

CLASS : XII<sup>th</sup>  
DATE :

**SOLUTIONS**

SUBJECT : MATHS  
DPP NO. :7

**Topic :-INTEGRALS**

1      **(a)**

$$\text{Let } I = \int \frac{\log(x+1) - \log x}{x(x+1)} dx = \int \frac{\log\left(1 + \frac{1}{x}\right)}{x(x+1)} dx$$

$$= \int \frac{\log\left(1 + \frac{1}{x}\right)}{x^2\left(1 + \frac{1}{x}\right)} dx$$

$$\text{Put } 1 + \frac{1}{x} = t \Rightarrow -\frac{1}{x^2} dx = dt$$

$$\therefore I = - \int \frac{\log t}{t} dt = -\frac{1}{2}(\log t)^2 + c$$

$$= -\frac{1}{2} \left[ \log\left(1 + \frac{1}{x}\right) \right]^2 + c$$

$$= -\frac{1}{2} [\log(x+1) - \log x]^2 + c$$

$$= -\frac{1}{2} \{ \log(x+1) \}^2 - \frac{1}{2} (\log x)^2 + \log(x+1) \cdot \log x + c$$

2      **(c)**

$$\text{Let } I = \int_0^1 x|x - \frac{1}{2}| dx$$

$$= - \int_0^{1/2} x\left(x - \frac{1}{2}\right) dx + \int_{1/2}^1 x\left(x - \frac{1}{2}\right) dx$$

$$= \left[ \frac{x^2}{4} - \frac{x^3}{3} \right]_0^{1/2} + \left[ \frac{x^3}{3} - \frac{x^2}{4} \right]_{1/2}^1$$

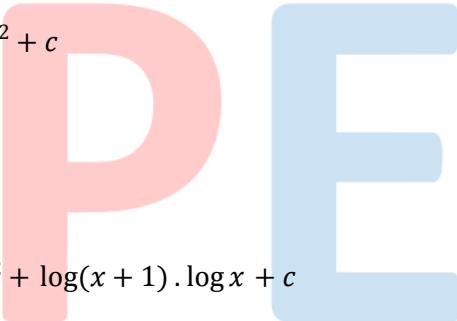
$$= \left( \frac{1}{16} - \frac{1}{24} \right) + \left( \frac{1}{3} - \frac{1}{4} - \frac{1}{24} + \frac{1}{16} \right) = \frac{1}{8}$$

3      **(a)**

Let

$$I = \int \frac{a^{x/2}}{\sqrt{a^{-x} - a^x}} dx = \int \frac{a^x}{\sqrt{1 - (a^x)^2}} dx$$

$$\Rightarrow I = \frac{1}{\log a} \int \frac{1}{\sqrt{1^2 - (a^x)^2}} d(a^x) = \frac{1}{\log a} \sin^{-1}(a^x) + C$$



4 (c)

$$\text{Let } I = \int_{\log 2}^x \frac{e^u}{e^u(e^u - 1)^{1/2}} du$$

$$\text{Put } e^u - 1 = t^2 \Rightarrow e^u du = 2t dt$$

$$\therefore I = \int_1^{\sqrt{e^x-1}} \frac{2t}{(t^2 + 1)t} dt$$

$$= 2 \int_1^{\sqrt{e^x-1}} \frac{dt}{(1 + t^2)}$$

$$= 2[\tan^{-1} t]_1^{\sqrt{e^x-1}}$$

$$= 2\left[\tan^{-1}\sqrt{e^x-1} - \frac{\pi}{4}\right] = \frac{\pi}{6} \quad [\text{given}]$$

$$\Rightarrow \tan^{-1}\sqrt{e^x-1} = \frac{\pi}{12} + \frac{\pi}{4} = \frac{\pi}{3}$$

$$\Rightarrow \sqrt{e^x-1} = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$$\Rightarrow e^x = 3 + 1 = 4$$

5 (c)

We have,

$$I = \int \{f(x)g''(x) - f''(x)g(x)\} dx$$

$$\Rightarrow I = \int f(x)g''(x) dx - \int f''(x)g(x) dx$$

$$\Rightarrow I = \left\{f(x)g'(x) - \int f'(x)g'(x) dx\right\} - \left\{g(x)f'(x) - \int g'(x)f'(x) dx\right\}$$

$$\Rightarrow I = f(x)g'(x) - f'(x)g(x)$$

6 (d)

$$\text{Let } I = \int_{-\pi/4}^{\pi/4} x^3 \sin^4 x dx$$

Since,  $x^3 \sin^4 x$  is an odd function.

$$\therefore I = 0$$

7 (a)

$$\text{Let } I = \int_0^{\pi/2} \sin^8 x dx = \frac{7.5.3.1}{8.6.4.2.2} \frac{\pi}{2} \quad [\text{by Walli's formula}]$$

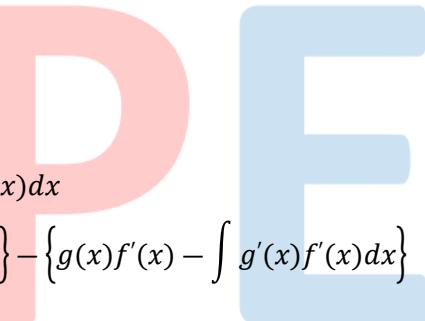
$$= \frac{105\pi}{32(4.3.2.1)}$$

$$= \frac{105\pi}{32.4!}$$

8 (b)

We have,

$$I = \int_{-\pi/2}^{\pi/2} \frac{|x|}{8 \cos^2 2x + 1} dx$$



$$= 2 \int_0^{\pi/2} \frac{x}{8 \cos^2 2x + 1} dx$$

$$= 2I_1, \text{ where } I_1 = \int_0^{\pi/2} \frac{x}{8 \cos^2 2x + 1} dx$$

Now,

$$I_1 = \int_0^{\pi/2} \frac{x}{8 \cos^2 2x + 1} dx \dots (\text{i})$$

$$\Rightarrow I_1 = \int_0^{\pi/2} \frac{\frac{\pi}{2} - x}{8 \cos^2(\pi - 2x) + 1} dx$$

$$\Rightarrow I_1 = \int_0^{\pi/2} \frac{\frac{\pi}{2} - x}{8 \cos^2 2x + 1} dx \dots (\text{ii})$$

Adding (i) and (ii), we get

$$2I_1 = \frac{\pi}{2} \int_0^{\pi/2} \frac{x}{8 \cos^2 2x + 1} dx$$

$$\Rightarrow I_1 = \frac{\pi}{4} \int_0^{\pi/2} \frac{\sec^2 2x}{9 + \tan^2 2x} dx = 2 \times \frac{\pi}{4} \int_0^{\pi/2} \frac{\sec^2 2x}{9 + \tan^2 2x} dx$$

$$\Rightarrow I_1 = \frac{\pi}{4} \int_0^{\pi/4} \frac{2 \sec^2 2x}{9 + \tan^2 2x} dx$$

$$\Rightarrow I_1 = \frac{\pi}{4} \int_0^{\pi/4} \frac{1}{3^2 + \tan^2 2x} d(\tan 2x)$$

$$\Rightarrow I_1 = \frac{\pi}{4} \times \frac{1}{3} \left[ \tan^{-1} \left( \frac{\tan 2x}{3} \right) \right]_0^{\pi/4} = \frac{\pi}{12} \times \frac{\pi}{2} = \frac{\pi^2}{24}$$

Hence,  $I = 2I_1 = \frac{\pi^2}{12}$

9      **(b)**

$$\because f(x) \cos x = \frac{1}{2} \cdot 2f(x)f'(x)$$

Then,  $f'(x) = \cos x$

$$\therefore f(x) = \sin x + c$$

10      **(d)**

On putting  $y^2 = f(x) = \frac{x+2}{2x+3}$ , we have

$$x = \frac{3y^2 - 2}{1 - 2y^2} \text{ and } dx = -\frac{2y}{(1 - 2y^2)^2} dy$$

$$\therefore \int (f(x))^{1/2} \frac{dx}{x} = - \int y \frac{2y}{(1 - 2y^2)^2} \cdot \frac{1 - 2y^2}{3y^2 - 2} dy$$

$$= 2 \int \frac{y^2}{(2y^2 - 1)(3y^2 - 2)} dy$$

$$\begin{aligned}
&= -2 \int \left[ \frac{1}{2y^2 - 1} - \frac{2}{3y^2 - 2} \right] dy \\
&= \frac{1}{\sqrt{2}} \log \left| \frac{1 + \sqrt{2y}}{1 - \sqrt{2y}} \right| - \frac{\sqrt{2}}{\sqrt{3}} \log \left| \frac{\sqrt{3y} + \sqrt{2}}{\sqrt{3y} - \sqrt{2}} \right| + C
\end{aligned}$$

Thus,  $g(x) = \log|x|$  and  $h(x) = \log|x|$

11 (c)

Let  $5^{5^x} = t \cdot \cdot$ . Then,  $5^{5^x} 5^{5^x} 5^x (\log 5)^3 dx = dt$ ,

$$\therefore I = \int 5^{5^x} \cdot 5^{5^x} \cdot 5^x dx = \frac{1}{(\log 5)^3} \int 1 \cdot dt = \frac{5^{5^x}}{(\log 5)^3} + C$$

12 (a)

$$\begin{aligned}
f(t) &= \int_{-t}^t \frac{e^{-[x]}}{2} dx = 2 \int_0^t \frac{e^{-x}}{2} dx \\
&= -[e^{-x}]_0^t = -e^{-t} + 1
\end{aligned}$$

$$\text{Now, } \lim_{t \rightarrow \infty} f(t) = -\lim_{t \rightarrow \infty} e^{-t} + 1 = 1$$

13 (b)

We have,

$$I = \int \frac{\cos 2x}{(\sin x + \cos x)^2} dx = \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} dx$$

$$\Rightarrow I = \log(\sin x + \cos x) + C$$

14 (c)

The primitive of the given function is

$$\begin{aligned}
\int \frac{dx}{\sqrt{\sin^3 x \sin(x + \theta)}} &= \int \frac{dx}{\sqrt{\sin^3 x (\sin x \cos \theta + \cos x \sin \theta)}} \\
&= \int \frac{\operatorname{cosec}^2 x dx}{\sqrt{\cos \theta + \cot x \sin \theta}}
\end{aligned}$$

Put  $\cot x = t \Rightarrow -\operatorname{cosec}^2 x dx = dt$

$$\begin{aligned}
&= -\frac{1}{\sqrt{\sin \theta}} \int \frac{dt}{\sqrt{\cot \theta + 1}} \\
&= -\frac{2}{\sqrt{\sin \theta}} (\cot \theta + t)^{1/2} + C \\
&= -\frac{2}{\sin \theta} (\cos \theta + t \sin \theta)^{1/2} + C \\
&= \frac{-2 \operatorname{cosec} \theta (\sin x \cos \theta + \sin \theta \cos x)^{1/2}}{\sqrt{\sin x}} + C \\
&= -2 \operatorname{cosec} \theta \left( \frac{\sin(\theta + x)}{\sin x} \right)^{1/2} + C
\end{aligned}$$

15 (b)

We have,

$$\begin{aligned}
I &= \int \frac{x^4 - 1}{x^2 \sqrt{x^4 + x^2 + 1}} dx = \int \frac{(x^2 - 1)(x^2 + 1)}{x^2 \sqrt{x^4 + x^2 + 1}} dx \\
\Rightarrow I &= \int \frac{\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x^2}\right)}{\sqrt{x^2 + \frac{1}{x^2} + 1}} dx = \int \frac{\left(x + \frac{1}{x}\right)}{\sqrt{\left(x + \frac{1}{x}\right)^2 - 1^2}} d\left(x + \frac{1}{x}\right) \\
\Rightarrow I &= \frac{1}{2} \int \frac{2\left(x + \frac{1}{x}\right)}{\sqrt{\left(x + \frac{1}{x}\right)^2 - 1^2}} d\left(x + \frac{1}{x}\right) \\
\Rightarrow I &= \frac{1}{2} \int \frac{1}{\sqrt{\left(x + \frac{1}{x}\right)^2 - 1^2}} d\left\{\left(x + \frac{1}{x}\right)^2 - 1^2\right\} \\
\Rightarrow I &= \sqrt{\left(x + \frac{1}{x}\right)^2 - 1^2} + C = \sqrt{x^2 + \frac{1}{x^2} + 1} + C \\
\Rightarrow I &= \sqrt{\frac{x^4 + x^2 + 1}{x}} + C
\end{aligned}$$

16      (b)

$$\begin{aligned}
&\int 32x^3(\log x)^2 dx \\
&= 32\{(\log x)^2 \frac{x^4}{4} - \int 2 \log x \cdot \frac{1}{x} \cdot \frac{x^4}{4} dx\} \\
&= 8x^4(\log x)^2 - 16 \int x^3 \log x dx \\
&= 8x^4(\log x)^2 - 16 \left\{ \log x \cdot \frac{x^4}{4} - \int \frac{1}{x} \cdot \frac{x^4}{4} dx \right\} \\
&= 8x^4(\log x)^2 - 4x^4 \log x + 4 \int x^3 dx \\
&= 8x^4(\log x)^2 - 4x^4 + \log x + x^4 + C \\
&= x^4[8(\log x)^2 - 4 \log x + 1] + C
\end{aligned}$$

17      (a)

$$\begin{aligned}
\text{Let } I &= \int \frac{e^x(1 + \sin x)}{1 + \cos x} dx \\
&= \int \frac{1}{2} e^x \sec^2 \frac{x}{2} dx + \int e^x \tan \frac{x}{2} dx \\
&= \frac{1}{2} \left[ 2e^x \tan \frac{x}{2} - \int 2e^x \tan \frac{x}{2} dx \right] + \int e^x \tan \frac{x}{2} dx \\
&= e^x \tan \frac{x}{2} + C
\end{aligned}$$

18      (a)

We have,

$$\begin{aligned}
 I &= \int \log(\sqrt{1-x} + \sqrt{1+x}) \cdot 1 \, dx \\
 &\stackrel{\text{I}}{=} x \log(\sqrt{1-x} + \sqrt{1+x}) - \int \frac{1}{2} \frac{1}{\sqrt{1-x^2}} \left( \frac{\sqrt{1-x^2}}{x} - \frac{1}{x} \right) \cdot x \, dx \\
 &\Rightarrow I = x \log(\sqrt{1-x} + \sqrt{1+x}) - \frac{1}{2} \int \left( 1 - \frac{1}{\sqrt{1-x^2}} \right) dx \\
 &\Rightarrow I = x \log(\sqrt{1-x} + \sqrt{1+x}) - \frac{x}{2} + \frac{1}{2} \sin^{-1} x + c
 \end{aligned}$$

Hence,  $f(x) = \log(\sqrt{1-x} + \sqrt{1+x})$ ,  $A = -\frac{1}{2}$  and  $B = \frac{1}{2}$

19 (a)

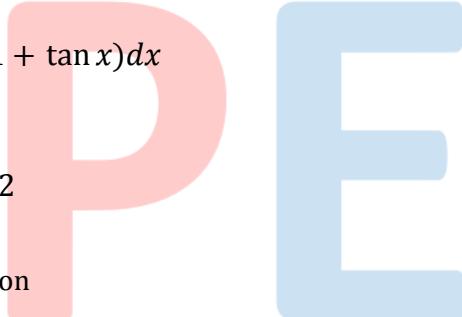
$$\text{Let } I = \int_0^{\pi/4} \log(1 + \tan x) \, dx \quad \dots \text{(i)}$$

$$\begin{aligned}
 \Rightarrow I &= \int_0^{\pi/4} \log \left[ 1 + \tan \left( \frac{\pi}{4} - x \right) \right] \, dx \\
 &= \int_0^{\pi/4} \log \left[ 1 + \frac{1 - \tan x}{1 + \tan x} \right] \, dx \\
 &= \int_0^{\pi/4} \log 2 \, dx - \int_0^{\pi/4} \log(1 + \tan x) \, dx \\
 \Rightarrow I &= \log 2 [x]_0^{\pi/4} - I \\
 \Rightarrow 2I &= \frac{\pi}{4} \log_e 2 \Rightarrow I = \frac{\pi}{8} \log_e 2
 \end{aligned}$$

20 (a)

$f(x) = |x| \sin^3 x$  is an odd function

$$\therefore \int_{-2}^2 |x| \sin^3 x \, dx = 0$$



**ANSWER-KEY**

Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	A	C	C	D	A	B	B	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	B	C	B	B	A	A	A	A

P  
C