

CLASS : XII<sup>th</sup>  
DATE :

**SOLUTIONS**

SUBJECT : MATHS  
DPP NO. :4

**Topic :-INTEGRALS**

1       **(b)**

Let

$$\begin{aligned}
 I &= \int_0^{\pi/2} \operatorname{cosec}(x - \pi/3) \operatorname{cosec}(x - \pi/6) dx \\
 \Rightarrow I &= 2 \int_0^{\pi/2} \frac{\sin[(x - \pi/6) - (x - \pi/3)]}{\sin(x - \pi/6) \sin(x - \pi/3)} dx \\
 \Rightarrow I &= 2 \int_0^{\pi/2} [\cot(x - \pi/3) - \cot(x - \pi/6)] dx \\
 \Rightarrow I &= 2 \left[ \log \sin \left( x - \frac{\pi}{3} \right) - \log \sin \left( x - \frac{\pi}{6} \right) \right]_0^{\pi/2} \\
 \Rightarrow I &= 2 \left[ \log \left( \frac{\sin(x - \pi/3)}{\sin(x - \pi/6)} \right) \right]_0^{\pi/2} \\
 \Rightarrow I &= 2 \left[ \log \left( \frac{1/2}{\sqrt{3}/2} \right) - \log \left( \frac{\sqrt{3}/2}{1/2} \right) \right] \\
 \Rightarrow I &= 2[-\log \sqrt{3} - \log \sqrt{3}] = -4 \log \sqrt{3} = -2 \log 3
 \end{aligned}$$

2       **(c)**

Primitive function means indefinite integral.

∴ Primitive function of  $f(x)$

$$\begin{aligned}
 I &= \int \frac{\sqrt{(a^2 - x^2)}}{x^4} dx \quad (\text{say}) \\
 &= \int \frac{x \sqrt{\frac{a^2}{x^2} - 1}}{x \cdot x^3} dx \\
 &= \int \frac{1}{x^3} \cdot \sqrt{\left(\frac{a^2}{x^2} - 1\right)} dx
 \end{aligned}$$

$$\text{Put } \frac{a^2}{x^2} - 1 = t^2$$

$$\Rightarrow -\frac{2a^2}{x^3} dx = 2t dt$$

$$\Rightarrow \frac{1}{x^3} dx = -\frac{1}{a^2} t dt$$

$$\text{Then, } I = -\frac{1}{a^2} \int t^2 dt$$

$$= -\frac{1}{3a^2} t^3 + c$$

$$= -\frac{1}{3a^2} \left( \frac{a^2}{x^2} - 1 \right)^{3/2} + c$$

$$= -\frac{(a^2 - x^2)^{3/2}}{3a^2 x^3} + c$$

3      **(c)**

We have,

$$(x) = \begin{cases} x, & \text{for } x < 1 \\ x - 1, & \text{for } x \geq 1 \end{cases}$$

$$\Rightarrow x^2 f(x) = \begin{cases} x^3, & \text{for } x < 1 \\ x^3 - x^2, & \text{for } x \geq 1 \end{cases}$$

$$\Rightarrow \int_0^2 x^2 f(x) dx = \int_0^1 x^3 dx + \int_1^2 x^3 - x^2 dx = \frac{5}{3}$$

4      **(d)**

$$\begin{aligned} \int \frac{x^2 + x - 6}{(x-2)(x-1)} dx &= \int \frac{x+3}{x-1} dx \\ &= \int 1 dx + \int \frac{4}{(x-1)} dx \\ &= x + 4 \log(x-1) + c \end{aligned}$$

5      **(b)**

We have,

$$I = \int \frac{x^3}{(1+x^2)^{1/3}} dx = \frac{1}{2} \int \frac{x^2}{(1+x^2)^{1/3}} \cdot 2x dx$$

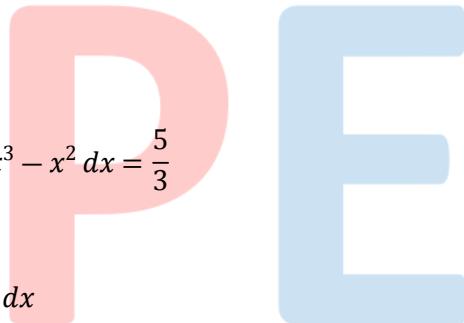
$$\Rightarrow I = \frac{1}{2} \int \frac{(1+x^2)-1}{(1+x^2)^{1/3}} d(1+x^2)$$

$$\Rightarrow I = \frac{1}{2} \int \{(1+x^2)^{2/3} - (1+x^2)^{-1/3}\} d(1+x^2)$$

$$\Rightarrow I = \frac{1}{2} \left\{ \frac{3}{5} (1+x^2)^{5/3} - \frac{3}{2} (1+x^2)^{2/3} \right\} + C$$

$$\Rightarrow I = \frac{1}{2} (1+x^2)^{2/3} \left\{ \frac{3}{5} (1+x^2) - \frac{2}{3} \right\} + C$$

$$\Rightarrow I = \frac{1}{20} (1+x^2)^{2/3} (6x^2 - 9) + C = \frac{3}{20} (1+x^2)^{2/3} (2x^2 - 3) + C$$



6       **(b)**

We have,

$$\begin{aligned} & \int_{-\pi/4}^{\pi/4} \left\{ a|\sin x| + \frac{b \sin x}{1 + \cos x} + c \right\} dx = 0 \\ \Rightarrow & a \int_{-\pi/4}^{\pi/4} |\sin x| dx + b \int_{-\pi/4}^{\pi/4} \frac{\sin x}{1 + \cos x} dx + c \int_{-\pi/4}^{\pi/4} dx = 0 \\ \Rightarrow & 2a \int_0^{\pi/4} |\sin x| dx + b \times 0 + 2c \int_0^{\pi/4} dx = 0 \\ \Rightarrow & 2a \int_0^{\pi/4} \sin x dx + 2c \times \frac{\pi}{4} = 0 \\ \Rightarrow & -2a \left( \cos \frac{\pi}{4} - \cos 0 \right) + \frac{\pi c}{2} = 0 \\ \Rightarrow & -2a \left( \frac{1}{\sqrt{2}} - 1 \right) + \frac{\pi c}{2} = 0 \Rightarrow a(2 - \sqrt{2}) + \frac{\pi c}{2} = 0 \end{aligned}$$

7       **(c)**

$$\begin{aligned} \text{Let } I &= \int_2^4 \{|x-2| + |x-3|\} dx \\ &= \int_2^3 \{(x-2) + (3-x)\} dx + \int_3^4 \{(x-2) + (x-3)\} dx \\ &= \int_2^3 dx + \int_3^4 (2x-5) dx \\ &= [x]_2^3 + [x^2 - 5x]_3^4 \\ &= 3 - 2 + [16 - 20 - (9 - 15)] \\ &= 1 + 2 = 3 \end{aligned}$$

8       **(a)**

$$g(x) = \int_0^x \cos^4 t dt \quad (\text{given})$$

$$\begin{aligned} g(x+\pi) &= \int_0^{\pi+x} \cos^4 t dt \\ &= \int_0^{\pi} \cos^4 t dt + \int_{\pi}^{\pi+x} \cos^4 t dt = I_1 + I_2 \\ \Rightarrow I_1 &= g(\pi) \end{aligned}$$

$$I_2 = \int_{\pi}^{\pi+x} \cos^4 t dt$$

Put  $t = \pi + y \Rightarrow dt = dy$

$$\begin{aligned} I_2 &= \int_0^x \cos^4(y+\pi) dy = \int_0^x [\cos(\pi+y)]^4 dy \\ &= \int_0^x (-\cos y)^4 dy = \int_0^x \cos^4 y dy = g(x) \end{aligned}$$

$$\therefore g(x + \pi) = g(x) + g(\pi)$$

9      **(b)**

We have,

$$\Rightarrow I = \int \cos^3 x e^{\log(\sin x)} dx = \int \cos^3 x \sin x \, dx$$

$$\Rightarrow I = - \int \cos^3 x \, d(\cos x) = -\frac{\cos^4 x}{4} + C$$

10      **(a)**

$$\text{Put } x + 1 = t^2 \Rightarrow dx = 2t \, dt$$

At  $x = 8, t = 3$  and  $x = 15, t = 4$

$$\begin{aligned}\therefore I &= \int_3^4 \frac{2t \, dt}{(t^2 - 1 - 3)t} \\ &= \int_3^4 \frac{2dt}{(t^2 - 4)} = 2 \cdot \frac{1}{4} \left[ \log \frac{t-2}{t+2} \right]_3 \\ &= \frac{1}{2} \left[ \log \frac{1}{3} - \log \frac{1}{5} \right] = \frac{1}{2} \log \frac{5}{3}\end{aligned}$$

11      **(a)**

$$\text{Using } \sin^2 x = \frac{1 - \cos x}{2} \text{ and } \cos^2 x = \frac{1 + \cos 2x}{2},$$

We get

$$I = \int \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx = \int \frac{(1 - \cos 2x)^2}{2(1 + \cos^2 2x)} dx$$

$$\Rightarrow I = \frac{1}{2} \int 1 - \frac{2 \cos 2x}{2 - \sin^2 2x} dx$$

$$\Rightarrow I = \frac{1}{2} \left\{ x - \frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{2} + \sin 2x}{\sqrt{2} - \sin 2x} \right| \right\} + C$$

12      **(c)**

We have,

$$f(x) = \begin{cases} \int_{-1}^x -t \, dt, & -1 \leq x \leq 0 \\ \int_{-1}^0 -tdt + \int_0^x tdt, & x \geq 0 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} \frac{1}{2}(1 - x^2), & -1 \leq x \leq 0 \\ \frac{1}{2}(1 + x^2), & x \geq 0 \end{cases}$$

13      **(b)**

Since  $\sqrt{1 + x^2} > x^2$  for all  $x \in [1, 2]$ . Therefore,

$$\frac{1}{\sqrt{1 + x^2}} < \frac{1}{x} \text{ for all } x \in [1, 2]$$



$$\Rightarrow \int_1^2 \frac{1}{\sqrt{1+x^2}} dx < \int_1^2 \frac{1}{x} dx \Rightarrow I_1 < I_2$$

14 (a)

Let  $f(x) = (1-x^2)\sin x \cos^2 x$

$$\begin{aligned} f(-x) &= [1 - (-x)^2][\sin(-x)]\cos^2(-x) \\ &= -(1-x^2)\sin x \cos^2 x = -f(x) \end{aligned}$$

$\Rightarrow f(x)$  is an odd function.

$$\therefore \int_{-\pi}^{\pi} (1-x^2)\sin x \cos^2 x dx = 0$$

15 (c)

We have,

$$I_n = \int_0^{\pi/2} x^n \sin x dx$$

I      II

$$\Rightarrow I_n = [-x^n \cos x]_0^{\pi/2} + n \int_0^{\pi/2} x^{n-1} \cos x dx$$

I      II

$$\Rightarrow I_n = n[x^{n-1} \sin x]_0^{\pi/2} - n(n-1) \int_0^{\pi/2} x^{n-2} \sin x dx$$

$$\Rightarrow I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}$$

Putting  $n = 4$ , we get

$$I_4 + 12I_2 = 4\left(\frac{\pi}{2}\right)^3$$

16 (b)

We have,

$$\int_0^2 x[x] dx = \int_0^1 x \times 0 dx + \int_1^2 x dx = \frac{3}{2}$$

17 (b)

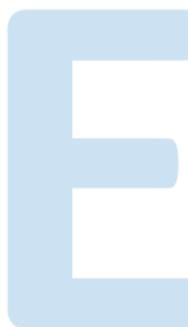
$$\begin{aligned} \int_{-1}^0 \frac{dx}{x^2 + 2x + 2} &= \int_{-1}^0 \frac{dx}{(x+1)^2 + 1} \\ &= [\tan^{-1}(x+1)]_{-1}^0 \\ &= [\tan^{-1} 1 - \tan^{-1} 0] = \frac{\pi}{4} - 0 = \frac{\pi}{4} \end{aligned}$$

18 (d)

$$\text{Let } I = \int_{-\pi/4}^{\pi/4} x^3 \sin^4 x dx$$

Since,  $x^3 \sin^4 x$  is an odd function

$$\therefore I = 0$$



19      (c)

$$\begin{aligned} I_1 &= \int_{1-k}^k xf\{x(1-x)\}dx \\ &= \int_{1-k}^k (1-x)f[(1-x)\{1-(1-x)\}]dx \text{(put } x = 1-x) \\ &= \int_{1-k}^k (1-x)f\{x(1-x)\}dx \\ &= \int_{1-k}^k f\{x(1-x)\}dx - \int_{1-k}^k xf\{x(1-x)\}dx \\ &= I_2 - I_1 \\ \therefore 2I_1 &= I_2 \\ \Rightarrow \frac{I_1}{I_2} &= \frac{1}{2} \end{aligned}$$

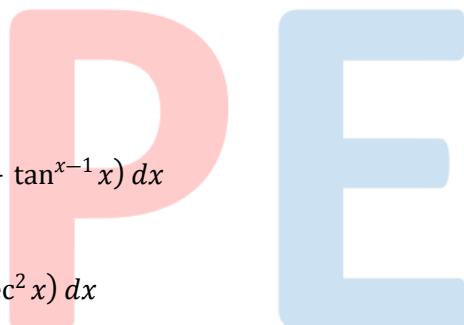
20      (a)

We have,

$$\begin{aligned} I_n &= \int_0^{\pi/4} \tan^n x dx \\ \therefore I_{n+1} + I_{n-1} &= \int_0^{\pi/4} (\tan^{n+1} x + \tan^{n-1} x) dx \\ \Rightarrow I_{n+1} + I_{n-1} &= \int_0^{\pi/4} (\tan^{n-1} x \sec^2 x) dx \\ \Rightarrow I_{n+1} + I_{n-1} &= \left[ \frac{\tan^n x}{n} \right]_0^{\pi/4} = \frac{1}{n} \end{aligned}$$

$$\Rightarrow n(I_{n+1} + I_{n-1}) = 1$$

$$\Rightarrow \lim_{n \rightarrow \infty} (I_{n+1} + I_{n-1}) = 1$$



**ANSWER-KEY**

Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	C	D	B	B	C	A	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	B	A	C	B	B	D	C	A

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