DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :6

Topic :-DIFFERENTITATION

1 **(a)**

Let $u=\cos^3 x, v=\sin^3 x$

$\frac{du}{dx}=-3\cos^2 x\sin x, \frac{dv}{dx}=3\sin^2 x\cos x$

Now, $\frac{du}{dv}=\frac{-3\cos^2 x\sin x}{3\sin^2 x\cos x}=-\cot x$

2 **(b)**

We have,

$$y=\log\left(\frac{1+x}{1-x}\right)^{1/4}-\frac{1}{2}\tan^{-1}x$$

$$\Rightarrow y=\frac{1}{4}\log(1+x)-\frac{1}{4}\log(1-x)-\frac{1}{2}\tan^{-1}x$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{4(1+x)}+\frac{1}{4(1-x)}-\frac{1}{2(1+x^2)}$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2(1-x^2)}-\frac{1}{2(1+x^2)}=\frac{x^2}{1-x^4}$$

3 **(a)**

Given, $y=\left(x+\sqrt{1+x^2}\right)^n$

$$\frac{dy}{dx}=n[x+\sqrt{1+x^2}]^{n-1}\left(1+\frac{x}{\sqrt{x^2+1}}\right)$$

$$=\frac{n[x+\sqrt{1+x^2}]^n}{\sqrt{1+x^2}}$$

$$\Rightarrow(1+x^2)\left(\frac{dy}{dx}\right)^2=n^2y^2$$

$$\Rightarrow 2\frac{dy}{dx}\frac{d^2y}{dx^2}(1+x^2)+2x\left(\frac{dy}{dx}\right)^2=2n^2y\frac{dy}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2}(1+x^2)+x\frac{dy}{dx}=n^2y$$

4 **(d)**

Let $y=\tan^{-1}\left\{\frac{3\sqrt{x}-x^{3/2}}{1-3x}\right\}$

Again let $\sqrt{x}=\tan t$

$$\therefore\ y = \tan^{-1}\left\{\frac{3\tan t - \tan^3 t}{1-3\tan^2 t}\right\} = \tan^{-1}(\tan 3t)$$

$$\Rightarrow\ y = 3\tan^{-1}\sqrt{x}$$

$$\Rightarrow\ \frac{dy}{dx} = \frac{3}{1+x}.\frac{1}{2\sqrt{x}} = \frac{3}{2(1+x)\sqrt{x}}$$

5 **(d)**

We have, $y = \tan^{-1}\left[\frac{\sin x + \cos x}{\cos x - \sin x}\right]$

$$\Rightarrow y = \tan^{-1}\left[\frac{1+\tan x}{1-\tan x}\right]$$

$$\Rightarrow y = \tan^{-1}\left[\frac{\tan\left(\frac{\pi}{4}\right) + \tan x}{1-\tan\left(\frac{\pi}{4}\right)\tan x}\right]$$

$$\Rightarrow y = \tan^{-1}\tan\left(\frac{\pi}{4} + x\right)$$

$$\Rightarrow y = \left(\frac{\pi}{4}\right) + x$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 1$$

6 **(d)**

Given, $x = \cos\theta,\quad y = \sin 5\theta$

$$\Rightarrow\ \frac{dx}{d\theta} = -\sin\theta,\quad \frac{dy}{d\theta} = 5\cos 5\theta$$

$$\therefore\ \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{5\cos 5\theta}{\sin\theta}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right).\frac{d\theta}{dx}$$

$$= \frac{d}{d\theta}\left(\frac{-5\cos 5\theta}{\sin\theta}\right)\frac{1}{-\sin\theta}$$

$$= \left(\frac{\sin\theta\sin 5\theta.25 + 5\cos 5\theta\cos\theta}{\sin^2\theta}\right).\frac{1}{-\sin\theta}$$

$$= -\frac{25\sin 5\theta}{\sin^2\theta} - \frac{5\cos 5\theta\cos\theta}{\sin^3\theta}$$

$$\therefore (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}$$

$$= (1-\cos^2\theta)\left(\frac{-25\sin 5\theta}{\sin^2\theta} - \frac{5\cos 5\theta\cos\theta}{\sin^3\theta}\right)$$

$$-\cos\theta\left(\frac{-5\cos 5\theta}{\sin\theta}\right)$$

$$= \sin^2\theta\left(\frac{-25\sin 5\theta}{\sin^2\theta} - \frac{5\cos\theta\cos 5\theta}{\sin^3\theta}\right) + \frac{5\cos\theta\cos 5\theta}{\sin\theta}$$

$$= -25 \sin 5\theta - \frac{5 \cos \theta \cos 5\theta}{\sin \theta} + \frac{5 \cos \theta \cos 5\theta}{\sin \theta}$$

$= -25y$

7 **(d)**

$$\because f(x) = a + bx$$
$$f\{f(x)\} = a + b(a + bx)$$
$$= ab + a + b^2x = a(1 + b) + b^2x$$
$$f[f\{f(x)\}] = f\{a(1 + b) + b^2x\}$$
$$= a + b\{a(1 + b) + b^2x\}$$
$$= a(1 + b + b^2) + b^3x$$
$$\therefore f'(x) = a(1 + b + b^2 + \ldots + b^{r-1}) + b^r x$$
$$= a\left(\frac{b^r - 1}{b - 1}\right) + b^r x$$

8 **(c)**

We have,

$x^y = e^{x-y} \Rightarrow y \log x = (x - y) \Rightarrow y = \dfrac{x}{1 + \log x}$

Differentiating w.r.t. x, we get $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$

9 **(d)**

Let $u = \sin^2 x$ and $v = \cos^2 x$

On differentiating w.r.t. x, we get

$$\frac{du}{dx} = 2 \sin x \cos x = \sin 2x$$

and $\frac{dv}{dx} = -2\cos x \sin x = -\sin 2x$

$$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{\sin 2x}{-\sin 2x} = -1$$

10 **(a)**

We have,

$x^p y^q = (x + y)^{p+q}$

$\Rightarrow p \log x + q \log y = (p + q) \log(x + y)$

Diff w.r.t. x, we get

$\dfrac{p}{x} + \dfrac{q}{y}\dfrac{dy}{dx} = \dfrac{p + q}{x + y}\left(1 + \dfrac{dy}{dx}\right)$

$\Rightarrow \dfrac{dy}{dx}\left(\dfrac{q}{y} - \dfrac{p + q}{x + y}\right) \Rightarrow \dfrac{p + q}{x + y} - \dfrac{p}{x} \Rightarrow \dfrac{dy}{dx} = \dfrac{y}{x}$

12 **(a)**

Let $y = \sin^{-1}\left(\frac{\sqrt{1 + x} + \sqrt{1 - x}}{2}\right)$

Putting $x = \cos \theta$, we get

$y = \sin^{-1}\left(\dfrac{1}{\sqrt{2}} \cos \dfrac{\theta}{2} + \dfrac{1}{\sqrt{2}} \sin \dfrac{\theta}{2}\right) = \sin^{-1}\left\{\sin\left(\dfrac{\pi}{4} + \dfrac{\theta}{2}\right)\right\}$

$\Rightarrow y = \dfrac{\pi}{4} + \dfrac{1}{2}\theta = \dfrac{\pi}{4} + \dfrac{1}{2} \cos^{-1} x$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{2\sqrt{1-x^2}}$$

13 **(c)**

Let $y = \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

Put $x = \cos 2\theta$

$$\therefore \quad y = \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right) = \tan^{-1}\left\{\tan\left(\frac{\pi}{4}-\theta\right)\right\}$$

$$= \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 0 + \frac{1}{2}.\frac{1}{\sqrt{1-x}} = \frac{1}{2\sqrt{1-x^2}}$$

14 **(c)**

$y = \tan^{-1} x + \cot^{-1} x + \sec^{-1} x + \operatorname{cosec}^{-1} x$

$= \frac{\pi}{2} + \frac{\pi}{2} = \pi$

$\frac{dy}{dx} = 0$

15 **(c)**

$$\because y = \left(\frac{ax+b}{cx+d}\right)$$

or $c\,xy + dy = ax + b$

On differentiating both sides w.r.t. x, we get

$$c\left\{x\frac{dy}{dx} + y.1\right\} + d\frac{dy}{dx} = a$$

or $x\frac{dy}{dx} + y + \left(\frac{d}{c}\right)\frac{dy}{dx} = \left(\frac{a}{c}\right)$

Again differentiating both sides w.r.t. x, we get

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} + \left(\frac{d}{c}\right)\frac{d^2y}{dx^2} = 0$$

or $x + \frac{2\frac{dy}{dx}}{\left(\frac{d^2y}{dx^2}\right)} + \frac{d}{c} = 0$

Again, on differentiating both sides w.r.t. x, we get

$$1 + \frac{\left(\frac{d^2y}{dx^2}.2\frac{d^2y}{dx^2} - 2\frac{dy}{dx}.\frac{d^3y}{dx^3}\right)}{\left(\frac{d^2y}{dx^2}\right)} + 0 = 0$$

$$\Rightarrow 2\frac{dy}{dx}\cdot\frac{d^3y}{dx^3}=3\left(\frac{d^2y}{dx^2}\right)^2$$

16 **(d)**

Given, $y=(\log_{\cos x}\sin x)(\log_{\sin x}\cos x)+\sin^{-1}\frac{2x}{1+x^2}$

At $x=\frac{\pi}{2}$,$\log_{\sin x}$ cos x is not defined.

Hence, we cannot determined the derivative at $x=\frac{\pi}{2}$.

17 **(b)**

$$y=\tan^{-1}\left(\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right)$$

$$=\tan^{-1}\left(\frac{\frac{a}{b}-\tan x}{1+\frac{a}{b}\tan x}\right)$$

$$=\tan^{-1}\left[\tan\left\{\tan^{-1}\left(\frac{a}{b}\right)-x\right\}\right]$$

$$\Rightarrow y=\tan^{-1}\left(\frac{a}{b}\right)-x$$

$$\therefore \frac{dy}{dx}=0-1=-1$$

18 **(d)**

We have, $x\cos\theta$, $y=\sin 5\theta$

$$\therefore \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=-\frac{5\cos 5\theta}{\sin\theta}$$

$$\Rightarrow \frac{d^2y}{dx^2}=-5\frac{d}{d\theta}\left(\frac{\cos 5\theta}{\sin\theta}\right)\cdot\frac{d\theta}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2}=\frac{-25\sin\theta\sin 5\theta-5\cos\theta\cos 5\theta}{\sin^3\theta}$$

$$\therefore (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=-25\sin 5\theta=-25y$$

19 **(c)**

Let $y=\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

Put $x=\cos 2\theta \Rightarrow \theta=\frac{1}{2}\cos^{-1}x$

$$\therefore y=\tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$$

$$\Rightarrow y=\tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$$

$$\Rightarrow y=\tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)$$

$$\Rightarrow y=\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \theta\right)\right)$$

$$\Rightarrow y = \frac{\pi}{4} - \theta$$

$$\Rightarrow y = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = -\frac{1}{2}\left(\frac{-1}{\sqrt{1-x^2}}\right) = \frac{1}{2\sqrt{1-x^2}}$$

20 **(b)**

$\because\ f(x) = (x-2)(x-4)(x-6)\ldots(x-2n)$

Taking log on both sides in the given equation, we get

$\log f(x) = \log(x-2) + \log(x-4) + \ldots + \log(x-2n)$

on differentiating w.r.t. x, we get

$\frac{1}{f(x)} f'(x) = \frac{1}{(x-2)} + \frac{1}{(x-4)} + \ldots + \frac{1}{(x-2n)}$

$$\Rightarrow\ f'(x) = (x-4)(x-6)\ldots(x-2n) + (x-2)(x-6)\ldots(x-2n) + \ldots + (x-2)(x-6)\ldots(x-2(n-1))$$

$\therefore\ f'(2) = (-2)(-4)\ldots(2-2n)$

$= (-2)^{n-1}(1.2\ldots..(n-1) = (-2)^{n-1}(n-1)!$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	A	D	D	D	D	C	D	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	C	C	C	D	B	D	C	B