DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :7

Topic :-INTEGRALS

1 **(a)**

Let $I=\int \frac{\log(x+1)-\log x}{x(x+1)}dx=\int \frac{\log\left(1+\frac{1}{x}\right)}{x(x+1)}dx$

$$=\int \frac{\log\left(1+\frac{1}{x}\right)}{x^2\left(1+\frac{1}{x}\right)}dx$$

Put $1+\frac{1}{x}=t \Rightarrow -\frac{1}{x^2}dx=dt$

$$\therefore I=-\int \frac{\log t}{t}dt=-\frac{1}{2}(\log t)^2+c$$

$$=-\frac{1}{2}\left[\log\left(1+\frac{1}{x}\right)\right]^2+c$$

$$=-\frac{1}{2}[\log(x+1)-\log x]^2+c$$

$$=-\frac{1}{2}\{\log(x+1)\}^2-\frac{1}{2}(\log x)^2+\log(x+1).\log x+c$$

2 **(c)**

Let $I=\int_0^1 x\left|x-\frac{1}{2}\right|dx$

$$=-\int_0^{1/2} x\left(x-\frac{1}{2}\right)dx+\int_{1/2}^1 x\left(x-\frac{1}{2}\right)dx$$

$$=\left[\frac{x^2}{4}-\frac{x^3}{3}\right]_0^{1/2}+\left[\frac{x^3}{3}-\frac{x^2}{4}\right]_{1/2}^1$$

$$=\left(\frac{1}{16}-\frac{1}{24}\right)+\left(\frac{1}{3}-\frac{1}{4}-\frac{1}{24}+\frac{1}{16}\right)=\frac{1}{8}$$

3 **(a)**

Let

$$I=\int \frac{a^{x/2}}{\sqrt{a^{-x}-a^x}}dx=\int \frac{a^x}{\sqrt{1-(a^x)^2}}dx$$

$$\Rightarrow I=\frac{1}{\log a}\int \frac{1}{\sqrt{1^2-(a^x)^2}}d(a^x)=\frac{1}{\log a}\sin^{-1}(a^x)+C$$

4 **(c)**

Let $I = \int_{\log 2}^{x} \frac{e^u}{e^u(e^u-1)^{1/2}} du$

Put $e^u - 1 = t^2 \Rightarrow e^u du = 2t\, dt$

$$\therefore I = \int_{1}^{\sqrt{e^x-1}} \frac{2t}{(t^2+1)t} dt$$

$$= 2\int_{1}^{\sqrt{e^x-1}} \frac{dt}{(1+t^2)}$$

$$= 2[\tan^{-1}\text{t}]_{1}^{\sqrt{e^x-1}}$$

$$= 2\left[\tan^{-1}\sqrt{e^x-1} - \frac{\pi}{4}\right] = \frac{\pi}{6} \quad \text{[given]}$$

$$\Rightarrow \tan^{-1}\sqrt{e^x-1} = \frac{\pi}{12} + \frac{\pi}{4} = \frac{\pi}{3}$$

$$\Rightarrow \sqrt{e^x-1} = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$$\Rightarrow e^x = 3 + 1 = 4$$

5 **(c)**

We have,

$$I = \int \{f(x)g''(x) - f''(x)g(x)\}dx$$

$$\Rightarrow I = \int f(x)g''(x)dx - \int f''(x)g(x)dx$$

$$\Rightarrow I = \left\{f(x)g'(x) - \int f'(x)g'(x)dx\right\} - \left\{g(x)f'(x) - \int g'(x)f'(x)dx\right\}$$

$$\Rightarrow I = f(x)g'(x) - f'(x)g(x)$$

6 **(d)**

Let $I = \int_{-\pi/4}^{\pi/4} x^3 \sin^4 x\, dx$

Since, $x^3 \sin^4 x$ is an odd function.

$\therefore I = 0$

7 **(a)**

Let $I = \int_0^{\pi/2} \sin^8 x\, dx = \frac{7.5.3.1}{8.6.4.2} \cdot \frac{\pi}{2}$ [by Walli's formula]

$$= \frac{105\pi}{32(4.3.2.1)}$$

$$= \frac{105\pi}{32.4!}$$

8 **(b)**

We have,

$$I = \int_{-\pi/2}^{\pi/2} \frac{|x|}{8\cos^2 2x + 1} dx$$

$$= 2\int_0^{\pi/2} \frac{x}{8\cos^2 2x + 1} dx$$

$$= 2I_1, \text{ where} I_1 = \int_0^{\pi/2} \frac{x}{8\cos^2 2x + 1} dx$$

Now,

$I_1 = \int_0^{\pi/2} \frac{x}{8\cos^2 2x + 1} dx$...(i)

$$\Rightarrow I_1 = \int_0^{\pi/2} \frac{\frac{\pi}{2} - x}{8\cos^2(\pi - 2x) + 1} dx$$

$\Rightarrow I_1 = \int_0^{\pi/2} \frac{\frac{\pi}{2} - x}{8\cos^2 2x + 1} dx$...(ii)

Adding (i) and (ii), we get

$$2I_1 = \frac{\pi}{2}\int_0^{\pi/2} \frac{x}{8\cos^2 2x + 1} dx$$

$$\Rightarrow I_1 = \frac{\pi}{4}\int_0^{\pi/2} \frac{\sec^2 2x}{9 + \tan^2 2x} dx = 2 \times \frac{\pi}{4}\int_0^{\pi/2} \frac{\sec^2 2x}{9 + \tan^2 2x} dx$$

$$\Rightarrow I_1 = \frac{\pi}{4}\int_0^{\pi/4} \frac{2\sec^2 2x}{9 + \tan^2 2x} dx$$

$$\Rightarrow I_1 = \frac{\pi}{4}\int_0^{\pi/4} \frac{1}{3^2 + \tan^2 2x} d(\tan 2x)$$

$$\Rightarrow I_1 = \frac{\pi}{4} \times \frac{1}{3}\left[\tan^{-1}\left(\frac{\tan 2x}{3}\right)\right]_0^{\pi/4} = \frac{\pi}{12} \times \frac{\pi}{2} = \frac{\pi^2}{24}$$

Hence, $I = 2I_1 = \frac{\pi^2}{12}$

9 **(b)**

$$\because f(x)\cos x = \frac{1}{2} \cdot 2f(x)f'(x)$$

Then, $f'(x) = \cos x$

$\therefore f(x) = \sin x + c$

10 **(d)**

On putting $y^2 = f(x) = \frac{x+2}{2x+3}$, we have

$x = \frac{3y^2 - 2}{1 - 2y^2}$ and $dx = -\frac{2y}{(1-2y^2)^2} dy$

$$\therefore \int (f(x))^{1/2} \frac{dx}{x} = -\int y \frac{2y}{(1-2y^2)^2} \cdot \frac{1-2y^2}{3y^2 - 2} dy$$

$$= 2\int \frac{y^2}{(2y^2 - 1)(3y^2 - 2)} dy$$

$$= -2\int\left[\frac{1}{2y^2-1}-\frac{2}{3y^2-2}\right]dy$$

$$= \frac{1}{\sqrt{2}}\log\left|\frac{1+\sqrt{2y}}{1-\sqrt{2y}}\right| - \sqrt{\frac{2}{3}}\log\left|\frac{\sqrt{3y}+\sqrt{2}}{\sqrt{3y}-\sqrt{2}}\right| + c$$

Thus, $g(x) = \log|x|$ and $h(x) = \log|x|$

11 **(c)**

Let $5^{5^{5^x}} = t \cdot \cdot$ Then, $5^{5^{5^x}}5^{5^x}5^x(\log 5)^3 dx = dt$,

$$\therefore I = \int 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x\, dx = \frac{1}{(\log 5)^3}\int 1 \cdot dt = \frac{5^{5^{5^x}}}{(\log 5)^3} + C$$

12 **(a)**

$$f(t) = \int_{-t}^{t}\frac{e^{-[x]}}{2}dx = 2\int_0^t \frac{e^{-x}}{2}dx$$

$$= -[e^{-x}]_0^t = -e^{-t} + 1$$

Now, $\lim_{t\to\infty} f(t) = -\lim_{t\to\infty} e^{-t} + 1 = 1$

13 **(b)**

We have,

$$I = \int\frac{\cos 2x}{(\sin x + \cos x)^2}dx = \int\frac{\cos x - \sin x}{(\sin x + \cos x)^2}dx$$

$\Rightarrow I = \log(\sin x + \cos x) + C$

14 **(c)**

The primitive of the given function is

$$\int\frac{dx}{\sqrt{\sin^3 x \sin(x+\theta)}} = \int\frac{dx}{\sqrt{\sin^3 x(\sin x\cos\theta + \cos x \sin\theta)}}$$

$$= \int\frac{\operatorname{cosec}^2 x\, dx}{\sqrt{\cos\theta + \cot x \sin\theta}}$$

Put $\cot x = t \Rightarrow -\operatorname{cosec}^2 x\, dx = dt$

$$= -\frac{1}{\sqrt{\sin\theta}}\int\frac{dt}{\sqrt{\cot\theta + 1}}$$

$$= -\frac{2}{\sqrt{\sin\theta}}(\cot\theta + t)^{1/2} + c$$

$$= -\frac{2}{\sin\theta}(\cos\theta + t\sin\theta)^{1/2} + c$$

$$= \frac{-2\operatorname{cosec}\theta(\sin x\cos\theta + \sin\theta\cos x)^{1/2}}{\sqrt{\sin x}} + c$$

$$= -2\operatorname{cosec}\theta\left(\frac{\sin(\theta + x)}{\sin x}\right)^{1/2} + c$$

15 **(b)**

We have,

$$I=\int \frac{x^4-1}{x^2\sqrt{x^4+x^2+1}}dx=\int \frac{(x^2-1)(x^2+1)}{x^2\sqrt{x^4+x^2+1}}dx$$

$$\Rightarrow I=\int \frac{\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x^2}\right)}{\sqrt{x^2+\frac{1}{x^2}+1}}dx=\int \frac{\left(x+\frac{1}{x}\right)}{\sqrt{\left(x+\frac{1}{x}\right)^2-1^2}}d\left(x+\frac{1}{x}\right)$$

$$\Rightarrow I=\frac{1}{2}\int \frac{2\left(x+\frac{1}{x}\right)}{\sqrt{\left(x+\frac{1}{x}\right)^2-1^2}}d\left(x+\frac{1}{x}\right)$$

$$\Rightarrow I=\frac{1}{2}\int \frac{1}{\sqrt{\left(x+\frac{1}{x}\right)^2-1^2}}d\left\{\left(x+\frac{1}{x}\right)^2-1^2\right\}$$

$$\Rightarrow I=\sqrt{\left(x+\frac{1}{x}\right)^2-1^2}+C=\sqrt{x^2+\frac{1}{x^2}+1}+C$$

$$\Rightarrow I=\sqrt{\frac{x^4+x^2+1}{x}}+C$$

16 **(b)**

$$\int 32x^3(\log x)^2\,dx$$

$$=32\{(\log x)^2\frac{x^4}{4}-\int 2\log x\cdot\frac{1}{x}\cdot\frac{x^4}{4}dx\}$$

$$=8x^4(\log x)^2-16\int x^3\log x\,dx$$

$$=8x^4(\log x)^2-16\{\log x\cdot\frac{x^4}{4}-\int\frac{1}{x}\cdot\frac{x^4}{4}dx\}$$

$$=8x^4(\log x)^2-4x^4\log x+4\int x^3dx$$

$$=8x^4(\log x)^2-4x^4+\log x+x^4+c$$

$$=x^4[\,8(\log x)^2-4\log x+1\,]+c$$

17 **(a)**

Let $I=\int\frac{e^x(1+\sin x)}{1+\cos x}dx$

$$=\int\frac{1}{2}e^x\sec^2\frac{x}{2}dx+\int e^x\tan\frac{x}{2}dx$$

$$=\frac{1}{2}\left[2e^x\tan\frac{x}{2}-\int 2e^x\tan\frac{x}{2}dx\right]+\int e^x\tan\frac{x}{2}dx$$

$$=e^x\tan\frac{x}{2}+c$$

18 **(a)**

We have,

$I = \int \log\left(\sqrt{1-x} + \sqrt{1+x}\right) \cdot 1\, dx$

I II

$\Rightarrow I = x\log\left(\sqrt{1-x} + \sqrt{1+x}\right) - \int \frac{1}{2}\frac{1}{\sqrt{1-x^2}}\left(\frac{\sqrt{1-x^2}}{x} - \frac{1}{x}\right) \cdot x\, dx$

$\Rightarrow I = x\log(\sqrt{1-x} + \sqrt{1+x}) - \frac{1}{2}\int \left(1 - \frac{1}{\sqrt{1-x^2}}\right) dx$

$\Rightarrow I = x\log(\sqrt{1-x} + \sqrt{1+x}) - \frac{x}{2} + \frac{1}{2}\sin^{-1}x + c$

Hence, $f(x) = \log(\sqrt{1-x} + \sqrt{1+x})$, $A = -\frac{1}{2}$ and $B = \frac{1}{2}$

19 **(a)**

Let $I = \int_0^{\pi/4} \log(1 + \tan x)dx$ ....(i)

$\Rightarrow I = \int_0^{\pi/4} \log\left[1 + \tan\left(\frac{\pi}{4} - x\right)\right]dx$

$= \int_0^{\pi/4} \log\left[1 + \frac{1-\tan x}{1+\tan x}\right]dx$

$= \int_0^{\pi/4} \log 2dx - \int_0^{\pi/4} \log(1 + \tan x)dx$

$\Rightarrow I = \log 2[x]_0^{\pi/4} - I$

$\Rightarrow 2I = \frac{\pi}{4}\log_e 2 \Rightarrow I = \frac{\pi}{8}\log_e 2$

20 **(a)**

$f(x) = |x|\sin^3 x$is an odd function

$\therefore \int_{-2}^{2} |x|\sin^3 x\, dx = 0$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	A	C	C	D	A	B	B	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	B	C	B	B	A	A	A	A