

DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :1

Topic :-INTEGRALS

1 (a)

We have,

$$f(x) = \int_0^x |x-2| dx \Rightarrow f'(x) = |x-2|$$

Clearly, $f'(x)$ is everywhere continuous and differentiable except at $x = 2$

2 (b)

If $f(t)$ is an odd function, then $\int_0^x f(t)dt$ is an even function

3 (c)

$$\int \frac{\sec x \operatorname{cosec} x}{2 \cot x - \sec x \operatorname{cosec} x} dx$$

$$= \int \frac{\frac{1}{\cos x \sin x}}{\frac{2 \cos x}{\sin x} - \frac{1}{\sin x \cos x}} dx$$

$$= \int \frac{dx}{2 \cos^2 x - 1}$$

$$= \int \frac{dx}{\cos 2x} = \int \sec 2x dx$$

$$= \frac{1}{2} \log |\sec 2x + \tan 2x| + c$$

4 (d)

$$\text{Let } I = \int_0^\pi \frac{\theta \sin \theta}{1 + \cos^2 \theta} d\theta \dots(i)$$

$$\Rightarrow I = \int_0^\pi \frac{(\pi - \theta) \sin \theta}{1 + \cos^2 \theta} d\theta \dots(ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^\pi \frac{\pi \sin \theta}{1 + \cos^2 \theta} d\theta$$

$$\text{Put } \cos \theta = t \Rightarrow -\sin \theta d\theta = dt$$

$$\therefore 2I = -\pi \int_1^{-1} \frac{1}{1+t^2} dt = 2\pi \int_0^1 \frac{1}{1+t^2} dt$$

$$= 2\pi[\tan^{-1} t]_0^1 = 2\pi \cdot \frac{\pi}{4}$$

$$\Rightarrow I = \frac{\pi^2}{4}$$

5 (d)

$$\text{Let } f(\theta) = \left[\int_0^{\sin^2 \theta} \sin^{-1} \sqrt{\phi} d\phi + \int_0^{\cos^2 \theta} \cos^{-1} \sqrt{\phi} d\phi \right]$$

$$f'(\theta) = \left(\frac{d}{d\theta} \sin^2 \theta \right) [\sin^{-1} \sqrt{\sin^2 \theta}] + \left(\frac{d}{d\theta} \cos^2 \theta \right) [\cos^{-1} \sqrt{\cos^2 \theta}]$$

$$(2 \sin \theta \cos \theta) \theta - (2 \sin \theta \cos \theta) \theta = 0$$

$$\therefore f(\theta) = \text{constant} = a \quad [\text{say}]$$

$$\therefore f\left(\frac{\pi}{4}\right) = a$$

$$\Rightarrow \int_0^{1/2} (\sin^{-1} \sqrt{\phi} + \cos^{-1} \sqrt{\phi}) d\phi = a$$

$$\Rightarrow \frac{\pi}{2} [\phi]_0^{1/2} = a \Rightarrow \frac{\pi}{4} = a$$

6 (c)

$$\text{Let } I = \int_0^{\pi/2} \frac{dx}{1 + \tan^3 x} = \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x + \cos^3 x} dx \dots (i)$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\cos^3 \left(\frac{\pi}{2} - x\right)}{\sin^3 \left(\frac{\pi}{2} - x\right) + \cos^3 \left(\frac{\pi}{2} - x\right)} dx$$

$$I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx \dots (ii)$$

On adding Eqs.(i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin^3 x + \cos^3 x} dx$$

$$= \int_0^{\pi/2} 1 dx = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

7 (c)

$$\text{Since, } I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx < \int_0^1 \frac{x}{\sqrt{x}} dx$$

Because in $x \in (0,1), x > \sin x$

$$I < \int_0^1 \sqrt{x} dx = \frac{2}{3} [x^{3/2}]_0^1 \Rightarrow I < \frac{2}{3}$$

For $x \in (0,1)$, $\frac{\cos x}{\sqrt{x}} < \frac{1}{\sqrt{x}}$

And $J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx < \int_0^1 x^{-\frac{1}{2}} dx = 2 \Rightarrow J < 2$

8 (c)

$$\begin{aligned} & \int_{-1}^1 |x| dx + \int_1^3 |x-2| dx \\ &= 2 \int_0^1 x dx + \int_1^2 (2-x) dx + \int_2^3 (x-2) dx \\ &= 2 \times \frac{1}{2} + \left[2x - \frac{x^2}{2} \right]_1^2 + \left[\frac{x^2}{2} - 2x \right]_2^3 \\ &= 1 + \frac{1}{2} + \frac{1}{2} = 2 \end{aligned}$$

9 (a)

Putting $x^n + 1 = t$ and $n x^{n-1} dx = dt$, we get

$$\begin{aligned} I &= \int \frac{1}{x(x^n + 1)} dx = \frac{1}{n} \int \frac{1}{t(t-1)} dt = \frac{1}{n} \int \left(\frac{1}{t-1} - \frac{1}{t} \right) dt \\ \Rightarrow I &= \frac{1}{n} \log \left(\frac{t-1}{t} \right) + C = \frac{1}{n} \log \left(\frac{x^n}{x^n + 1} \right) + C \end{aligned}$$

10 (d)

We have,

$$\begin{aligned} I &= \int_0^{1/2} |\sin \pi x| dx = \frac{1}{\pi} \int_0^{\pi/2} |\sin t| dt, \text{ where } t = \pi x \\ \Rightarrow I &= \frac{1}{\pi} \int_0^{\pi/2} \sin t dt = \frac{1}{\pi} \end{aligned}$$

12 (c)

We know that,

$$\left| \int_a^b f(x)g(x) dx \right| \leq \sqrt{\int_a^b f^2(x) dx} \cdot \sqrt{\int_a^b g^2(x) dx}$$

Putting $f(x) = 1$ and $g(x) = \sqrt{1+x^3}$, we get

$$\begin{aligned} I \int_0^1 \sqrt{1+x^3} dx &\leq \sqrt{\int_0^1 1 dx} \cdot \sqrt{\int_0^1 1+x^3 dx} = \sqrt{1 + \frac{1}{4}} = \frac{\sqrt{5}}{2} \\ \Rightarrow I &< 2 \text{ and } I < \frac{\sqrt{7}}{2} \end{aligned}$$

13 (b)

We have,

$$I = \frac{1}{c} \int_{ac}^{bc} f\left(\frac{x}{c}\right) dx$$

Putting $\frac{x}{c} = t$ and $dx = c dt$, we get

$$I = \frac{1}{c} \int_a^b f(t) c dt = \int_a^b f(t) dt = \int_a^b f(x) dx$$

14 (c)

$$\begin{aligned} \int e^x \left(\frac{1-x}{1+x^2} \right)^2 dx &= \int e^x \frac{(1+x^2-2x)}{(1+x^2)^2} dx \\ &= \int e^x \left(\frac{1}{1+x^2} - \frac{2x}{(1+x^2)^2} \right) dx \\ &= e^x \cdot \frac{1}{1+x^2} + \int \frac{2xe^x}{(1+x^2)^2} dx - \int \frac{2xe^x}{(1+x^2)^2} dx \\ &= \frac{e^x}{1+x^2} + c \end{aligned}$$

15 (a)

We have,

$$\begin{aligned} \frac{d}{dx}(f(x)) &= \frac{e^{\sin x}}{x} \Rightarrow \int \frac{e^{\sin x}}{x} dx = F(x) \\ \therefore \int_1^4 \frac{3}{x} e^{\sin x^3} dx &= \int_1^4 \frac{e^{\sin x^3}}{x^3} \cdot 3x^2 dx = \int_1^{64} \frac{e^{\sin x^3}}{x^3} d(x^3) \\ &= F(64) - F(1) \end{aligned}$$

Hence, $k = 64$

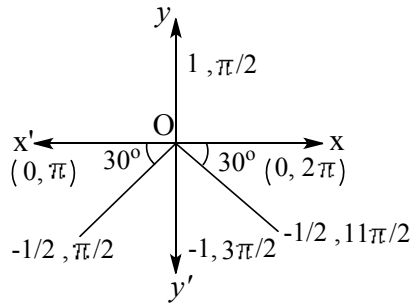
17 (a)

We have,

$$\begin{aligned} I &= \int \frac{\log(x+1) - \log x}{x(x+1)} dx = \int \frac{\log(1+1/x)}{(x^2+x)} dx \\ \Rightarrow I &= \int \frac{\log(1+1/x)}{(x^2(1+1/x))} dx = - \int \log\left(1 + \frac{1}{x}\right) d\left\{\log\left(1 + \frac{1}{x}\right)\right\} \\ \Rightarrow I &= -\frac{1}{2} \left\{ \log\left(1 + \frac{1}{x}\right) \right\}^2 + C \\ \Rightarrow I &= -\frac{1}{2} [\log(x+1) - \log x]^2 + C \\ \Rightarrow I &= -\frac{1}{2} [\{\log(x+1)\}^2 + (\log x)^2 - 2 \log(x+1) \cdot \log x] + C \\ \Rightarrow I &= -\frac{1}{2} \{\log(x+1)\}^2 - \frac{1}{2} (\log x)^2 + \log(x+1) \cdot \log x + C \end{aligned}$$

18 (a)

It is a question of greatest integer function. We have, subdivide the interval π to 2π as under keeping in view that we have to evaluate $[2\sin x]$



We know that, $\sin \frac{\pi}{6} = \frac{1}{2}$

$$\sin\left(\pi + \frac{\pi}{6}\right) = \sin \frac{7\pi}{6} = -\frac{1}{2}$$

$$\sin \frac{11\pi}{6} = \sin\left(2\pi - \frac{\pi}{6}\right) = -\sin \frac{\pi}{6} = -\frac{1}{2}$$

$$\sin \frac{9\pi}{6} = \sin \frac{3\pi}{6} = -1$$

Hence, we divide the interval π to 2π as $\left(\pi, \frac{7\pi}{6}\right), \left(\frac{7\pi}{6}, \frac{11\pi}{6}\right), \left(\frac{11\pi}{6}, 2\pi\right)$

$$\sin x = \left(0, -\frac{1}{2}\right), \left(-1, -\frac{1}{2}\right), \left(0, -\frac{1}{2}\right)$$

$$2 \sin x = (0, -1), (-2, -1), (0, -1)$$

$$[2 \sin x] = -1$$

$$\therefore I = I_1 + I_2 + I_3$$

$$= \int -1 dx + \int -2 dx + \int -1 dx$$

Between proper limits

$$= -\frac{\pi}{6} - 2\left(\frac{4\pi}{6}\right) - \frac{\pi}{6} = -\frac{10\pi}{6} = -\frac{5\pi}{3}$$

19 (c)

$$\int_0^1 \frac{d}{dx} \left[\sin^{-1} \left(\frac{2x}{1+x^2} \right) \right] dx$$

$$= \left[\sin^{-1} \left(\frac{2x}{1+x^2} \right) \right]_0^1 = \sin^{-1}(1) - \sin^{-1}(0) = \frac{\pi}{2}$$

20 (c)

$$\text{Let } F(x) = e^x$$

$$\text{Also, } f(x) + g(x) = x^2 \Rightarrow g(x) = x^2 - e^x$$

$$\text{Now, } \int_0^1 f(x)g(x)dx = \int_0^1 e^x(x^2 - e^x)dx$$

$$= \int_0^1 x^2 e^x dx - \int_0^1 e^{2x} dx$$

$$= [x^2 e^x - \int 2x e^x dx]_0^1 - \frac{1}{2} [e^{2x}]_0^1$$

$$= [x^2 e^x - 2x e^x + 2e^x]_0^1 - \frac{1}{2} (e^2 - 1)$$

$$\begin{aligned} &= [(1 - 2 + 2)e^1 - (0 - 0 + 2)e^0] - \frac{1}{2}e^2 + \frac{1}{2} \\ &= e - \frac{e^2}{2} - \frac{3}{2} \end{aligned}$$

PE

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	C	D	D	C	C	C	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	B	C	A	A	A	A	C	C

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