DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth | **SOLUTIONS** | SUBJECT : MATHS

DATE : | | DPP NO. :10

Topic :-INTEGRALS

1 **(c)**

Let $I=\int_{-\pi/2}^{\pi/2}(x^3+x\cos x+\tan^5 x+1)dx$

Since, x^3,$x\cos x$ and $\tan^5 x$ are odd functions,

therefore

$$I=\int_{-\pi/2}^{\pi/2}1dx=[x]_{-\pi/2}^{\pi/2}=\pi$$

2 **(a)**

$\because f'(x)=g(x)$

$$\Rightarrow \int_a^b f(x)g(x)dx=\int_a^b f(x)f'(x)dx$$

$$=\left[\frac{(f(x))^2}{2}\right]_a^b$$

$$=\frac{1}{2}[(f(b))^2-(f(a))^2]$$

3 **(c)**

We have,

$$I=\int\frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}dx=\int\frac{4e^{2x}+6}{9e^{2x}-4}dx$$

$$\Rightarrow I=\int\frac{4\,e^{2x}}{9\,e^{2x}-4}dx+6\int\frac{1}{9\,e^{2x}-4}dx$$

$$\Rightarrow I=\frac{2}{9}\int\frac{18\,e^{2x}}{9e^{2x}-4}dx+6\int\frac{e^{-2x}}{9-4e^{-2x}}dx$$

$$\Rightarrow I=\frac{2}{9}\log(9e^{2x}-4)+\frac{6}{8}\log(9-4e^{-2x})+C$$

$$\Rightarrow I=\frac{2}{9}\log(9\,e^{2x}-4)+\frac{3}{4}\log(9\,e^{2x}-4)-\frac{3}{4}\log e^{2x}+C$$

$$\Rightarrow I=\frac{-3}{2}x+\frac{35}{36}\log(9e^{2x}-4)+C$$

Hence, $A=-\frac{3}{2}$, $b=\frac{35}{36}$ and $C\in R$

4 **(b)**

Let $I=\int \frac{\cos 4x+1}{\cot x-\tan x}dx$

$$=\int \frac{2\cos^2 2x}{\frac{\cos^2 x-\sin^2 x}{\sin x\cos x}}dx$$

$$=\int \frac{2\cos^2 2x}{\cos 2x}.\sin x\cos x\,dx$$

$$=\int \cos 2x.\sin 2x\ \ dx$$

$$=\frac{1}{2}\int 2\cos 2x\sin 2x\ \ dx$$

$$=\frac{1}{2}\int \sin 4x\ \ dx=\frac{1}{2}\frac{(-\cos 4x)}{4}+c$$

$$\Rightarrow I=\frac{-\cos 4x}{8}+c=k\cos 4x+c$$

$$\therefore\ k=-\frac{1}{8}$$

5 **(c)**

Let $I=\int_{-2}^{4}|x+1|dx$

$$=\int_{-2}^{-1}-(x+1)dx+\int_{-1}^{4}(1+x)dx$$

$$=-\left[\frac{x^2}{2}+x\right]_{-2}^{-1}+\left[x+\frac{x^2}{2}\right]_{-1}^{4}$$

$$=-\left[\frac{1}{2}-1-(2-2)\right]+\left[4+8—\left(1+\frac{1}{2}\right)\right]$$

$$=\frac{1}{2}+\left(\frac{25}{2}\right)=13$$

6 **(c)**

$$\int_{2}^{3}\frac{dx}{x^2-x}$$

$$=\int_{2}^{3}\frac{dx}{x(x-1)}=\int_{2}^{3}\left[\frac{1}{x-1}-\frac{1}{x}\right]dx$$

$$=[\log(x-1)]_2^3-[\log x]_2^3$$

$$=[\ \log 2-\log 1]-[\ \log 3-\log 2]$$

$$=2\log 2-\log 3=\ \log\frac{4}{3}$$

7 **(d)**

Let $I=\int \frac{x^2+1}{x^4-x^2+1}dx=\int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}-1}dx$

$$= \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+1} dx$$

Put $x-\frac{1}{x}=t \Rightarrow \left(1+\frac{1}{x^2}\right)dx = dt$

$$\therefore I = \int \frac{dt}{t^2+1} = \tan^{-1} t + c$$

$$= \tan^{-1}\left(x-\frac{1}{x}\right) + c$$

$$= \tan^{-1}\left(\frac{x^2-1}{x}\right) + c$$

8 **(c)**

Let $I = \int e^x \sec x dx + \int e^x \sec x \tan x dx$

$$= e^x \sec x - \int e^x \sec x \tan x \, dx + \int e^x \sec x \tan x \, dx + c$$

$$= e^x \sec x + c$$

9 **(d)**

$f'(x) = 2xe^{-(x^2+1)^2} - 2xe^{-x^4}$

$= 2x\left(e^{-(x^2+1)^2} - e^{-x^4}\right)\left[\text{here} e^{-x^4} > e^{-(x^2+1)^2}\right]$

So, $f'(x) > 0$, when $x < 0$ $\left[\text{as} x^4 < (x^2+1)^2\right]$

11 **(b)**

Given, $\int_0^a x \, dx \le a + 4$

$$\Rightarrow \left[\frac{x^2}{2}\right]_0^a \le a+4 \Rightarrow \frac{a^2}{2} \le a+4$$

$\Rightarrow a^2 \le 2a + 8 \Rightarrow a^2 - 2a - 8 \le 0$

$\Rightarrow (a-4)(a+2) \le 0 \Rightarrow -2 \le a \le 4$

12 **(a)**

Given, $u_n = \int_0^{\pi/4} \tan^n x \, dx$

or $u_n = \int_0^{\pi/4} \tan^{n-2} x \,(\sec^2 x - 1)\, dx$

$$= \int_0^{\pi/4} \tan^{n-2} x \sec^2 x \, dx - \int_0^{\pi/4} \tan^{n-2} x \, dx$$

On putting $\tan x = t$, $\sec^2 x \, dx = dt$ in Ist integral, we get

$$u_n = \int_0^1 t^{n-2} \, dt - u_{n-2}$$

$$\Rightarrow u_n + u_{n-2} = \left[\frac{t^{n-1}}{n-1}\right]_0^1 = \left[\frac{1^{n-1}}{n-1} - 0\right] = \frac{1}{n-1}$$

13 **(b)**

Let $I = \int_0^\pi x \sin^4 x \, dx$...(i)

$I=\int_0^{\pi}(\pi-x)\sin^4 x\,dx$...(ii)

On adding Eqs. (i) and (ii), we get

$$2I=\pi\int_0^{\pi}\sin^4 x\ dx$$

$$=2\pi\int_0^{\pi/2}\sin^4 x\ dx$$

$$=2\pi.\frac{4-1}{4}.\frac{4-3}{4-2}.\frac{\pi}{2}$$

$$=2\pi.\frac{3}{4}.\frac{1}{2}.\frac{\pi}{2}=\frac{3\pi^2}{8}$$

$$\Rightarrow I=\frac{3\pi^2}{16}$$

14 **(a)**

We have,

$$\int\frac{\sin x-\cos x}{\sqrt{1-\sin 2x}}e^{\sin x}\cos x\ dx$$

$$\Rightarrow I=\int\frac{\sin x-\cos x}{\sin x-\cos x}e^{\sin x}\cos x\ dx=\int e^{\sin x}\cos x\ dx=e^{\sin x}+C$$

15 **(c)**

Let $I=\int\frac{dx}{\sqrt{x}+\sqrt[3]{x}}$

Now, let $x=t^6\Rightarrow dx=6t^5dt$

$$\Rightarrow I=\int\frac{6t^5}{t^3+t^2}dt=6\int\frac{t^3}{t+1}dt$$

$$=6\int\left(t^2-t+1-\frac{1}{t+1}\right)dt$$

$$=2t^3-3t^2+6t-6\log(t+1)+c$$

$$=2\sqrt{x}-3(\sqrt[3]{x})+6(\sqrt[6]{x})-6\log(\sqrt[6]{x}+1)+c$$

16 **(a)**

We have,

$$I=\int\sqrt[3]{x}\sqrt[7]{1+\sqrt[3]{x^4}}dx=\frac{3}{4}\int\sqrt[7]{1+x^{4/3}}\times\frac{4}{3}x^{1/3}dx$$

$$\Rightarrow I=\frac{3}{4}\int(1+x^4)^{1/7}\,d(1+x^{4/3})$$

$$\Rightarrow I=\frac{3}{4}\times\frac{7}{8}\times(1+x^{4/3})^{8/7}+C=\frac{21}{32}(1+x^{4/3})^{8/7}+C$$

17 **(a)**

Let $I=\int\sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

Put $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

$\therefore \quad I = \int \sin^{-1}(\sin 2\theta).\sec^2\theta\, d\theta$

$= 2\int \theta \sec^2\theta\, d\theta = 2[\theta\tan\theta - \int \tan\theta\, d\theta]$

$= 2[\theta\tan\theta + \log\cos\theta] + c$

$= 2\left[x\tan^{-1}x + \log\frac{1}{\sqrt{1+x^2}}\right] + c$

$= 2x\tan^{-1}x - \log(1+x^2) + c$

$= f(x) - \log(1+x^2) + c \quad$ [given]

$\therefore f(x) = 2x\tan^{-1}x$

18 **(b)**

Given $I_{10} = \int_0^{\pi/2} x^{10}\sin x dx$

$= [-x^{10}\cos x]_0^{\pi/2} + \int_0^{\pi/2} 10x^9.\cos x dx$

$= 0 + [10x^9\sin x]_0^{\pi/2} - \int_0^{\pi/2} 90x^8\sin x\, dx$

$= 10\left(\frac{\pi}{2}\right)^9 - 90I_8$

$\Rightarrow I_{10} + 90I_8 = 10\left(\frac{\pi}{2}\right)^9$

19 **(d)**

Let $I = \int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$...(i)

and $I = \int_0^{\pi} \frac{(\pi - x)dx}{a^2\cos^2(\pi - x) + b^2\sin^2(\pi - x)}$

$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)dx}{a^2\cos^2 x + b^2\sin^2 x}$...(ii)

On adding Eqs.(i) and (ii), we get

$2I = \int_0^{\pi} \frac{(x + \pi - x)dx}{a^2\cos^2 x + b^2\sin^2 x}$

$\Rightarrow 2I = \pi\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$

$\Rightarrow I = \frac{\pi}{2}\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$

$\Rightarrow I = 2\cdot\frac{\pi}{2}\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$

On dividing numerator and denominator be $\cos^2 x$, we get

$I = \pi\int_0^{\pi/2} \frac{\sec^2 x\, dx}{a^2 + b^2\tan^2 x}$

Put $b\tan x = t \Rightarrow b\sec^2 x\, dx = dt$

$$\therefore\ I = \frac{\pi}{b}\int_0^{\infty}\frac{dt}{a^2+t^2} = \frac{\pi}{b}\cdot\frac{1}{a}\left[\tan^{-1}\frac{t}{a}\right]_0^{\infty}$$

$$= \frac{\pi}{ab}\left[\frac{\pi}{2}-0\right] = \frac{\pi^2}{2ab}$$

20 **(a)**

$$\int_{-1}^{1}(x-[x])dx = \int_{-1}^{0}(x+1)dx + \int_{0}^{1}(x-0)dx$$

$$= \left[\frac{x^2}{2}+x\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}+\frac{1}{2} = 1$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	C	B	C	C	D	C	D	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	B	A	C	A	A	B	D	A