DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :4

Topic :-INTEGRALS

1 **(b)**

Let

$$I=\int_0^{\pi/2} \operatorname{cosec}(x-\pi/3)\operatorname{cosec}(x-\pi/6)dx$$

$$\Rightarrow I=2\int_0^{\pi/2}\frac{\sin[(x-\pi/6)-(x-\pi/3)]}{\sin(x-\pi/6)\sin(x-\pi/3)}dx$$

$$\Rightarrow I=2\int_0^{\pi/2}[\cot(x-\pi/3)-\cot(x-\pi/6)]\,dx$$

$$\Rightarrow I=2\left[\log\sin\left(x-\frac{\pi}{3}\right)-\log\sin\left(x-\frac{\pi}{6}\right)\right]_0^{\pi/2}$$

$$\Rightarrow I=2\left[\log\left(\frac{\sin(x-\pi/3)}{\sin(x-\pi/6)}\right)\right]_0^{\pi/2}$$

$$\Rightarrow I=2\left[\log\left(\frac{1/2}{\sqrt{3}/2}\right)-\log\left(\frac{\sqrt{3}/2}{1/2}\right)\right]$$

$$\Rightarrow I=2[-\log\sqrt{3}-\log\sqrt{3}]=-4\log\sqrt{3}=-2\log 3$$

2 **(c)**

Primitive function means indefinite integral.

$\therefore$ Primitive function of $f(x)$

$I=\int\frac{\sqrt{(a^2-x^2)}}{x^4}dx$ (say)

$$=\int\frac{x\sqrt{\frac{a^2}{x^2}-1}}{x\cdot x^3}dx$$

$$=\int\frac{1}{x^3}\cdot\sqrt{\left(\frac{a^2}{x^2}-1\right)}dx$$

Put $\frac{a^2}{x^2} - 1 = t^2$

$$\Rightarrow -\frac{2a^2}{x^3}dx = 2t\,dt$$

$$\Rightarrow \frac{1}{x^3}dx = -\frac{1}{a^2}t\,dt$$

Then, $I = -\frac{1}{a^2}\int t^2 dt$

$$= -\frac{1}{3a^2}t^3 + c$$

$$= -\frac{1}{3a^2}\left(\frac{a^2}{x^2} - 1\right)^{3/2} + c$$

$$= -\frac{(a^2 - x^2)^{3/2}}{3a^2x^3} + c$$

3 **(c)**

We have,

$$(x) = \begin{cases} x, \text{ for } x < 1 \\ x - 1 \text{ for } x \geq 1 \end{cases},$$

$$\Rightarrow x^2 f(x) = \begin{cases} x^3, & \text{for } x < 1 \\ x^3 - x^2 & \text{for } x \geq 1 \end{cases}$$

$$\Rightarrow \int_0^2 x^2 f(x)\,dx = \int_0^1 x^3 dx + \int_1^2 x^3 - x^2\,dx = \frac{5}{3}$$

4 **(d)**

$$\int \frac{x^2 + x - 6}{(x-2)(x-1)}dx = \int \frac{x+3}{x-1}dx$$

$$= \int 1\,dx + \int \frac{4}{(x-1)}dx$$

$$= x + 4\log(x-1) + c$$

5 **(b)**

We have,

$$I = \int \frac{x^3}{(1+x^2)^{1/3}}dx = \frac{1}{2}\int \frac{x^2}{(1+x^2)^{1/3}} \cdot 2x\,dx$$

$$\Rightarrow I = \frac{1}{2}\int \frac{(1+x^2) - 1}{(1+x^2)^{1/3}}d(1+x^2)$$

$$\Rightarrow I = \frac{1}{2}\int \left\{(1+x^2)^{2/3} - (1+x^2)^{-1/3}\right\}d(1+x^2)$$

$$\Rightarrow I = \frac{1}{2}\left\{\frac{3}{5}(1+x^2)^{5/3} - \frac{3}{2}(1+x^2)^{2/3}\right\} + C$$

$$\Rightarrow I = \frac{1}{2}(1+x^2)^{2/3}\left\{\frac{3}{5}(1+x^2) - \frac{2}{3}\right\} + C$$

$$\Rightarrow I = \frac{1}{20}(1+x^2)^{2/3}(6x^2 - 9) + C = \frac{3}{20}(1+x^2)^{2/3}(2x^2 - 3) + C$$

6 **(b)**

We have,

$$\int_{-\pi/4}^{\pi/4} \left\{a|\sin x| + \frac{b \sin x}{1+\cos x} + c\right\} dx = 0$$

$$\Rightarrow a \int_{-\pi/4}^{\pi/4} |\sin x|\, dx + b \int_{-\pi/4}^{\pi/4} \frac{\sin x}{1+\cos x} dx + c \int_{-\pi/4}^{\pi/4} dx = 0$$

$$\Rightarrow 2a \int_{0}^{\pi/4} |\sin x|\, dx + b \times 0 + 2c \int_{0}^{\pi/4} dx = 0$$

$$\Rightarrow 2a \int_{0}^{\pi/4} \sin x\; dx + 2c \times \frac{\pi}{4} = 0$$

$$\Rightarrow -2a\left(\cos\frac{\pi}{4} - \cos 0\right) + \frac{\pi c}{2} = 0$$

$$\Rightarrow -2a\left(\frac{1}{\sqrt{2}} - 1\right) + \frac{\pi c}{2} = 0 \Rightarrow a(2-\sqrt{2}) + \frac{\pi c}{2} = 0$$

7 **(c)**

Let $I = \int_2^4 \{|x-2| + |x-3|\} dx$

$$= \int_2^3 \{(x-2) + (3-x)\} dx + \int_3^4 \{(x-2) + (x-3)\} dx$$

$$= \int_2^3 dx + \int_3^4 (2x-5) dx$$

$$= [x]_2^3 + [x^2 - 5x]_3^4$$

$$= 3 - 2 + [16 - 20 - (9 - 15)]$$

$$= 1 + 2 = 3$$

8 **(a)**

$g(x) = \int_0^x \cos^4 t\, dt$ (given)

$$g(x+\pi) = \int_0^{\pi+x} \cos^4 t\, dt$$

$$= \int_0^{\pi} \cos^4 t\, dt + \int_{\pi}^{\pi+x} \cos^4 t\, dt = I_1 + I_2$$

$\Rightarrow I_1 = g(\pi)$

$$I_2 = \int_{\pi}^{\pi+x} \cos^4 t\, dt$$

Put $t = \pi + y \Rightarrow dt = dy$

$$I_2 = \int_0^x \cos^4(y+\pi) dy = \int_0^x [\cos(\pi+y)]^4 dy$$

$$= \int_0^x (-\cos y)^4 dy = \int_0^x \cos^4 y\, dy = g(x)$$

$\therefore\ g(x+\pi) = g(x) + g(\pi)$

9 **(b)**

We have,

$$\Rightarrow I = \int \cos^3 x\, e^{\log(\sin x)} dx = \int \cos^3 x \sin x \ dx$$

$$\Rightarrow I = -\int \cos^3 x \ d(\cos x) = -\frac{\cos^4 x}{4} + C$$

10 **(a)**

Put $x + 1 = t^2 \Longrightarrow dx = 2t\, dt$

At $x = 8, t = 3$ and $x = 15, t = 4$

$$\therefore I = \int_3^4 \frac{2t\, dt}{(t^2 - 1 - 3)t}$$

$$= \int_3^4 \frac{2dt}{(t^2 - 4)} = 2.\frac{1}{4}\left[\log\frac{t-2}{t+2}\right]_3^4$$

$$= \frac{1}{2}\left[\log\frac{1}{3} - \log\frac{1}{5}\right] = \frac{1}{2}\log\frac{5}{3}$$

11 **(a)**

Using $\sin^2 x = \frac{1-\cos x}{2}$ and $\cos^2 x = \frac{1+\cos 2x}{2}$,

We get

$$I = \int \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx = \int \frac{(1-\cos 2x)^2}{2(1+\cos^2 2x)} dx$$

$$\Rightarrow I = \frac{1}{2}\int 1 - \frac{2\cos 2x}{2 - \sin^2 2x} dx$$

$$\Rightarrow I = \frac{1}{2}\left\{x - \frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}+\sin 2x}{\sqrt{2}-\sin 2x}\right|\right\} + C$$

12 **(c)**

We have,

$$f(x) = \begin{cases} \int_{-1}^{x} -t\, dt, & -1 \le x \le 0 \\ \int_{-1}^{0} -t dt + \int_{0}^{x} t dt, & x \ge 0 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} \frac{1}{2}(1 - x^2), & -1 \le x \le 0 \\ \frac{1}{2}(1 + x^2), & x \ge 0 \end{cases}$$

13 **(b)**

Since $\sqrt{1+x^2} > x^2$ for all $x \in [1, 2]$. Therefore,

$\frac{1}{\sqrt{1+x^2}} < \frac{1}{x}$ for all $x \in [1, 2]$

$$\Rightarrow \int_1^2 \frac{1}{\sqrt{1+x^2}} dx < \int_1^2 \frac{1}{x} dx \Rightarrow I_1 < I_2$$

14 **(a)**

Let $f(x) = (1 - x^2)\sin x \cos^2 x$

$f(-x) = [1 - (-x)^2][\sin(-x)]\cos^2(-x)$

$= -(1 - x^2)\sin x \cos^2 x = -f(x)$

$\Rightarrow f(x)$ is an odd function.

$$\therefore \int_{-\pi}^{\pi} (1 - x^2)\sin x \cos^2 x \, dx = 0$$

15 **(c)**

We have,

$$I_n = \int_0^{\pi/2} x^n \sin x \, dx$$

I II

$$\Rightarrow I_n = [-x^n \cos x]_0^{\pi/2} + n \int_0^{\pi/2} x^{n-1} \cos x \, dx$$

I II

$$\Rightarrow I_n = n[x^{n-1}\sin x]_0^{\pi/2} - n(n-1) \int_0^{\pi/2} x^{n-2} \sin x \, dx$$

$$\Rightarrow I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}$$

Putting $n = 4$, we get

$$I_4 + 12I_2 = 4\left(\frac{\pi}{2}\right)^3$$

16 **(b)**

We have,

$$\int_0^2 x[x] \, dx = \int_0^1 x \times 0 \, dx + \int_1^2 x \, dx = \frac{3}{2}$$

17 **(b)**

$$\int_{-1}^{0} \frac{dx}{x^2 + 2x + 2} = \int_{-1}^{0} \frac{dx}{(x+1)^2 + 1}$$

$= [\tan^{-1}(x+1)]_{-1}^{0}$

$= [\tan^{-1} 1 - \tan^{-1} 0] = \frac{\pi}{4} - 0 = \frac{\pi}{4}$

18 **(d)**

Let $I = \int_{-\pi/4}^{\pi/4} x^3 \sin^4 x \, dx$

Since, $x^3 \sin^4 x$ is an odd function

$\therefore I = 0$

19 **(c)**

$$I_1=\int_{1-k}^{k} xf\{x(1-x)\}dx$$

$$=\int_{1-k}^{k}(1-x)f[(1-x)\{1-(1-x)\}]dx(\text{put} x=1-x)$$

$$=\int_{1-k}^{k}(1-x)f\{x(1-x)\}dx$$

$$=\int_{1-k}^{k} f\{x(1-x)\}dx-\int_{1-k}^{k} xf\{x(1-x)\}dx$$

$$=I_2-I_1$$

$$\therefore 2I_1=I_2$$

$$\Rightarrow \frac{I_1}{I_2}=\frac{1}{2}$$

20 **(a)**

We have,

$$I_n=\int_0^{\pi/4} \tan^n x\, dx$$

$$\therefore I_{n+1}+I_{n-1}=\int_0^{\pi/4}(\tan^{n+1} x+\tan^{x-1} x)\, dx$$

$$\Rightarrow I_{n+1}+I_{n-1}=\int_0^{\pi/4}(\tan^{n-1} x\sec^2 x)\, dx$$

$$\Rightarrow I_{n+1}+I_{n-1}=\left[\frac{\tan^n x}{n}\right]_0^{\pi/4}=\frac{1}{n}$$

$$\Rightarrow n(I_{n+1}+I_{n-1})=1$$

$$\Rightarrow \lim_{n\to\infty}(I_{n+1}+I_{n-1})=1$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	C	D	B	B	C	A	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	B	A	C	B	B	D	C	A