DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :5

Topic :-DIFFERENTITATION

1 **(a)**

Given that, $x = \exp\left\{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)\right\}$

Taking log on both sides, we get

$$\log x = \tan^{-1}\left(\frac{y-x^2}{x^2}\right)$$

$$\Rightarrow \frac{y-x^2}{x^2} = \tan(\log x)$$

$$\Rightarrow y = x^2 \tan(\log x) + x^2$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 2x\tan(\log x) + x^2\frac{\sec^2(\log x)}{x} + 2x$$

$$\Rightarrow \frac{dy}{dx} = 2x\tan(\log x) + x\sec^2(\log x) + 2x$$

$$\Rightarrow \frac{dy}{dx} = 2x[1+\tan(\log x)] + x\sec^2(\log x)$$

2 **(b)**

Given, $x = \frac{\sin y}{\sin(a+y)}$

$\Rightarrow \frac{dx}{dy} = \frac{\sin(a+y)\cos y \ - \sin\cos(a+y)}{\sin^2(a+y)}$

$$= \frac{\sin(a+y-y)}{\sin^2(a+y)}$$

$\Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

3 **(b)**

Given, $f(x) = e^x \sin x$

$\Rightarrow \ f'(x) = e^x\cos x + \sin x e^x$

$\Rightarrow \ f''(x) = \ e^x\cos x - e^x\sin x + e^x\sin x + e^x\cos x$

$= \ 2e^x\cos x$

4 **(b)**

We know that, $\cos A \cos 2A \cos 2^2 A \ldots \cos 2^{n-1} A = \frac{\sin(2^n A)}{2^n \sin A}$

$$\therefore \cos x \cos 2x \cos 4x \cos 8x \cos 16x = \frac{\sin 32x}{32 \sin x}$$

$$\Rightarrow f(x) = \frac{1}{32} \cdot \frac{\sin 32x}{\sin x}$$

$$\therefore f'(x) = \frac{1}{32} \times \frac{\sin x(32 \cos 32x) - \sin 32x \cos x}{\sin^2 x}$$

$$\Rightarrow f'\left(\frac{\pi}{4}\right) = \frac{1}{\sin \frac{\pi}{4}} = \sqrt{2}$$

5 **(a)**

Given, $\frac{1+x}{1-y} = \sec a$

$$\Rightarrow y \sec a = \sec a - 1 - x$$

$$\Rightarrow \frac{dy}{dx} \sec a = -1$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{\sec a} = \frac{-1}{\left(\frac{1+x}{1-y}\right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y-1}{x+1}$$

6 **(c)**

Let $u = a^{\sec x}$ and $v = a^{\tan x}$

$\Rightarrow \frac{du}{dx} = a^{\sec x} \log_e a . \sec x \tan x$

and $\frac{dv}{dx} = a^{\tan x} \log_e a . \sec^2 x$

$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{a^{\sec x} \log_e a \sec x \tan x}{a^{\tan x} \log_e a \sec^2 x}$

$= a^{\sec x - \tan x} \sin x$

7 **(b)**

On differentiating given curves w.r.t.θ respectively, we get

$$\frac{dx}{d\theta} = a\left(-\sin\theta + \frac{1}{\tan\left(\frac{\theta}{2}\right)} .\sec^2\frac{\theta}{2} . \frac{1}{2}\right)$$

and $\frac{dy}{d\theta} = a\cos\theta$

$\Rightarrow \frac{dx}{d\theta} = \frac{a\cos^2\theta}{\sin\theta}$ and $\frac{dy}{d\theta} = a\cos\theta$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\cos\theta}{a\cos^2\theta/\sin\theta} = \tan\theta$

8 **(c)**

Since, $\phi(x) = f^{-1}(x) \Rightarrow x = f\{\phi(x)\}$

On differentiating w.r.t. x, we get

$$1 = f'\{\phi(x)\}.\phi'(x)$$

$\Rightarrow \quad \phi'(x) = \frac{1}{f'\{\phi(x)\}}$ ...(i)

But $f'\{\phi(x)\} = \frac{1}{1+\{\phi(x)\}^5} \quad \left(\because f'(x) = \frac{1}{1+x^5}\right)$

$\therefore$ From Eq.(i),

$\phi'(x) = \frac{1}{f'\{\phi(x)\}} = 1 + \{\phi(x)\}^5$

9 **(b)**

We have,

$f(x) = 3\, e^{x^2} \Rightarrow f'(x) = 6x\, e^{x^2}$

$\therefore f(0) = 3$ and $f'(0) = 0$

Now,

$f'(x) - 2\,x\,f(x) + \frac{1}{3}f(0) - f'(0) = 6\,x\,e^{x^2} - 6\,x\,e^{x^2} + \frac{1}{3}(3) - 0 = 1$

10 **(c)**

On differentiating partially the given equation w.r.t. x and y

$\frac{\partial u}{\partial x} = \frac{(x+y)}{(x^2+\ y^2)} \times \frac{x^2 - y^2 + 2xy}{(x+y)^2}$

$\Rightarrow x\frac{\partial u}{\partial x} = \frac{x}{(x^2+y^2)} \times \frac{x^2-y^2+2xy}{(x+y)}$ ...(i)

and $\frac{\partial u}{\partial y} = \frac{(x+y)}{(x^2+y^2)} \times \frac{y^2-x^2+2xy}{(x+y)^2}$

$\Rightarrow y\frac{\partial u}{\partial y} = \frac{y}{(x^2+y^2)} \times \frac{(y^2-x^2+2xy)}{(x+y)}$ ...(ii)

On adding Eqs. (i) and (ii), we get

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{(x^2+y^2)(x+y)}$

$[x(x^2-y^2+2xy) + y(y^2-x^2+2xy)]$

$= \frac{1}{(x^2+y^2)(x+y)} \times (x^2+y^2)(x+y)$

$= 1$

11 **(a)**

We have,

$F(x) = \frac{1}{x^2}\int_4^x \left(4t^2 - 2F'(t)\right) dt$

$\Rightarrow x^2F(x) = \int_4^x \left(4t^2 - 2F'(t)\right) dt$

Differentiating both sides with respect to x, we get

$2x\,F(x) + x^2F'(x) = 4x^2 - 2F'(x)$

Putting $x = 4$, we get

$8F(4) + 16F'(4) = 64 - 2F'(4)$

$\Rightarrow 18F'(4) = 64 \quad [\because F(4) = 0]$

$\Rightarrow F'(4) = \frac{32}{9}$

12 **(d)**

Put $x^2 = \cos 2\theta$ in the given equation, we get

$y = \tan^{-1}\frac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}$

$= \tan^{-1}\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

$= \tan^{-1}\tan\left(\frac{\pi}{4} - \theta\right)$

$\Rightarrow y = \frac{\pi}{4} - \theta = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x^2$

$\Rightarrow \frac{dy}{dx} = 0 - \frac{1}{2}\left(-\frac{(2x)}{\sqrt{1-x^4}}\right) = \frac{x}{\sqrt{1-x^4}}$

13 **(b)**

We have,

$f(x) = |x^2 - 5x + 6|$

$\Rightarrow f(x) = \begin{cases} x^2 - 5x + 6, & \text{if } x \geq 3 \text{ or } x \leq 2 \\ -(x^2 - 5x + 6), & \text{if } 2x < x < 3 \end{cases}$

$\therefore f'(x) = \begin{cases} (2x - 5), & \text{if } x > 3 \text{ or } x < 2 \\ -(2x - 5), & \text{if } 2 < x < 3 \end{cases}$

15 **(c)**

Let $y = x^6 + 6^x$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 6x^5 + 6^x \log 6$$

16 **(b)**

We have,

$f(x) = \cos^2 x + \cos^2(x + \pi/3) + \sin x \sin(x + \pi/3)$

$\Rightarrow f(x) = \frac{1}{2}\left[1 + \cos 2x + 1 + \cos\left(2x + \frac{2\pi}{3}\right) + \cos\frac{\pi}{3} - \cos\left(2x + \frac{\pi}{3}\right)\right]$

$\Rightarrow f(x) = \frac{1}{2}\left[\frac{5}{2} + \cos 2x + \cos\left(2x + \frac{2\pi}{3}\right) - \cos\left(2x + \frac{\pi}{3}\right)\right]$

$\Rightarrow f(x) = \frac{1}{2}\left[\frac{5}{2} + 2\cos\left(2x + \frac{\pi}{3}\right)\cos\frac{\pi}{3} - \cos\left(2x + \frac{\pi}{3}\right)\right]$

$\Rightarrow f(x) = \frac{5}{4}$

$\therefore gof(x) = g(5/4) = 3$ for all x

$\Rightarrow \frac{d}{dx}(gof(x)) = 0$ for all x

17 **(c)**

We have,

$y = \sin^{-1}x + \cos^{-1}y \Rightarrow y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$

18 **(b)**

Given, $z = y + f(v)$ ...(i)

Where $v = \left(\frac{x}{y}\right)$

On differentiating partially Eq. (i) w.r.t. x, we get

$\frac{\partial z}{\partial x} = f'\left(\frac{x}{y}\right).\left(\frac{1}{y}\right)$

$\Rightarrow v\frac{\partial z}{\partial x} = f'\left(\frac{x}{y}\right).\left(\frac{1}{y}\right)\left(\frac{x}{y}\right) = f'\left(\frac{x}{y}\right)\left(\frac{x}{y^2}\right)$ ...(ii)

Now, differentiating partially Eq. (i) w.r.t. y, we get

$\frac{\partial z}{\partial y} = 1 + f'\left(\frac{x}{y}\right)\left(\frac{-x}{y^2}\right)$ ...(iii)

On adding Eqs. (ii) and (iii), we get

$v\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = f'\left(\frac{x}{y}\right)\left(\frac{x}{y^2}\right) + 1 + f'\left(\frac{x}{y}\right)\left(\frac{-x}{y^2}\right) = 1$

19 **(d)**

$$y = \tan^{-1}\left(\frac{\sqrt{x - x}}{1 + x^{3/2}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{x} - x}{1 + \sqrt{x}.x}\right)$$

$$= \tan^{-1}(\sqrt{x}) - \tan^{-1}(x)$$

On differentiating w.r.t. x, we get

$$y' = \frac{1}{1 + x}.\frac{1}{2\sqrt{x}} - \frac{1}{1 + x^2}$$

$$\Rightarrow y'(1) = \frac{1}{2}.\frac{1}{2} - \frac{1}{2} = -\frac{1}{4}$$

20 **(c)**

We have,

$f(1) = 1$ and $f'(1) = 2$

$\therefore \lim_{x \to 1} \frac{\sqrt{f(x)} - 1}{\sqrt{x} - 1}$

$= \lim_{x \to 1} \frac{\frac{f'(x)}{2\sqrt{f(x)}}}{\frac{1}{2\sqrt{x}}} = \lim_{x \to 1} \frac{\sqrt{x}\, f'(x)}{\sqrt{f(x)}} = \frac{f'(1)}{\sqrt{f(1)}} = 2$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	B	B	A	C	B	C	B	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	D	B	C	C	B	C	B	D	C