DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :7

Topic :-DIFFERENTITATION

1 **(a)**

Given, $x = 2\cos\theta - \cos 2\theta$

and $y = 2\sin\theta - \sin 2\theta$

$\frac{dx}{d\theta} = -2\sin\theta + 2\sin 2\theta$

and $\frac{dy}{d\theta} = 2\cos\theta - 2\cos 2\theta$

$\therefore \quad \frac{dy}{dx} = \frac{2\cos\theta - 2\cos 2\theta}{-2\sin\theta + 2\sin 2\theta}$

$= \frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$

$= \frac{2\sin\left(\frac{\theta + 2\theta}{2}\right)\sin\left(\frac{2\theta - \theta}{2}\right)}{2\cos\left(\frac{\theta + 2\theta}{2}\right)\sin\left(\frac{2\theta - \theta}{2}\right)} = \tan\frac{3\theta}{2}$

2 **(c)**

Since, $f'(x) > \phi'(x)$

$\Rightarrow 2^{2x-1} 2\log 2 > -2^x \log 2 + 2\log 2$

$\Rightarrow 2^{2x} > -2^x + 2$

$\Rightarrow 2^{2x} + 2^x - 2 > 0$

$\Rightarrow (2^x - 1)(2^x + 2) > 0$

$\Rightarrow 2^x - 1 > 0 \qquad [\because 2^x + 2 > 0 \text{ for all } x]$

$\Rightarrow 2^x > 1$

$\therefore \; x > 0$

3 **(b)**

We have,

$x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow x^2(1+y) = y^2(1+x)$

$\Rightarrow x^2 - y^2 = -x^2 y + xy^2$

$\Rightarrow (x-y)(x+y) = -xy(x-y)$

$\Rightarrow x + y = -xy$

$\Rightarrow y = -\frac{x}{1+x} \Rightarrow \frac{dy}{dx} = -\left\{\frac{(1+x) - x}{(1+x)^2}\right\} = -\frac{1}{(1+x)^2}$

4 **(c)**

$\because \; y = e^{(1/2)\log(1+\tan^2 x)}$

$$\Rightarrow\ y = (\sec^2 x)^{1/2} = \sec x$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \sec x \tan x$$

5 **(a)**

Given,

$$f(x) = \frac{1}{4}\left[\frac{x-1}{1} + \frac{(x-1)^3}{3} + \frac{(x-1)^5}{5} + \frac{(x-1)^7}{7} + \dots\right]$$

$$\Rightarrow\ f(x) = \frac{1}{4}\left[\frac{1}{2}\log(\frac{1+(x-1)}{1-(x-1)}\right] = \frac{1}{8}\log\left(\frac{x}{2-x}\right)$$

$$\Rightarrow f'(x) = \frac{1}{8} \times \frac{1}{\left(\frac{x}{2-x}\right)}\left[\frac{(2-x)1 - x(-1)}{(2-x)^2}\right] = \frac{1}{4x(2-x)}$$

6 **(d)**

$$f(x) = \begin{vmatrix} x^3 & x^2 & 3x^2 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\Rightarrow f(x) = x^3(-6p^3 - 4p^2) - x^2(p^3 - 4p) + 3x^2(p^2 + 6p)$$

$$\Rightarrow f(x) = -6p^3x^3 - 4p^2x^3 - x^2p^3 + 4px^2 + 3p^2x^2 + 18px^2$$

On differentiating w.r.t. x, we get

$$\frac{d}{dx}f(x) = -18p^3x^2 - 12p^2x^2 - 2xp^3 + 8px + 6p^2x + 36px$$

Again differentiating w.r.t. x, we get

$$\frac{d^2}{dx^2}f(x) = -36p^3x - 24p^2x - 2p^3 + 8p + 6p^2 + 36p$$

Again differentiating w.r.t. x, we get

$\frac{d^3}{dx^3}f(x) = -36p^3 - 24p^2 = \text{constant}$

7 **(d)**

We have,

$f(x) = \text{arc}\tan\left(\frac{x^x - x^{-x}}{2}\right)$

$\Rightarrow f(x) = \tan^{-1}\left\{\frac{x^{2x} - 1}{2x^x}\right\}$

$\Rightarrow f(x) = -\tan^{-1}\left\{\frac{1 - x^{2x}}{2x^x}\right\}$

$\Rightarrow f(x) = -\cot^{-1}\left\{\frac{2x^x}{1 - x^{2x}}\right\}$

$\Rightarrow f(x) = \frac{-\pi}{2} + \tan^{-1}\left\{\frac{2x^x}{1 - x^{2x}}\right\}$

$$\Rightarrow f(x)=\begin{cases}\dfrac{-\pi}{2}+2\tan^{-1}(x^x), & \text{if } 0<x<1\\ \dfrac{-\pi}{2}-\pi+2\tan^{-1}(x^x), & \text{if } x>1\end{cases}$$

$$\Rightarrow f(x)=\begin{cases}\dfrac{-\pi}{2}+2\tan^{-1}(x^x), & \text{if } 0<x<1\\ \dfrac{-3\pi}{2}+2\tan^{-1}(x^x), & \text{if } x>1\end{cases}$$

$\Rightarrow f'(x)=\frac{2}{1+x^{2x}}\times x^x(1+\log_e x)$ for all $x>0, x\neq 1$

Clearly, $f'(1)$ does not exist

8 **(a)**

We have,

$f(x)+f(y)+f(x)f(y)=1$ for all $x, y\in R$ ...(i)

Putting $x=y=0$, we get

$2f(0)+\{f(0)\}^2=1$

$\Rightarrow \{f(0)\}^2+2f(0)-1=0$

$\Rightarrow f(0)=\dfrac{-2\pm\sqrt{4+4}}{2}$

$\Rightarrow f(0)=-1\pm\sqrt{2}$

$\Rightarrow f(0)=\sqrt{2}-1$ $[\because f(x)>0$ for all $x]$

Putting $y=x$ in (i), we get

$\{f(x)\}^2+2f(x)-1=0$ for all x

$\Rightarrow 2f(x)f'(x)+2f'(x)=0$ for all x

$\Rightarrow 2\{f(x)+1\}f'(x)=0$ for all x

$\Rightarrow f'(x)=0$ for all $x[\because f(x)>0$ for all $x]$

9 **(b)**

Given, $f(x)=e^x$, $g(x)=\sin^{-1}x$

Since, $h(x)=f[g(x)]=e^{\sin^{-1}x}$

Now, $h'(x)=e^{\sin x}.\frac{1}{\sqrt{1-x^2}}$

$$\Rightarrow h'(x)=h(x).\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{h'(x)}{h(x)}=\frac{1}{\sqrt{1-x^2}}$$

10 **(b)**

$\because f(x,y)=\dfrac{\cos(x-4y)}{\cos(x+4y)}$

$\therefore f\left(x,\dfrac{\pi}{2}\right)=\dfrac{\cos(x-2\pi)}{\cos(x+2\pi)}=\dfrac{\cos x}{\cos x}=1$

$\therefore \dfrac{\partial f}{\partial x}=0$

11 **(d)**

Given, $y=\sin^n x\cos nx$

$$\frac{dy}{dx} = n\sin^{n-1}x\cos x\cos nx - n\sin^{n}x\sin nx$$

$= n\sin^{n-1}x[\cos x\cos nx - \sin x\sin nx]$

$= n\sin^{n-1}x\cos(n+1)x$

12 **(a)**

Let $f(x) = 3e^{2x}$

Now, $f'(x) = 6e^{2x} = 2f(x)$

Therefore, our assumption is true.

$\therefore (2) = 3e^{2\times2} = 3e^{4}$

13 **(c)**

$y^2 = P(x) \Rightarrow 2yy' = P'(x)$ ...(i)

$$\Rightarrow (2y)y'' + y'(2y') = P''(x)$$

$$\Rightarrow 2yy'' = P''(x) - 2(y')^2$$

$$\Rightarrow 2y^3y'' = y^2P''(x) - 2(yy')^2$$

$$= y^2P''(x) - 2\frac{\{P'(x)\}^2}{4}$$

[from Eq.(i)]

$$\Rightarrow 2y^3.y'' = P(x)P''(x) - \frac{1}{2}\{P'(x)\}^2$$

$$\therefore \frac{d}{dx}(2y^3.y'')$$

$$= P(x)P'''(x) + P''(x)P'(x) - P'(x)P''(x)]$$

$$= P(x).P'''(x)$$

$$\Rightarrow 2\frac{d}{dx}\left(y^3\frac{d^2y}{dx^2}\right) = P(x)P'''(x)$$

14 **(c)**

Given, $x = e^{y+x}$

$$\Rightarrow \log x = (y+x) \Rightarrow \frac{1}{x} = \frac{dy}{dx} + 1$$

$\Rightarrow \frac{dy}{dx} = \frac{1-x}{x}$

15 **(a)**

Let $y = a\sin^3 t$ and $x = a\cos^3 t$, then

On differentiating w.r.t. t, we get

$$\frac{dy}{dt} = 3a\sin^2 t\cos t$$

and $\frac{dx}{dt} = 3a\cos^2 t(-\sin t)$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{3a\sin^2 t\cos t}{3a\cos^2 t(-\sin t)} = -\tan t$$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = -\sec^2 t\frac{dt}{dx} = \frac{-\sec^2 t}{3a\cos^2 t(-\sin t)}$$

$$= \frac{1}{3a}\left(\frac{\sec^4 t}{\sin t}\right)$$

$$\therefore \left(\frac{d^2y}{dx^2}\right)_{t=\frac{\pi}{4}} = \frac{1}{3a}.\frac{4}{\frac{1}{\sqrt{2}}} = \frac{4\sqrt{2}}{3a}$$

16 **(c)**

Let $u = \sin x^3$ and $v = \cos x^3$.

On differentiating w.r.t. x, we get

$$\frac{du}{dx} = \cos x^3.3x^2 \text{ and } \frac{dv}{dx} = -\sin x^3.3x^2$$

$$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{3x^2\cos x^3}{-3x^2\sin x^3} = -\cot x^3$$

17 **(d)**

$$\frac{\cos x}{1+\sin x} = \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}$$

$$= \frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}} \quad = \frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}$$

$$= \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$\therefore = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right) = \frac{\pi}{4} - \frac{x}{2}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{1}{2}$$

18 **(a)**

Given, $= \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$

$$= \tan^{-1}\sqrt{\frac{1-\cos\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)}}$$

$$= \tan^{-1}\left|\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right|$$

$$= \frac{\pi}{4} - \frac{x}{2} \qquad \left[\because x = \frac{\pi}{6}\right]$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{2}$$

19 **(a)**

Given, $y = \frac{\log \sin x}{\log \cos x}$

$$\Rightarrow \frac{dy}{dx} = \frac{\cot x \log \cos x + \tan x \log \sin x}{(\log \cos x)^2}$$

20 **(d)**

Given, $y = 2^x.3^{2x-1}$

$$\Rightarrow \frac{dy}{dx} = 2^x.3^{2x-1}2\log 3 + 3^{2x-1}.2^x \log 2$$

$$= 2^x 3^{2x-1} \log 18 = y \log 18$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} \log 18$$

$$= y(\log 18)^2$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	B	C	A	D	D	A	B	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	A	C	C	A	C	D	A	A	D