DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :6

Topic :-INTEGRALS

1 **(b)**

On putting, $x = \tan\theta \Rightarrow\ dx = sce^2\theta\ d\theta$, we get

$$f(x) = \int \frac{\tan^2\theta.\sec^2\theta}{\sec^2\theta(1+\sec\theta)} d\theta$$

$$= \int \frac{\sec^2\theta - 1}{1+\sec\theta} d\theta = \int (\sec\theta - 1)\, d\theta$$

$$= \log(\sec\theta + \tan\theta) - \theta + c$$

$$\Rightarrow f(x) = \log(\sqrt{1+x^2} + x) - \tan^{-1} x + c$$

At $x = 0$, $f(0) = \log(1+0) - 0 + c \Rightarrow c = 0$

$$\therefore\ f(x) = \log(\sqrt{1+x^2} + x) - \tan^{-1} x$$

At $x = 1$, $f(1) = \log(1+\sqrt{2}) - \frac{\pi}{4}$

2 **(b)**

We have,

$I = \int_0^1 \frac{1}{(1+x^2)^{3/2}} dx = \int_0^{\pi/4} \cos\theta$, where $x = \tan\theta$

$$\Rightarrow I = \frac{1}{\sqrt{2}}$$

43 **(a)**

We have, $I(m,n) = I = \int_0^1 t^m (1+t)^n dt$

$$\Rightarrow I(m,n) = \left[(1+t)^n \cdot \frac{t^{m+1}}{m+1}\right]_0^1 - \frac{n}{m+1}\int_0^1 (1+t)^{n-1} t^{m+1} dt$$

$$\Rightarrow I(m,n) = \frac{2^n}{m+1} - \frac{n}{m+1} I(m+1, n-1)$$

4 **(b)**

Let $I = \int_0^1 \frac{\tan^{-1} x}{1+x^2}\ dx$

Put $\tan^{-1} x = t \Rightarrow \frac{1}{1+x^2} dx = dt$

$$\therefore\ I = \int_0^{\pi/4} t\ dt = \left[\frac{t^2}{2}\right]_0^{\pi/4}$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{4}\right)^4 - 0^2\right] = \frac{\pi^2}{32}$$

5 **(b)**

Let $I=\int_3^5 \frac{x^2}{x^2-4}\,dx=\int_3^5\left(\frac{x^2-4}{x^2-4}+\frac{4}{x^2-4}\right)dx$

$$=\int_3^5\left(1+\frac{4}{x^2-4}\right)dx$$

$$=\left[x+\frac{4}{2\times 2}\log_e\left(\frac{x-2}{x+2}\right)\right]_3^5$$

$$=\left[5+\log_e\left(\frac{5-2}{5+2}\right)-3-\log_e\left(\frac{3-2}{3+2}\right)\right]$$

$$=2+\log_e\left(\frac{3}{7}\right)-\log_e\left(\frac{1}{5}\right)$$

$$=2+\log_e\left(\frac{3}{7}\times\frac{5}{1}\right)=2+\log_e\left(\frac{15}{7}\right)$$

6 **(b)**

$$\int\sqrt{x^2+a^2}\,dx=\int\sqrt{x^2+a^2}\cdot 1\,dx$$

$$=\sqrt{x^2+a^2}\int 1dx-\int\left[\frac{d}{dx}\left(\sqrt{x^2+a^2}\right)\int 1dx\right]dx$$

$$=x\sqrt{x^2+a^2}-\int\left[\frac{2x}{2\sqrt{x^2+a^2}}x\right]dx$$

$$=x\sqrt{x^2+a^2}-\int\left[\frac{x^2+a^2-a^2}{\sqrt{x^2+a^2}}\right]dx$$

$$=x\sqrt{x^2+a^2}-\int\left[\sqrt{x^2+a^2}-\frac{a^2}{\sqrt{x^2+a^2}}\right]dx$$

$$=x\sqrt{x^2+a^2}-\int\sqrt{x^2+a^2}dx+a^2\int\frac{dx}{\sqrt{x^2+a^2}}$$

$$=x\sqrt{x^2+a^2}-I+a^2\log[x+\sqrt{x^2+a^2}]+c$$

$$\Rightarrow 2I=x\sqrt{x^2+a^2}+a^2\log[x+\sqrt{x^2+a^2}]+c$$

$$\Rightarrow I=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left[x\sqrt{x^2+a^2}\right]+c$$

7 **(b)**

$$I_n+I_{n+2}=\int_0^{\pi/4}\tan^n x(1+\tan^2 x)dx=\int_0^{\pi/4}\tan^n x\sec^2 x\,dx$$

Put $\tan x=t\Rightarrow\sec^2 x dx=dt$

$$\therefore I_n+I_{n+2}=\int_0^1 t^n\,dt=\frac{1}{n+1}$$

$$\Rightarrow\lim_{n\to\infty}n[I_n+I_{n+2}]=\lim_{n\to\infty}\frac{n}{n+1}=\lim_{n\to\infty}\frac{1}{\left(1+\frac{1}{n}\right)}=1$$

8 **(b)**

Put $x+1=t^2$

$\Rightarrow dx = 2t\,dt$

$\Rightarrow x^2+1=(t^2-1)^2+1$

$= t^4-2t^2+2$

$\therefore$ Given integral

$$= \int (t^4-2t^2+2)t.2t\,dt$$

$$= 2\int (t^6-2t^4+2t^2)dt$$

$$= 2\left[\frac{t^7}{7}-2\frac{t^5}{5}+2\frac{t^3}{3}\right]+c$$

$$= 2\left[\frac{(x+1)^{\frac{7}{2}}}{7}-2\frac{(x+1)^{\frac{5}{2}}}{5}+2\frac{(x+1)^{\frac{3}{2}}}{3}\right]+c$$

9 **(c)**

$$\int \frac{f'(x)}{f(x)\log[f(x)]}dx = \log[\log f(x)]+c$$

10 **(d)**

We have, $0<x<1$

$\therefore \frac{1}{2}x^2<x^2<x$

$\Rightarrow -x^2>-x$

$\Rightarrow e^{-x^2}>e^{-x}$

$\Rightarrow e^{-x^2}\cos^2 x > e^{-x}\cos^2 x$

$$\Rightarrow \int_0^1 e^{-x^2}\cos^2 x\,dx > \int_0^1 e^{-x}\cos^2 x\,dx$$

$\Rightarrow I_2>I_1$...(i)

Also, $\cos^2 x \le 1$

$\Rightarrow e^{-x^2}\cos^2 x < e^{-x^2} < e^{-(1/2)x^2}\left[\because -\frac{1}{2}x^2>-x^2>-x\right]$

$$\Rightarrow \int_0^1 e^{-x^2}\cos^2 x\,dx < \int_0^1 e^{-x^2}dx < \int_0^1 e^{-(1/2)x^2}dx$$

$\Rightarrow I_2<I_3<I_4$...(ii)

From (i) and (ii), we get $I_1 < I_2 < I_3 < I_4$
Hence, I_4 is the greatest integral
11 **(c)**
We have,
$$I = \int \frac{1}{(1+x^2)\sqrt{1-x^2}} dx$$
$$\Rightarrow I = \int \frac{-t\,dt}{(t^2+1)\sqrt{t^2-1}}, \text{ where } x = \frac{1}{t}, dx = -\frac{1}{t^2}dt$$
$$\Rightarrow I = \int \frac{-u\,du}{(u^2+2)\sqrt{u^2}}, \text{ where } t^2 - 1 = u^2$$
$$\Rightarrow I = -\int \frac{1}{u^2+(\sqrt{2})^2} du = -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + K$$
$$\Rightarrow I = -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}\,x}\right) + K$$
$$\Rightarrow I = -\frac{1}{\sqrt{2}}\left\{\frac{\pi}{2} - \cot^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}\,x}\right)\right\} + K \left[\because \tan^{-1}x + \cot^{-1}x = \pi/2\right]$$
$$\Rightarrow I = -\frac{1}{\sqrt{2}}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}\,x}\right) + \left(K - \frac{\pi}{2\sqrt{2}}\right)$$
$$\Rightarrow I = -\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right) + C, \text{ where } C = K - \frac{\pi}{2\sqrt{2}}$$
12 **(c)**
$$I = \int_{1/n}^{(an-1)/n} \frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}} dx$$
$$= \int_{1/n}^{a-(1/n)} \frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}} dx \ldots\text{(i)}$$
$$= \int_{1/n}^{a-(1/n)} \frac{\sqrt{\frac{1}{n}+a-\frac{1}{n}-x}dx}{\sqrt{a-\left(\frac{1}{n}+a-\frac{1}{n}-x\right)}+\sqrt{\left(\frac{1}{n}+a-\frac{1}{n}-x\right)}}$$
$$\Rightarrow I = \int_{\frac{1}{n}}^{a-\frac{1}{n}} \frac{\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}} dx \ldots\text{(ii)}$$
On adding Eqs.(i) and (ii), we get
$$2I = \int_{1/n}^{a-(1/n)} 1\,dx = [x]_{\frac{1}{n}}^{a-\frac{1}{n}}$$
$$\Rightarrow 2I = a - \frac{1}{n} - \frac{1}{n} = \frac{na-2}{n}$$
$$\Rightarrow I = \frac{na-2}{2n}$$
13 **(b)**
Let $I = \int e^x(x^5 + 5x^4 + 1)dx$

$= \int e^x x^5 dx + 5\int e^x x^4 dx + \int e^x\, dx$

$= x^5 e^x - \int 5x^4 e^x dx + 5\int e^x x^4 dx + e^x$

$= x^5 e^x + e^x + c$

14 **(a)**

$\cos x \log\left(\frac{1+x}{1-x}\right)$is an odd function

$$\therefore \int_{-1/2}^{1/2} \cos x \log\left(\frac{1+x}{1-x}\right) dx = 0$$

$\therefore\ k = 0$

15 **(b)**

Let $I = \int \sin\sqrt{3}\, dx$

Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$$\therefore\ I = \int 2t \sin t\, dt$$

$$= 2\left[-t\cos t + \int \cos t\, dt\right]$$

$$= 2[-t\cos t + \sin t] + c$$

$$= 2\left[-\sqrt{x}\cos\sqrt{x} + \sin\sqrt{x}\right] + c$$

16 **(d)**

$f'(x) = (x^2 - 1)(2x) - (x - 1) = (x - 1)(2x^2 + 2x - 1)$

Which is positive for $x > 1$. Hence, f increases in $[1, 2]$

Hence, global maximum of f is $f(2) = 4$

17 **(b)**

$$\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \frac{\left(\frac{r}{n}\right)}{\sqrt{1 + \left(\frac{r}{n}\right)^2}} = \frac{1}{2}\int_0^1 \frac{2x}{\sqrt{1+x^2}} dx$$

$$= \frac{1}{2}\left[\frac{\sqrt{1+x^2}}{1/2}\right]_0^1 = \sqrt{2} - 1$$

18 **(d)**

We have,

$$I = \int_0^{2\pi} \cos^{99} x\, dx,$$

$$\Rightarrow I = 2\int_0^{\pi} \cos^{99} x\, dx \quad [\because \cos^{99}(2\pi - x) = \cos^{99} x]$$

But, $\int_0^{\pi} \cos^{99} x dx = 0 \quad [\because \cos^{99}(\pi - x) = -\cos^{99} x]$

$\therefore I = 2 \times 0 = 0$

19 **(b)**

Let $I=\int_{-2}^{2}(ax^3+bx+c)dx$

In the given integral, $ax^3$and bx are odd functions. Hence, it depends only on the value of c.

20 **(b)**

Let $I=\int_0^{\pi} x\, f(\sin x)dx$

$$\Rightarrow I=\int_0^{\pi}(\pi-x)f\{\sin(\pi-x)\}dx$$

$$=\pi\int_0^{\pi} f(\sin x)dx-x\int_0^{\pi} f(\sin x)dx$$

$$\Rightarrow I=\pi\int_0^{\pi} f(\sin x)dx-I$$

$$\Rightarrow 2I=2\pi\int_0^{\pi/2} f(\sin x)dx$$

$$\Rightarrow I=\pi\int_0^{\pi/2} f(\sin x)dx$$

PE

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	A	B	B	B	B	B	C	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	B	A	B	D	B	D	B	B