)**

$$\text{Let } I = \int \frac{f'(x)}{f(x) \log f(x)} dx$$

$$\text{Put } \log f(x) = t \Rightarrow \frac{f'(x)}{f(x)} dx = dt$$

$$\therefore I = \int \frac{1}{t} dt = \log t + c$$

$$= \log \log f(x) + c$$

8      **(a)**

$$\text{Let } I = \frac{2}{3} \int_0^{\pi/2} \frac{\sqrt{\cos \theta}}{(\sqrt{\sin \theta} + \sqrt{\cos \theta})} d\theta \dots (\text{i})$$

$$\text{And } I = \frac{2}{3} \int_0^{\pi/2} \frac{\sqrt{\cos(\pi/2 - \theta)}}{\sqrt{\sin(\pi/2 - \theta)} + \sqrt{\cos(\pi/2 - \theta)}} d\theta$$

$$\Rightarrow I = \frac{2}{3} \int_0^{\pi/2} \frac{\sqrt{\sin \theta}}{\sqrt{\sin \theta} + \sqrt{\cos \theta}} d\theta \dots (\text{ii})$$

On adding Eqs.(i) and (ii), we get

$$2I = \frac{2}{3} \int_0^{\pi/2} 1 d\theta = \frac{2}{3} [\theta]_0^{\pi/2} \Rightarrow I = \frac{1}{3} \cdot \frac{\pi}{2} = \frac{\pi}{6}$$

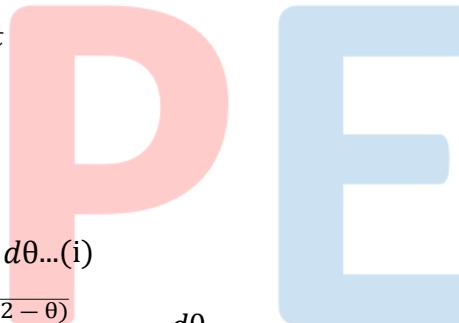
9      **(a)**

$$\lim_{n \rightarrow \infty} \left[ \frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \dots + \frac{n}{n^2 + n^2} \right]$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{n}{n^2 + r^2}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \frac{1}{1 + \left(\frac{r}{n}\right)^2}$$

$$= \int_0^1 \frac{dx}{1 + x^2} = [\tan^{-1} x]_0^1 = \frac{\pi}{4}$$



10 (d)

Integrand is an odd function. So, the value of the integral is zero

11 (a)

$$\text{Let } I = \int \frac{\sin x}{\cos x(1 + \cos x)} dx$$

Put  $\cos x = t \Rightarrow -\sin x dx = dt$

$$\therefore I = \int \frac{-dt}{t(1+t)} = - \int \left[ \frac{1}{t} - \frac{1}{(1+t)} \right] dt$$

$$= \log\left(\frac{t+1}{t}\right) + c = f(x) + c \quad [\text{given}]$$

$$\therefore f(x) = \log\left(\frac{t+1}{t}\right) = \log\left(\frac{1+\cos x}{\cos x}\right)$$

12 (a)

Since  $f(x)$  satisfies conditions of Rolle's theorem on  $[1, 2]$ . Therefore,  $f(1) = f(2)$

Hence,  $\int_1^2 f'(x) dx = f(2) - f(1) = 0$

13 (a)

$$\int e^{2x}(2 \sin 3x + 3 \cos 3x) dx$$

$$= 2 \int e^{2x} \sin 3x dx + 3 \int e^{2x} \cos 3x dx$$

$$= e^{2x} \sin 3x - 3 \int e^{2x} \cos 3x dx + 3 \int e^{2x} \cos 3x dx$$

$$= e^{2x} \sin 3x + c$$

14 (d)

$$\int_{-2}^2 |[x]| dx$$

$$= \int_{-2}^{-1} |[x]| dx + \int_{-1}^0 |[x]| dx + \int_0^1 |[x]| dx + \int_1^2 |[x]| dx$$

$$= \int_{-2}^{-1} 2 dx + \int_{-1}^0 1 dx + \int_0^1 0 dx + \int_1^2 1 dx$$

$$= 2[x]_{-2}^{-1} + [x]_{-1}^0 + 0 + [x]_1^2$$

$$= 2(-1 + 2) + (0 + 1) + (2 - 1) = 2 + 1 + 1 = 4$$

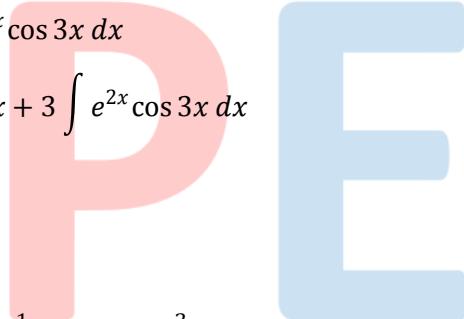
15 (c)

We have,

$$\int_0^{\pi/2} \cos^n x \sin^n x dx = \lambda \int_0^{\pi/2} \sin^n x dx$$

$$\Rightarrow \int_0^{\pi/2} (\sin 2x)^n dx = 2^n \lambda \int_0^{\pi/2} \sin^n x dx$$

$$\Rightarrow \int_0^{\pi} (\sin t)^n dt = 2^{n+1} \lambda \int_0^{\pi/2} \sin^n x dx$$



$$\Rightarrow 2 \int_0^{\pi/2} \sin^n t dt = 2^{n+1} \lambda \int_0^{\pi/2} \sin^n x dx$$

$$\left[ \because \int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx \text{ if } f(2a-x) = f(x) \right]$$

$$\Rightarrow 2 = 2^{n+1} \lambda \Rightarrow \lambda = \frac{1}{2^n}$$

16      **(a)**

Clearly,  $\frac{t^2 \sin 2t}{t^2 + 1}$  is an odd function

$$\therefore \int_{-3}^3 \frac{t^2 \sin 2t}{t^2 + 1} dt = 0$$

Also,

$$\int_0^1 \frac{1}{t^2 + 2t \cos \alpha + 1} dt = \frac{1}{\sin \alpha} \left[ \tan^{-1} \left( \frac{t + \cos \alpha}{\sin \alpha} \right) \right]_0^1 = \frac{\alpha}{2 \sin \alpha}$$

Thus the given equation reduces to

$$x^2 \frac{\alpha}{2 \sin \alpha} - 2 = 0 \Rightarrow x = \pm 2 \sqrt{\frac{\sin \alpha}{\alpha}}$$

17      **(a)**

$$\text{Let } I = \int_{\pi/3}^{\pi/2} \frac{dx}{1 + \sqrt{\tan x}}$$

$$\int_{\pi/6}^{\pi/3} \left( \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} \right) dx \dots (\text{i})$$

$$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} \dots (\text{ii})$$

$$\left( \because \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_{\pi/6}^{\pi/3} 1 dx = [x]_{\pi/6}^{\pi/3} = \frac{\pi}{6}$$

$$\Rightarrow I = \frac{\pi}{12}$$

18      **(d)**

$$\int_0^2 [x^2] dx$$

$$= \int_0^1 [x^2] dx + \int_1^{\sqrt{2}} [x^2] dx = \int_{\sqrt{2}}^{\sqrt{3}} [x^2] dx + \int_{\sqrt{3}}^2 [x^2] dx$$

$$\begin{aligned}
&= \int_0^1 0 \, dx + \int_1^{\sqrt{2}} 1 \, dx + \int_{\sqrt{2}}^{\sqrt{3}} 2 \, dx + \int_{\sqrt{3}}^2 3 \, dx \\
&= [x]_1^{\sqrt{2}} + [2x]_{\sqrt{2}}^{\sqrt{3}} + [3x]_{\sqrt{3}}^2 \\
&= 5 - \sqrt{3} - \sqrt{2}
\end{aligned}$$

19 (a)

Given,  $\int f(x) \sin x \cos x \, dx = \frac{1}{2(b^2 - a^2)} \log\{f(x)\} + c$

On differentiating both sides, we get

$$\begin{aligned}
f(x) \sin x \cos x &= \frac{1}{2(b^2 - a^2)} \cdot \frac{1}{f(x)} f'(x) \\
\Rightarrow 2(b^2 - a^2) \sin x \cos x &= \frac{f'(x)}{[f(x)]^2} \\
\Rightarrow \int (2b^2 \sin x \cos x - 2a^2 \sin x \cos x) \, dx &= \int \frac{f'(x)}{[f(x)]^2} \, dx
\end{aligned}$$

$$I_1 - I_2 = \int \frac{f'(x)}{[f(x)]^2} \dots (i)$$

Where  $I_1 = \int 2b^2 \sin x \cos x \, dx$  and  $I_2 = \int 2a^2 \sin x \cos x \, dx$

Now,  $I_1 = 2b^2 \sin x \cos x \, dx$

Put  $\cos x = t \Rightarrow -\sin x \, dx = dt$

$$\therefore I_1 = - \int 2b^2 t \, dt = -b^2 t^2 = -b^2 \cos^2 x$$

and  $I_2 = \int 2a^2 \sin x \cos x \, dx$

Put  $\sin x = t \Rightarrow \cos x \, dx = dt$

$$\therefore I_2 = \int 2a^2 t \, dt = a^2 t^2 = a^2 \sin^2 x$$

$\therefore$  From Eq. (i),

$$-b^2 \cos^2 x - a^2 \sin^2 x = -\frac{1}{f(x)}$$

$$\Rightarrow f(x) = \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x}$$

20 (c)

Let  $I = \int_0^a x f(x) \, dx \dots (i)$

$$\Rightarrow I = \int_0^a (a-x) f(a-x) \, dx$$

$$I = \int_0^a (a-x) f(x) \, dx$$

[ $\because f(x) = f(a-x)$  given]

$$\Rightarrow I = a \int_0^a f(x) \, dx - \int_0^a x f(x) \, dx \dots (ii)$$

On adding Eqs.(i) and (ii), we get

$$2I = a \int_0^a f(x)dx$$

$$\Rightarrow I = \frac{a}{2} \int_0^a f(x)dx$$

**ANSWER-KEY**

<b>Q.</b>	1	2	3	4	5	6	7	8	9	10
<b>A.</b>	C	B	C	A	C	D	C	A	A	D
<b>Q.</b>	11	12	13	14	15	16	17	18	19	20
<b>A.</b>	A	A	A	D	C	A	A	D	A	C

P E