DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :9

Topic :-DIFFERENTITATION

1 **(a)**

$$y=\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\times\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}+\sqrt{1+\sin x}}$$

$$=\frac{2(1+\cos x)}{-2\sin x}=-\cot\frac{x}{2}$$

$$\therefore\frac{dy}{dx}=\frac{1}{2}\text{cosec}^2\frac{x}{2}$$

2 **(a)**

Since, $y=x^{\sin x}+\sqrt{x}$

Let $y_1=x^{\sin x}$ and $y_2=\sqrt{x}$

Now, $y_1=x^{\sin x}\Rightarrow\log y_1=\sin x\log x$

On differentiating w.r.t. x, we get

$$\frac{1}{y_1}.\frac{dy_1}{dx}=\cos x\log x+\frac{1}{x}\sin x$$

$$\Rightarrow\frac{dy_1}{dx}=x^{\sin x}[\cos x\log x+\frac{1}{x}\sin x]$$

$$\Rightarrow\left(\frac{dy_1}{dx}\right)_{x=\frac{\pi}{2}}=\left(\frac{\pi}{2}\right)^{\sin\frac{\pi}{2}}\left[\cos\frac{\pi}{2}\log\frac{\pi}{2}+\frac{2}{\pi}\sin\frac{\pi}{2}\right]$$

$$=\frac{\pi}{2}\times\frac{2}{\pi}=1$$

Now, $y_2=\sqrt{x}\Rightarrow\frac{dy_2}{dx}=\frac{1}{2\sqrt{x}}$

$$\Rightarrow\left(\frac{dy_2}{dx}\right)_{x=\frac{\pi}{2}}=\frac{1}{2\sqrt{\frac{\pi}{2}}}=\sqrt{\frac{1}{2\pi}}$$

Since, $y=y_1+y_2$

$$\therefore\text{ At }x=\frac{\pi}{2},\quad\frac{dy}{dx}=\frac{dy_1}{dx}+\frac{dy_2}{dx}\Rightarrow\frac{dy}{dx}=1+\frac{1}{\sqrt{2\pi}}$$

3 **(a)**

Since, $y=\frac{3at^2}{1+t^2}$ and $x=\frac{3at}{1+t^3}$

On differentiating given curves w.r.t. t respectively

$$\frac{dy}{dt}=\frac{(1+t^3)(6at)-3at^2(3t^2)}{(1+t^3)^2}=\frac{6at-3at^4}{(1+t^3)^2}$$

and $\frac{dx}{dt}=\frac{(1+t^3)(3a)-3at(3t^2)}{(1+t^3)^2}=\frac{3a-6at^3}{(1+t^3)^2}$

$\therefore \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{3at(2-t^3)}{3a(1-2t^3)}=\frac{t(2-t^3)}{(1-2t^3)}$

4 **(d)**

Given, $y=\frac{\log x}{\log a}+\frac{\log a}{\log x}+1+1$

$\Rightarrow \frac{dy}{dx}=\frac{1}{x\log a}-\frac{\log a}{x(\log x)^2}$

25 **(c)**

We have,

$8f(x)+6f\left(\frac{1}{x}\right)=x+5$ ...(i)

Replacing x by $\frac{1}{x}$, we get

$6f(x)+8f\left(\frac{1}{x}\right)=\frac{1}{x}+5$ ....(ii)

Eliminating $f\left(\frac{1}{x}\right)$ from these two equations, we get

$f(x)=\frac{1}{28}\left(8x-\frac{6}{x}+10\right)$

$\therefore y=x^2f(x)=\frac{1}{28}(8x^3-6x+10x^2)$

$\Rightarrow \frac{dy}{dx}=\frac{1}{28}(24x^2-6+20x)$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=-1}=\frac{1}{28}(24-6-20)=-\frac{1}{14}$

6 **(d)**

Since, $y=\sqrt{\frac{1-x}{1+x}}$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{\sqrt{1+x}\times\frac{(-1)}{2\sqrt{1-x}}-\sqrt{1-x}\times\frac{1}{2\sqrt{1+x}}}{(\sqrt{1+x})^2}$$

$$\Rightarrow \frac{dy}{dx}=-\frac{1}{(1+x)\sqrt{1-x^2}}\times\frac{1-x}{1-x}$$

$$\Rightarrow (1-x^2)\frac{dy}{dx}+y=0$$

7 **(a)**

$x^y=e^{2(x-y)}$

$\therefore y\ \log x=2(x-y)$

$\Rightarrow y\ (\log x+2)=2x$

$y=\frac{2x}{\log x+2}$

$$\frac{dy}{dx} = \frac{(\log x + 2)(2) - 2x.\frac{1}{x}}{(\log x + 2)^2}$$

$$= \frac{2\log x + 4 - 2}{(\log x + 2)^2} = \frac{2(\log x + 1)}{(\log x + 2)^2}$$

8 **(a)**

$x = a(1 + \cos\theta), \quad y = a(\theta + \sin\theta)$

$$\frac{dx}{d\theta} = -a\sin\theta, \ \frac{dy}{d\theta} = a\,(1 + \cos\theta)$$

$$\therefore \ \frac{dy}{dx} = \frac{1 + \cos\theta}{-\sin\theta} = \frac{2\cos^2\frac{\theta}{2}}{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$\Rightarrow \frac{dy}{dx} = -\cot\frac{\theta}{2}$$

$$\therefore \ \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{d\theta}\left(\frac{dy}{dx}\right).\frac{d\theta}{dx}$$

$$= \frac{d}{d\theta}\left(-\cot\frac{\theta}{2}\right).\frac{1}{-a\sin\theta}$$

$$= \frac{1}{2}\text{cosec}^2\frac{\theta}{2}.\frac{1}{-a\sin\theta}$$

$$\therefore \ \left(\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{2}} = \frac{1}{2}.2.\frac{1}{-a} = -\frac{1}{a}$$

9 **(a)**

Given, $y^2 = ax^2 + bx + c \Rightarrow 2y\frac{dy}{dx} = 2ax + b$

$$\Rightarrow 2\left(\frac{dy}{dx}\right)^2 + 2y\left(\frac{d^2y}{dx^2}\right) = 2a$$

$$\Rightarrow y\frac{d^2y}{dx^2} = a - \left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow y\frac{d^2y}{dx^2} = a - \left(\frac{2ax + b}{2y}\right)^2$$

$$\Rightarrow y\frac{d^2y}{dx^2} = \frac{4ay^2 - (2ax + b)^2}{4y^2}$$

$$\Rightarrow 4y^3\frac{d^2y}{dx^2} = 4a(ax^2 + bx + c) - (4a^2x^2 + 4abx + b^2)$$

$$\Rightarrow 4y^3\frac{d^2y}{dx^2} = 4ac - b^2$$

$$\Rightarrow y^3\frac{d^2y}{dx^2} = \frac{4ac - b^2}{4} = \text{constant}$$

10 **(d)**

Given, $y = 1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + ... = \frac{1}{1 - \frac{1}{x}}$

$\Rightarrow \ y = \frac{x}{x-1}$(GP series) ...(i)

$\frac{dy}{dx} = \frac{1(x-1) - x.1}{(x-1)^2} = -\frac{1}{(x-1)^2}$

$\Rightarrow \frac{dy}{dx} = -\frac{y^2}{x^2}$ [from Eq.(i)]

11 **(c)**

Given, $f(x,y) = 2(x-y)^2 - x^4 - y^4$

On differentiating partially w.r.t. x, twicely

$f_x = 4(x-y) - 4x^3$

$\Rightarrow f_{xx} = 4 - 12x^2$

$\Rightarrow (f_{xx})_{(0,0)} = 4 - 0 = 4$

Similarly, $f_{yy} = 4 - 12y^2$

$\Rightarrow (f_{yy})_{(0,0)} = 4 - 0 = 4$

and $f_{xy} = -4 + 0$

$\Rightarrow (f_{xy})_{(0,0)} = -4$

$\therefore \left|f_{xx} f_{yy} - f_{xy}^2\right|_{(0,0)} = |4(4) - (-4)^2| = 0$

12 **(c)**

$$y = \tan^{-1}\frac{1}{1+x+x^2} + \tan^{-1}\frac{1}{x^2+3x+3} +$$

...+upto n terms

$$= \tan^{-1}\frac{(x+1)-x}{1+x(x+1)} + \tan^{-1}\frac{(x+2)-(x+1)}{1+(x+1)(x+2)} +$$

...+ upto n terms

$$= [\tan^{-1}(x+1) - \tan^{-1}x + \tan^{-1}(x+2) - \tan^{-1}(x+1)$$
$$+ ... + \tan^{-1}(x+n) - \tan^{-1}\{x+(n-1)\}]$$
$$= \tan^{-1}(x+n) - \tan^{-1}x$$
$$\therefore y'(x) = \frac{1}{1+(x+n)^2} - \frac{1}{1+x^2}$$
$$\Rightarrow y'(0) = \frac{1}{1+n^2} - 1 = \frac{n^2}{1+n^2}$$

13 **(b)**

Let $f(x) = x^2$

On differentiating w.r.t. x, we get

$f'(x) = 2x$

Given that, $f'(a+b) = f'(a) + f'(b)$

$$\Rightarrow 2(a+b) = 2a + 2b$$
$$\Rightarrow 2a + 2b = 2a + 2b$$

14 **(c)**

Let $y = (x+1)^n$

$\therefore \frac{dy}{dx} = n(x+1)^{n-1}$

$\frac{d^2y}{dx^2} = n(n-1)(x+1)^{n-2}$

Similarly, $\frac{d^2y}{dx^n} = n(n-1)(n-2)...3.2.1 = n!$

15 **(d)**

Given, $y = 2^x.3^{2x-1}$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 2^x.3^{2x-1}\log 3(2) + 2^x.3^{2x-1}\log 2$$

$$\Rightarrow \frac{dy}{dx} = 2^x.3^{2x-1}[2\log 3 + \log 2]$$

$$\Rightarrow \frac{dy}{dx} = y\log 18$$

16 **(a)**

Given, $y = x^2 + \frac{1}{y} \quad \Rightarrow \quad y^2 = x^2y + 1 \quad \Rightarrow 2y\frac{dy}{dx} = y.2x + x^2\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{2xy}{2y - x^2}$

17 **(d)**

$$y = \frac{\frac{(e^x + e^{-x})}{2}}{\frac{(e^x - e^{-x})}{2}} = \frac{\cosh x}{\sinh x}$$

$\Rightarrow y = \coth x \Rightarrow \frac{dy}{dx} = -\text{cosech}^2\, x$

18 **(c)**

$$f(f(x)) = f(|x-2)$$
$$= ||x-2| - 2|$$
$$= x - 4 \quad (\because x > 20)$$
$$\Rightarrow g(x) = x - 4$$
$$\therefore g'(x) = 1$$

19 **(b)**

$$5f(x) + 3f\left(\frac{1}{x}\right) = x + 2 \text{ ...(i)}$$

On replacing x by $\frac{1}{x}$, we get

$$5f\left(\frac{1}{x}\right) + 3f(x) = \frac{1}{x} + 2 \text{ ...(ii)}$$

On multiplying Eq. (i) by 5 and Eq. (ii) by 3 and then on subtracting, we get

$$\therefore 16f(x) = 5x - \frac{3}{x} + 4$$

$$\Rightarrow xf(x) = \frac{5x^2 - 3 + 4x}{16} = y$$

$$\therefore \frac{dy}{dx} = \frac{10x + 4}{16}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{10 + 4}{16} = \frac{7}{8}$$

20 **(b)**

$f(\log x) = \log\log(x)$

$\Rightarrow \frac{d}{dx}\{f(\log x)\} = \frac{1}{x}.\frac{1}{\log x} = (x\log x)^{-1}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	A	D	C	D	A	A	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	B	C	D	A	D	C	B	B