DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :2

Topic :-INTEGRALS

1 **(b)**

We have,

$$I=\int_0^{\pi}\log(1+\cos x)\,dx$$

$$\Rightarrow I=\int_0^{\pi}\log\left(2\cos^2\frac{x}{2}\right)dx$$

$$\Rightarrow I=\int_0^{\pi}\left(\log 2+2\log\cos\frac{x}{2}\right)dx$$

$$\Rightarrow I=\pi\log 2+2\int_0^{\pi}\log\cos\frac{x}{2}\,dx$$

$$\Rightarrow I=\pi\log 2+2\times 2\int_0^{\pi/2}\log\cos t\,dt,\text{ where } t=\frac{x}{2}\text{ and } dx=2dt$$

$$\Rightarrow I=\pi\log 2+4\times-\frac{\pi}{2}\log 2=-\pi\log 2$$

2 **(a)**

Putting $\tan^{-1}x=t$, we have

$$I=\int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)dx$$

$$\Rightarrow I=\int e^t(\tan t+\sec^2 t)dt=e^t\tan t+C=x\,e^{\tan^{-1}x}+C$$

3 **(c)**

Given,

$$I_1=\int_a^{\pi-a}x\,f(\sin x)\,dx$$

and $I_2=\int_a^{\pi-a}f(\sin x)\,dx$

Now, $I_1=\int_a^{\pi-a}x\,f(\sin x)\,dx$

$$=\int_a^{\pi-a}(\pi-x)f[\sin(\pi-x)]\,dx$$

$$= \int_a^{\pi-a} (\pi - x) f(\sin x)\, dx$$

$$= \int_a^{\pi-a} \pi\, f(\sin x) dx - I_1$$

$$\Rightarrow 2I_1 = \pi I_2 \Rightarrow I_2 = \frac{2}{\pi} I_1$$

4 **(b)**

$$\int \cos^{-1}\left(\frac{1}{x}\right) dx = \int \sec^{-1} x \cdot 1\, dx$$

$$= \sec^{-1} x \int dx - \int \left[\frac{d}{dx} \sec^{-1} x \int dx\right] dx$$

$$= x \sec^{-1} x - \int \frac{1}{x\sqrt{x^2 - 1}} x\, dx$$

$$= x \sec^{-1} x - \int \frac{1}{\sqrt{x^2 - 1}}\, dx$$

$$= x \sec^{-1} x - \cosh^{-1} x + c$$

5 **(d)**

Let $I = \int_0^1 \frac{dx}{x + \sqrt{1 - x^2}}$

Put $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

$I = \int_0^{\pi/2} \frac{\cos\theta\, d\theta}{\sin\theta + \cos\theta}$...(i)

$$\Rightarrow I = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2} - \theta\right) d\theta}{\sin\left(\frac{\pi}{2} - \theta\right) + \cos\left(\frac{\pi}{2} - \theta\right)}$$

$= \int_0^{\pi/2} \frac{\sin\theta}{\cos\theta + \sin\theta} d\theta$...(ii)

On adding Eqs.(i) and (ii), we get

$$2I = \int_0^{\pi/2} 1\, d\theta \Rightarrow I = \frac{\pi}{4}$$

6 **(b)**

Let

$$I = \int_0^{\sqrt{n}} [x^2] dx$$

$$\Rightarrow I = \sum_{r=1}^{n} \int_{\sqrt{r-1}}^{\sqrt{r}} [x^2]\, dx$$

$$\Rightarrow I = \sum_{r=1}^{n} \int_{\sqrt{r-1}}^{\sqrt{r}} (r - 1)\, dx$$

$$\Rightarrow I = \sum_{r=1}^{n} (r - 1)(\sqrt{r} - \sqrt{r - 1})$$

$\Rightarrow I = (\sqrt{2} - \sqrt{1}) + 2(\sqrt{3} - \sqrt{2}) + 3(\sqrt{4} - \sqrt{3})$
$+ \ldots + (n-1)(\sqrt{n} - \sqrt{n-1})$

$\Rightarrow I = n\sqrt{n} - (\sqrt{1} + \sqrt{2} + \ldots + \sqrt{n}) = n\sqrt{n} - \sum_{r=1}^{n} \sqrt{r}$

7 **(d)**

Let $I = \int \frac{1}{x}(\log_{ex} e) dx = \int \frac{1}{x(1 + \log_e x)} dx$

Put $\log_e x = t \Rightarrow \frac{1}{x} dx = dt$

$\therefore\ I = \int \frac{dt}{(1+t)} = \log_e(1+t) + c$

$= \log_e(1 + \log_e x) + c$

8 **(a)**

$I_1 - I_2 = \int_0^{\pi/2} (\cos\theta - \sin 2\theta) f(\sin\theta + \cos^2\theta) d\theta$

Put $\sin\theta + \cos^2\theta = t$

$\Rightarrow (\cos\theta - \sin 2\theta) d\theta = dt$

Then, $I_1 - I_2 = \int_1^1 f(t) dt = 0$

$\therefore\ I_1 = I_2$

9 **(b)**

(1)$I = \int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x}\, dx$

$= \int_{-\pi/2}^{\pi/2} \sqrt{\cos x} \sin x\, dx$

$= 2\int_0^{\pi/2} \sqrt{\cos x} \sin x\, dx$

$= -\frac{4}{3}[\cos^{3/2} x]_0^{\pi/2} = \frac{4}{3}$

(2)$I = \int_0^1 |x-1| dx + \int_1^4 |x-1| dx + \int_0^3 |x-3| dx + \int_3^4 |x-3| dx$

$= \int_0^1 -(x-1) dx + \int_1^4 (x-1) dx + \int_0^3 -(x-3) dx + \int_3^4 (x-3) dx$

$= 10$

10 **(a)**

Let $I = \int_{-\pi/4}^{\pi/4} \sin^{-4} x\, dx = \int_{-\pi/4}^{\pi/4} \text{cosec}^{-4} x\, dx$

$$= \int_{-\pi/4}^{\pi/4} (1 + \cot^2 x)\, \text{cosec}^2 x\, dx$$

Put $\cot x = t$

$\Rightarrow -\text{cosec}^2 x\, dx = dt$

$\therefore\ I = -\int_{-1}^{1} (1 + t^2)\, dt = -2\int_0^1 (1 + t^2)\, dt$

$$= -2\left[t + \frac{t^3}{3}\right]_0^1 = -2\left[1 + \frac{1}{3}\right] = -\frac{8}{3}$$

11 **(d)**

We have,

$$\lim_{x\to\infty} \frac{\left(\int_0^x e^x\,dx\right)^2}{\int_0^x e^{2x^2}\,dx} = \lim_{x\to\infty} \frac{(e^x - 1)^2}{\int_0^x e^{2x^2}\,dx}$$

$$= \lim_{x\to\infty} \frac{2(e^x - 1)e^x}{\int_0^x e^{2x^2}}$$ [Using L'Hospital's rule]

$$= 2 \lim_{x\to\infty} \frac{e^x - 1}{e^{2x^2 - x}}$$

$$= 2 \lim_{x\to\infty} \frac{e^x}{e^{2x^2 - x}(4x - 1)}$$ [Using L'Hospital's rule]

$$= 2 \lim_{x\to\infty} \frac{1}{e^{2x^2 - 2x}(4x - 1)} = 0$$

12 **(a)**

Given, $\int_{\sin x}^{1} t^2 f(t)dt = 1 - \sin x$

Now, $\frac{d}{dx}\int_{\sin x}^{1} t^2 f(t)dt = \frac{d}{dx}(1 - \sin x)$

$\Rightarrow [1^2 f(1)].(0) - (\sin^2 x).f(\sin x).\cos x = -\cos x$

[by Leibnitz formula]

$\Rightarrow$ Put $\sin x = 1/\sqrt{3}$

$\therefore f\left(\frac{1}{\sqrt{3}}\right) = (\sqrt{3})^2 = 3$

13 **(b)**

We have,

$f(a - x) + f(a + x) = 0$

$\Rightarrow f(2a - x) + f(x) = 0$ [On replacing $x$by $x - a$]

$\Rightarrow f(2a - x) = -f(x)$

$$\because \int_0^{2a} f(x)dx = \int_0^a \{f(x) + f(2a - x)\}\,dx$$

$$\therefore \int_0^{2a} f(x)\,dx = \int_0^a \{f(x) - f(x)\}\,dx = 0$$

14 **(a)**

$$\int \frac{dx}{\sin(x - a)\sin(x - b)}$$

$$= \frac{1}{\sin(a - b)} \int \frac{\sin\{(x - b) - (x - a)\}}{\sin(x - a)\sin(x - b)}\,dx$$

$$= \frac{1}{\sin(a - b)}$$

$$\int \frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)}\,dx$$

$$=\frac{1}{\sin(a-b)}\left[\int \cot(x-a)dx-\int \cot(x-b)dx\right]$$

$$=\frac{1}{\sin(a-b)}\left[\log\sin(x-a)-\log\sin(x-b)\right]+c$$

$$=\frac{1}{\sin(a-b)}\log\left|\frac{\sin(x-a)}{\sin\,(x-b)}\right|+c$$

15 **(b)**

Putting $2+x=t^2$, we get

$$I=\int\sqrt{\frac{5-x}{2+x}}\,dx=2\int\sqrt{7-t^2}\,\,dt$$

$$\Rightarrow I=t\sqrt{7-t^2}+7\sin^{-1}\frac{t}{\sqrt{7}}+C$$

$$\Rightarrow I=\sqrt{x+2}\sqrt{5-x}+7\sin^{-1}\frac{\sqrt{x+2}}{7}+C$$

16 **(a)**

We have,

$$I=\int\frac{2x^2+3}{(x^2-1)(x^2+4)}dx=\int\frac{1}{x^2-1}dx+\int\frac{1}{x^2+4}dx$$

$$\Rightarrow I=\frac{1}{2}\log\left(\frac{x-1}{x+1}\right)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+C$$

$$\Rightarrow I=-\frac{1}{2}\log\left(\frac{x+1}{x-1}\right)+\frac{1}{2}\tan^{-1}x+C$$

$\therefore a=-1/2 \text{and} b=1/2$

17 **(c)**

$$\int e^x\frac{(x-1)}{x^2}\,dx$$

$$=\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$$

$$=\frac{e^x}{x}+c$$

18 **(d)**

We have,

$$I=\int_0^{3\alpha}\operatorname{cosec}(x-\alpha)\operatorname{cosec}(x-2\alpha)dx$$

$$\Rightarrow I=\frac{1}{\sin\alpha}\int_0^{3\alpha}\frac{\sin\alpha}{\sin(x-\alpha)\,\sin(x-2\alpha)}dx$$

$$\Rightarrow I=\frac{1}{\sin\alpha}\int_0^{3\alpha}\frac{\sin\{(x-\alpha)-(x-2\alpha)\}}{\sin(x-\alpha)\,\sin(x-2\alpha)}dx$$

$$\Rightarrow I = \frac{1}{\sin\alpha}\int_0^{3\alpha} \cot(x-2\alpha) - \cot(x-\alpha)\,dx$$

$$\Rightarrow I = \frac{1}{\sin\alpha}\left[\log\frac{\sin(x-2\alpha)}{\sin(x-\alpha)}\right]_0^{3\alpha}$$

$$\Rightarrow I = \frac{1}{\sin\alpha}\left[\log\frac{\sin\alpha}{\sin 2\alpha} - \log\frac{\sin 2\alpha}{\sin\alpha}\right]$$

$$\Rightarrow I = \frac{1}{\sin\alpha}\left[\log\left(\frac{\sin\alpha}{2\sin\alpha\cos\alpha}\right)\right] = \frac{2}{\sin\alpha}\log\left(\frac{1}{2}\sec\alpha\right)$$

$$\Rightarrow I = 2\ \mathrm{cosec}\,\alpha\log\left(\frac{1}{2}\sec\alpha\right)$$

19 **(a)**

$$\int_0^3 \frac{3x+1}{x^2+9}\,dx = \frac{3}{2}\int_0^3 \frac{2x}{x^2+9}\,dx + \int_0^3 \frac{1}{x^2+9}\,dx$$

$$= \frac{3}{2}\left[\log\left(x^2+9\right)\right]_0^3 + \frac{1}{3}\left[\tan^{-1}\frac{x}{3}\right]_0^3$$

$$= \frac{3}{2}[\log 18 - \log 9] + \frac{1}{3}[\tan^{-1}(1) - \tan^{-1}(0)]$$

$$= \frac{3}{2}[\log 2] + \frac{\pi}{12}$$

$$= \log(2\sqrt{2}) + \frac{\pi}{12}$$

20 **(a)**

Let $I = \int \frac{dx}{\sqrt{(1-x)(x-2)}}$

$$= \int \frac{dx}{\sqrt{-x^2+3x-2)}} = \int \frac{dx}{\sqrt{\frac{1}{4} - \left(x-\frac{3}{2}\right)^2}}$$

$$= \sin^{-1}\left(\frac{\left(x-\frac{3}{2}\right)}{\frac{1}{2}}\right) + c = \sin^{-1}(2x-3) + c$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	C	B	D	B	D	A	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	A	B	A	B	A	C	D	A	A