DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :8

Topic :-DIFFERENTITATION

1 **(b)**

$\because \quad y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

$$= \cos^{-1}\left(\frac{x-1}{x+1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$$

$\Rightarrow y = \frac{\pi}{2} \quad \Rightarrow \frac{dy}{dx} = 0$

2 **(a)**

Since, $f(x) = e^{g(x)}$

$\Rightarrow e^{g(x+1)} = f(x+1) = xf(x) = xe^{g(x)}$

and $\quad g(x+1) = \log x + g(x)$

$\Rightarrow \quad g(x+1) - g(x) = \log x \quad$ ...(i)

Replacing x by $x - \frac{1}{2}$, we get

$g\left(x+\frac{1}{2}\right) - g\left(x-\frac{1}{2}\right) = \log(x - \frac{1}{2})$

$= \log(2x-1) - \log 2$

$\therefore \; g''\left(x+\frac{1}{2}\right) - g''\left(x-\frac{1}{2}\right) = -\frac{4}{(2x-1)^2} \quad$ ...(ii)

On substituting, $x = 1,2,3,...N$ in Eq. (ii) and adding, we get

$g''\left(N+\frac{1}{2}\right) - g''\left(\frac{1}{2}\right) = -4\left\{1 + \frac{1}{9} + \frac{1}{25} + ... + \frac{1}{(2N-1)^2}\right\}$

3 **(b)**

We have,

$$y = \log_{x^2+4}(7x^2-5x+1) = \frac{\log_e(7x^2-5x+1)}{\log_e(x^2+4)} = \frac{\log_e f(x)}{\log_e g(x)}$$

$$\therefore \frac{dy}{dx} = \frac{\log_e(g(x) \cdot \frac{f'(x)}{f(x)} - \log_e f(x) \cdot \frac{g'(x)}{g(x)}}{(\log_e g(x))^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\log_e g(x)}\left\{\frac{f'(x)}{f(x)} - y\frac{g'(x)}{g(x)}\right\}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\log_e(x^2+4)}\left\{\frac{14x-5}{7x^2-5x+1} - \frac{2xy}{x^2+4}\right\}$$

5 **(a)**

We have, $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$, where $x = \tan\theta$

$\Rightarrow y = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{1}{2}\theta = \frac{1}{2}\tan^{-1}x$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2(1+x^2)} \Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{2}$

6 **(b)**

Let $y = \sqrt{\sec\sqrt{x}}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{\sec\sqrt{x}}}.\sec\sqrt{x}.\tan\sqrt{x}.\frac{1}{2\sqrt{x}}$$

$$= \frac{1}{4\sqrt{x}}(\sec\sqrt{x})^{3/2}.\sin x$$

7 **(c)**

We have,

$Sgn\,x = \begin{cases} 1, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -1, & \text{for } x < 0 \end{cases}$

$\therefore g(x) = Sgn\sin x = \begin{cases} \sin x, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -\sin x, & \text{for } x < 0 \end{cases}$

$\Rightarrow g'(x) = \begin{cases} \cos x, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -\cos x, & \text{for } x < 0 \end{cases}$

$\Rightarrow g'(1) = \cos 1$

8 **(c)**

$$\because y = x^{\text{In}\,x}$$

On taking log on both sides, we get

$\text{In}\,y = (\text{In}\,x)^2$

On differentiating w.r.t. x,we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{2\,\text{In}\,x}{x}$$

$$\Rightarrow \frac{dy}{dx} = y\frac{2\,\text{In}\,x}{x} = \frac{2(x^{\text{In}\,x})\text{In}\,x}{x}$$

$$\Rightarrow \frac{dy}{dx} = 2x^{\text{In}\,x-1}\text{In}\,x$$

9 **(d)**

Since, $x = e^t\sin t$ and $y = e^t\cos t$

$\Rightarrow \frac{dx}{dt} = e^t\cos t + \sin t\, e^t$

and $\frac{dy}{dt} = -e^t\sin t + e^t\cos t$

$$\therefore \quad \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{(\cos t-\sin t)}{(\cos t+\sin t)}$$

$$\frac{d^2y}{dx^2}=\frac{\begin{bmatrix}(\cos t+\sin t)(-\sin t-\cos t)\\-(\cos t-\sin t)(-\sin t+\cos t)\end{bmatrix}}{(\cos t+\sin t)^2}\frac{dt}{dx}$$

$$=\frac{-(\sin t+\cos t)^2-(\cos t-\sin t)^2}{(\cos t+\sin t)^2}$$

$$\times\frac{1}{e^t(\cos t+\sin t)}$$

$$=-\frac{2}{e^t(\cos t+\sin t)^3}$$

$$\Rightarrow\left(\frac{d^2y}{dx^2}\right)_{(x=\pi)}=\frac{-2}{e^\pi(\cos\pi+\sin\pi)^3}=\frac{2}{e^\pi}$$

10 **(b)**

We have,

$f'(x)=\sin(\log x)$ and $y=f\left(\frac{2x+3}{3-2x}\right)$

$$\therefore\frac{dy}{dx}=f'\left(\frac{2x+3}{3-2x}\right)\times\frac{d}{dx}\left(\frac{2x+3}{3-2x}\right)$$

$$\Rightarrow\frac{dy}{dx}=\sin\left\{\log\left(\frac{2x+3}{3-2x}\right)\right\}\times\frac{12}{(3-2x)^2}\quad[\because f'(x)=\sin(\log x)]$$

11 **(d)**

Given, $r=\left[2\phi+\cos^2\left(2\phi+\frac{\pi}{4}\right)\right]^{1/2}$

$$\frac{dr}{d\phi}=\frac{\left[2-2\cos\left(2\phi+\frac{\pi}{4}\right)\sin\left(2\phi+\frac{\pi}{4}\right).2\right]}{2\sqrt{2\phi+\cos^2\left(2\phi+\frac{\pi}{4}\right)}}$$

$$=\frac{\left[1-\sin\left(4\phi+\frac{\pi}{2}\right)\right]}{\sqrt{2\phi+\cos^2\left(2\phi+\frac{\pi}{4}\right)}}$$

$$\Rightarrow\left(\frac{dr}{d\phi}\right)_{\phi=\pi/4}=\frac{\left[1-\sin\left(\pi+\frac{\pi}{2}\right)\right]}{\sqrt{2.\frac{\pi}{4}+\cos^2\left(\frac{\pi}{2}+\frac{\pi}{4}\right)}}$$

$$=\frac{1+1}{\sqrt{\frac{\pi}{2}+\frac{1}{2}}}=2\sqrt{\frac{2}{1+\pi}}$$

12 **(d)**

$$\because y=1+x+x^2+\dots\infty$$

$$\therefore y=\frac{1}{1-x}=(1-x)^{-1}$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = -\frac{1}{(1-x)^2}(-1) = \frac{1}{(1-x)^2}$$

$$\therefore \frac{dy}{dx} - y = \frac{1}{(1-x)^2} - \frac{1}{(1-x)}$$

$$= \frac{1-1+x}{(1-x)^2} = \frac{x}{(1-x)^2}$$

$$\Rightarrow \frac{dy}{dx} - y = xy^2$$

$$\Rightarrow \frac{dy}{dx} = xy^2 + y$$

13 **(a)**

$f'(5) = \lim_{h\to 0} \frac{f(5+h) - f(5)}{h}$

$$= \lim_{h\to 0} \frac{2f(5)f(h) - f(5)}{h}$$

$$= \lim_{h\to 0} 2f(5)[\frac{f(h) - \frac{1}{2}}{h}]$$

$$\Rightarrow 1024 \log 2 = 2f(5)f'(0)$$

Again now, $f(2+3) = 2f(2)f(3)$ ...(i)

$$\Rightarrow \frac{1024 \log 2}{2f'(0)} = 2 \times 8 \times f(3)$$

$\Rightarrow \quad f(3) = \frac{32 \log 2}{f'(0)}$ ...(ii)

$\therefore \ f'(3) = \lim_{h\to 0} \log \frac{f(3+h) - f(3)}{h}$

$$= \lim_{h\to 0} \frac{2f(3)f(h) - f(3)}{h}$$

$$= 2f(3)f'(0)$$

$$= 2 \times \frac{32 \log 2 f'(0)}{f'(0)}$$

$=64 \log 2$ [from Eq. (ii)]

14 **(d)**

Let $u = e^{x^2}$ and $v = e^{2x-1}$

On differentiating w.r.t. x, we get

$\frac{du}{dx} = e^{x^2}.2x$ and $\frac{dv}{dx} = e^{2x-1}(2)$

$\therefore \frac{du}{dv} = \frac{e^{x^2}.2x}{e^{2x-1}.2}$

$$\Rightarrow \frac{du}{dv} = xe^{x^2-2x+1}$$

$\Rightarrow \left(\frac{du}{dv}\right)_{(x=1)} = 1.e^{1-2+1} = 1$

15 **(b)**

Given, $x = log_e t$

$\Rightarrow e^x = t$ and $y + 1 = t^2 \Rightarrow$ y $= e^{2x} - 1$

On differentiating w.r.t. y, we get

$2e^{2x}\frac{dx}{dy}=1$

$\Rightarrow \frac{dx}{dy}=\frac{1}{2e^{2x}}$

Again, differentiating w.r.t. y, we get

$\frac{d^2x}{dy^2}=\frac{1}{2}e^{-2x}(-2)\frac{dx}{dy}$

$=-e^{-2x}.\frac{1}{2e^{2x}}$

$=-\frac{1}{2}e^{-4x}$

16 **(a)**

Given, $y=\cot^{-1}(\cos 2x)^{1/2}$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{1+\cos 2x}\times\frac{1}{2\sqrt{\cos 2x}}\times -2\sin 2x$$

$$= \frac{2\sin x\cos x}{2\cos^2 x\sqrt{\cos 2x}}$$

$\Rightarrow \frac{dy}{dx}=\frac{\tan x}{\sqrt{\cos 2x}}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{6}}=\frac{1/\sqrt{3}}{\sqrt{1/2}}=\sqrt{\frac{2}{3}}$

17 **(c)**

Polynomial $P(x)$, satisfying the given relation can be taken as x

$$ie, P(x)=x$$

$$\therefore P'(x)=1$$

$$\Rightarrow P'(0)=1$$

18 **(c)**

$y=(\cos x^2)^2$

On differentiating w.r.t. x, we get

$\frac{dy}{dx}=2\cos x^2(-\sin x^2)2x=-2x\sin 2x^2$

19 **(c)**

We have,

$f(x)=2^{2x-1}$ and $g(x)=-2^x+2x\log 2$

$\therefore f'(x)>g'(x)$

$\Rightarrow 2\times 2^{2x-1}\log 2>-2^x\log 2+2\log 2$

$\Rightarrow 2^{2x}>-2^x+2$

$\Rightarrow 2^{2x}+2^x-2>0$

$\Rightarrow (2^x-1)(2^x+2)>0$

$\Rightarrow 2^x-1>0$

$\Rightarrow 2^x>1\Rightarrow x>0\Rightarrow x\in(0,\infty)$

20 **(c)**

Let $I = \frac{d}{dx}\left\{\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) - \tan^{-1}\left(\frac{4x-4x^3}{1-6x^2+x^4}\right)\right\}$

Put $x = \tan\theta$ the given equation

$\therefore \quad I = \frac{d}{dx}\{\tan^{-1}(\tan 2\theta) + \tan^{-1}(\tan 3\theta) - \tan^{-1}(\tan 4\theta)\}$

$= \frac{d}{dx}(\theta) = \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	B	C	A	B	C	C	D	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	D	A	D	B	A	C	C	C	C