DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :5

Topic :-INTEGRALS

1 **(c)**

$$\int_1^e \frac{1}{x}\,dx = [\log x]_1^e = \log e - \log 1 = 1$$

2 **(b)**

We have,

$$\lim_{n\to\infty} \frac{1^{99}+2^{99}+\ldots+n^{99}}{n^{100}} = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{r^{99}}{n^{100}} = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^{99}$$

$$= \int_0^1 x^{99}\,dx = \left[\frac{x^{100}}{100}\right]_0^1 = \frac{1}{100} - 0 = \frac{1}{100}$$

3 **(a)**

We have,

$$I = \int \frac{\cos 2x}{\cos x}\,dx = \int 2\cos x - \sec x\,dx$$

$\Rightarrow I = 2\sin x - \log(\sec x + \tan x) + C$

$\Rightarrow I = 2\sin x + \log(\sec x - \tan x) + C$

4 **(c)**

We have,

$$I = \int f'(ax+b)\{f(ax+b)\}^n\,dx$$

$$\Rightarrow I = \frac{1}{a}\int \{f(ax+b)\}^n\,d\{f(ax+b)\} = \frac{1}{a}\times\frac{\{(ax+b)\}^{n+1}}{n+1} + C$$

5 **(c)**

$$\because 2^{x^3} < 2^{x^2}, 0 < x < 1 \quad \text{and} 2^{x^3} > 2^{x^2}, \qquad x > 1$$

$\therefore I_4 > I_3 \text{and} I_2 < I_1$

6 **(c)**

Let $I = \int \frac{\sqrt{\tan x}}{\sin x \cos x}\,dx$

$$= \int \frac{\sqrt{\sin x}}{\sqrt{\cos x}\sin x\cos x}\,dx$$

$$= \int \frac{1}{\sqrt{\sin x}\cos^{3/2} x}\,dx$$

$$= \int \frac{1}{\sqrt{\tan x}\cos^2 x}\, dx$$

$$= \int \frac{\sec^2 x}{\sqrt{\tan x}}\, dx$$

Put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$$\therefore\ I = \int \frac{1}{\sqrt{t}}\, dt = 2\sqrt{t} = 2\sqrt{\tan x}$$

7 **(b)**

Let $I = \lim\limits_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2+r^2}}$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{n\sqrt{1+(r/n)^2}}$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r/n}{\sqrt{1+(r/n)^2}}$$

Put $\frac{r}{n} = x, \frac{1}{n} dx, \lim\limits_{n\to\infty} \sum_{r=1}^{2n} = \int_0^2$

$$\therefore I = \int_0^2 \frac{x}{\sqrt{1+x^2}}\, dx = [\sqrt{1+x^2}]_0^2 = \sqrt{5} - 1$$

8 **(a)**

$$\because I = \int_0^\infty \frac{x \log x}{(1+x^2)^2}\, dx$$

Put $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

$$\therefore I = \int_0^{\pi/2} \frac{\tan\theta \log(\tan\theta)}{\sec^4\theta} \sec^2\theta\, d\theta$$

$$= \int_0^{\pi/2} \frac{\sin\theta \log(\tan\theta)}{\cos\theta \cdot \frac{1}{\cos^2\theta}}\, d\theta$$

$$= \int_0^{\pi/2} \sin\theta\cos\theta \log(\tan\theta)\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} \sin 2\theta \log(\tan\theta)\, d\theta = 0 \left(\because \int_0^{\pi/2} \sin\ 2\theta \log\tan\theta\, d\theta = 0\right)$$

9 **(a)**

$$\int_{-1}^{1} [x[1+\sin\pi x]+1]dx$$

$$= \int_{-1}^{0} [x[1+\sin\pi x]+1]dx + \int_{0}^{1} [x[1+\sin\pi x]+1]dx$$

Now, $-1 < x < 0 \Rightarrow [1+\sin\pi x] = 0$

$0 < x < 1 \Rightarrow [1+\sin\pi x] = 1$

$\Rightarrow [x[1 + \sin \pi x] + 1] = 1$

$\therefore \int_{-1}^{1} [x[1 + \sin \pi x] + 1] dx = 2$

10 **(d)**

We have,

$$I = \int_0^\infty e^{-ax^2} dx$$

$$\Rightarrow I = \frac{1}{\sqrt{a}} \int_0^\infty e^{-(\sqrt{a}\,x)^2} d(\sqrt{a}x)$$

$$\Rightarrow I = \frac{1}{\sqrt{a}} \times \frac{\sqrt{\pi}}{2} \Rightarrow I = \frac{1}{2}\frac{\sqrt{\pi}}{a}\left[\because \int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}\right]$$

11 **(d)**

Let $I = \int \frac{2x}{\sqrt{1-4^x}} dx \cdot$ Then,

$$I = \frac{1}{\log 2} \int \frac{1}{1-(2^x)^2} d(2^x) = \frac{1}{\log 2} \sin^{-1}(2^x) + C$$

$$\therefore K = \frac{1}{\log 2}$$

12 **(d)**

We have,

$$I = \int_{-\pi}^{\pi} (\cos ax - \sin bx)^2 dx$$

$$\Rightarrow I = \int_{-\pi}^{\pi} (\cos^2 ax - \sin^2 bx - 2\cos ax \sin bx) dx$$

$$\Rightarrow I = \int_{-\pi}^{\pi} (\cos^2 ax - \sin^2 bx) dx - 2\int_{-\pi}^{\pi} \cos ax \sin bx\, dx$$

$$\Rightarrow I = 2\int_{-\pi}^{\pi} (\cos^2 ax + \sin^2 bx) dx - 0$$

$$\Rightarrow I = 2\int_0^{\pi} \frac{1+\cos 2ax}{2} + \frac{1-\cos 2bx}{2} dx$$

$$\Rightarrow I = \int_0^{\pi} 2 + \cos 2ax - \cos 2bx\, dx = 2\pi$$

13 **(d)**

Let $I = \int \frac{mx^{m+2n-1} - nx^{n-1}}{x^{2m+2n} + 2x^{m+n} + 1}\, dx$

$$= \int \frac{mx^{m+2n-1} - nx^{n-1}}{(1+x^{m+n})^2} dx$$

$$\therefore I = \int \frac{mx^{m-1} - \frac{n}{x^{n+1}}}{\left(x^m + \frac{1}{x^n}\right)^2} dx$$

Let $x^m + \frac{1}{x^n} = t$

$$\Rightarrow mx^{m-1} - \frac{n}{x^{n+1}} dx = dt$$

$$\therefore I = \int \frac{1}{t^2} dt = -\frac{1}{t} + c$$

$$= \frac{-1}{\left(x^m + \frac{1}{x^n}\right)} + c = \frac{-x^n}{x^{m+n}+1} + c$$

14 **(b)**

We have,

$$I = \int_0^{16\pi/3} |\sin x| dx = \int_0^{5\pi} |\sin x| dx + \int_{5\pi}^{16\pi/3} |\sin x| dx$$

$$\Rightarrow I = 5\int_0^{\pi} |\sin x|\, dx + \int_0^{\pi/3} |\sin x|\, dx \quad [\because |\sin x| \text{is periodic with period } \pi]$$

$$\Rightarrow I = 5\int_0^{\pi} \sin x\, dx + \int_0^{\pi/3} \sin x\, dx = 5 \times 2 + \left(-\frac{1}{2} + 1\right) = \frac{21}{2}$$

15 **(d)**

Let $I = \int_{-\pi/2}^{\pi/2} \{f(x) + f(-x)\}\{g(x) - g(-x)\}\, dx$

Again, let

$h(x) = \{f(x) + f(-x)\}\{g(x) - g(-x)\}$

$\Rightarrow h(-x) = \{f(-x) + f(x)\}\{g(-x) - g(x)\}$

$\Rightarrow h(-x) = -h(x)$

Hence, $h(x)$ is an odd function

$\therefore\ I = 0$

16 **(a)**

We have,

$$I = \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx = \int \frac{\sqrt{\tan x}}{\tan x} \sec^2 x\, dx$$

$\Rightarrow I = \int (\tan x)^{-1/2}\, d(\tan x) = 2\sqrt{\tan x} + C$

18 **(c)**

We have,

$$\int_{-\pi/3}^{\pi/3} \left\{\frac{a}{3}|\tan x| + \frac{6\tan x}{1+\sec x} + c\right\} dx = 0$$

$$\Rightarrow \frac{a}{3}\int_{-\pi/3}^{\pi/3} |\tan x|\, dx + b\int_{-\pi/3}^{\pi/3} \frac{\tan x}{1+\sec x}\, dx + c\int_{-\pi/3}^{\pi/3} dx = 0$$

$$\Rightarrow 2a\int_{0}^{\pi/3} \tan x\, dx + b \times 0 + c\left(\frac{2\pi}{3}\right) = 0$$

$$\Rightarrow \frac{2a}{3}[\log_e|\sec x|]_0^{\pi/3} + \frac{2\pi}{3}c = 0$$

$$\Rightarrow \frac{2a}{3}\ln 2 + \frac{2\pi}{3}c = 0 \Rightarrow c = -\frac{a}{\pi}\ln 2$$

19 **(d)**

Given, $f'(2) = \tan\frac{\pi}{4} = 1$, $f'(4) = \tan\frac{\pi}{3} = \sqrt{3}$

$$\therefore \int_2^4 f'(x)f''(x)dx = \left[\frac{1}{2}[f'(x)]^2\right]_2^4$$

$$= \frac{1}{2}[f'(4)]^2 - \frac{1}{2}[f'(2)]^2$$

$$= \frac{3}{2} - \frac{1}{2} = 1$$

20 **(a)**

Let $I = \int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$

$$= \int \frac{dx}{\cos^3 x\sqrt{4\sin x\cos x}} = \frac{1}{2}\int \frac{dx}{\cos^{7/2} x \sin^{1/2} x}$$

$$= \frac{1}{2}\int \frac{\sec^4 x}{\sqrt{\tan x}}dx = \frac{1}{2}\int \frac{(1+\tan^2 x)\sec^2 x}{\sqrt{\tan x}}dx$$

Put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$$\therefore I = \frac{1}{2}\int \frac{1+t^2}{\sqrt{t}}dt$$

$$= \frac{1}{2}\int t^{-1/2}dt + \frac{1}{2}\int t^{3/2}dt = t^{1/2} + \frac{t^{5/2}}{5} + c$$

$$= \sqrt{\tan x} + \frac{1}{5}\tan^{5/2}x + c$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	A	C	C	C	B	A	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	D	D	B	D	A	B	C	D	A