# DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

**SOLUTIONS**

SUBJECT : MATHS
DPP NO. :4

## Topic :-DIFFERENTITATION

1 **(b)**

Let $D=\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$

$$\Rightarrow D=\begin{vmatrix} \sin px & p\cos px & -p^2\sin px \\ -p^3\cos px & p^4\sin px & p^5\cos px \\ -p^6\sin px & -p^7\cos px & p^8\sin px \end{vmatrix}$$

Taking $p^3$ and $p^6$common from $R_2$ and $R_3$row

$$=p^9\begin{vmatrix} \sin px & p\cos px & -p^2\sin px \\ -\cos px & p\sin px & p^2\cos px \\ -\sin px & -p\cos px & p^2\sin px \end{vmatrix}$$

$$=-p^9\begin{vmatrix} \sin px & p\cos px & -p^2\sin px \\ -\cos px & p\sin px & p^2\cos px \\ \sin px & p\cos px & -p^2\sin px \end{vmatrix}$$

$=0$ ($R_1$and $R_3$rows are identical)

2 **(d)**

$$y=1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\dots$$
$$\Rightarrow y=e^{-x}$$

On differentiating w.r.t. $x$, we get

$$\frac{dy}{dx}=-e^{-x}$$

Again differentiating w.r.t. $x$, we get

$$\frac{d^2y}{dx^2}=-e^{-x}(-1)=e^{-x}=y$$

3 **(b)**

We have, $f(4)=4$ and $f'(4)=1$

$$\therefore \lim_{x\to 4}\frac{2-\sqrt{f(x)}}{2-\sqrt{x}}=\lim_{x\to 4}\frac{\frac{-f'(x)}{2\sqrt{f(x)}}}{-\frac{1}{2\sqrt{x}}}\text{ [Using L'Hospital's Rule]}$$

$$\Rightarrow \lim_{x\to 4}\frac{2-\sqrt{f(x)}}{2-\sqrt{x}}=\lim_{x\to 4}\frac{\sqrt{x}\,f'(x)}{\sqrt{f(x)}}=\frac{2f'(4)}{\sqrt{f(4)}}=\frac{2}{2}=1$$

4 **(c)**

$\because\ x=\dfrac{2at}{1+t^3}$ and $y=\dfrac{2at^2}{(1+t^3)^2}$

$\therefore 2ay = x^2$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{x}{a}$

5 **(c)**

Given, $y=e^{a\sin^{-1}x}$

On differentiating w.r.t. $x$, we get

$y_1 = e^{a\sin^{-1}x} a.\frac{1}{\sqrt{1-x^2}}$

$\Rightarrow y_1\sqrt{1-x^2} = ay$

$\Rightarrow (1-x^2)y_1^2 = a^2y^2$

Again, differentiating w.r.t. $x$, we get

$(1-x^2)2y_1y_2 - 2xy_1^2 = a^2 2yy_1$

$\Rightarrow (1-x^2)y_2 - xy_1 - a^2y = 0$

Using Leibnitz's rule,

$(1-x^2)y_{n+2} + {}^nC_1 y_{n+1}(-2x) + {}^nC_2 y_n(-2)$

$-xy_{n+1} - {}^nc_1 y_n - a^2y_n = 0$

$\Rightarrow\ (1-x^2)y_{n+2} + xy_{n+1}(-2n-1)$

$+ y_n[-n(n-1)-n-a^2] = 0$

$\Rightarrow\ (1-x^2)y_{n+2} - (2n+1)xy_{n+1} = (n^2+a^2)y_n$

6 **(a)**

Since, $\frac{x-y}{x+y} = \sec^{-1} a$

$\Rightarrow \frac{(x+y)\left(1-\frac{dy}{dx}\right)-(x-y)\left(1+\frac{dy}{dx}\right)}{(x+y)^2}=0$

$$\Rightarrow x+y-x+y-(x+y+x-y)\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\frac{y}{x}$$

7 **(c)**

Given, $y = x + x^2 + x^3 + ... \Rightarrow y = \frac{x}{1-x}$

$$\Rightarrow x = \frac{y}{1+y} = y - y^2 + y^3 - ...$$

On differentiating w.r.t. $y$, we get

$$\frac{dx}{dy} = 1 - 2y + 3y^2 - ...$$

8 **(b)**

Let $y=\sqrt{\frac{1-\sin 2x}{1+\sin 2x}} = \frac{\cos x - \sin x}{\cos x + \sin x}$

$$= \frac{1-\tan x}{1+\tan x} = \tan\left(\frac{\pi}{4}-x\right)$$

$\Rightarrow \frac{dy}{dx} = -\sec^2\left(\frac{\pi}{4} - x\right)$

9 **(b)**

Let $u = \log_{10} x$ and $v = x^2$

$$\therefore \frac{du}{dx} = \frac{\log_{10} e}{x} \text{ and } \frac{dv}{dx} = 2x$$

$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{\log_{10} e}{x}/2x$

$$= \frac{\log_{10} e}{2x^2}$$

10 **(d)**

$$\because y = x \text{ In}\left(\frac{x}{a+bx}\right) = x(\text{In } x - \text{In}(a+bx))$$

$$\text{or } \left(\frac{y}{x}\right) = \text{In } x - \text{In }(a+bx)$$

On differentiating both sides w.r.t. $x$, we get

$$\left(\frac{x\frac{dy}{dx} - y.1}{x^2}\right) = \frac{1}{x} - \frac{b}{a+bx} = \frac{a}{x(a+bx)} \quad \text{...(i)}$$

or $\left(x\frac{dy}{dx} - y\right) = \frac{ax}{a+bx}$

On taking log on both sides, we get

$$\text{In}\left(x\frac{dy}{dx} - y\right) = \text{In }(ax) - \text{In }(a+bx)$$

On differentiating both sides w.r.t. $x$, we get

$$\frac{x\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx}}{\left(x\frac{dy}{dx} - y\right)} = \frac{1}{x} - \frac{b}{a+bx} = \frac{a}{x(a+bx)}$$

$$= \frac{\left(x\frac{dy}{dx} - y\right)}{x^2} \quad \text{[from Eq.(i)]}$$

or $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$

12 **(a)**

On differentiating given equation w.r.t. $x$, we get

$$4x - 3x\frac{dy}{dx} - 3y + 2y\frac{dy}{dx} + 1 + 2\frac{dy}{dx} - 0 = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{3y - 4x - 1}{2y - 3x + 2}$$

13 **(c)**

Given, $y = \sqrt{\sin x + y} \quad \Rightarrow \quad y^2 = \sin x + y$

$$\Rightarrow 2y\frac{dy}{dx} = \cos x + \frac{dy}{dx}$$

$$\Rightarrow(2y-1)\frac{dy}{dx}= \cos x$$

14 **(b)**

We have,

$2^x+2^y=2^{x+y}$

Differentiating with respect to $x$, we get

$2^x\log 2\,+2^y\log 2\,\frac{dy}{dx}=2^{x+y}\log 2\,\left(1+\frac{dy}{dx}\right)$

$\Rightarrow 2^x+2^y\frac{dy}{dx}=2^{x+y}\left(1+\frac{dy}{dx}\right)$

$\Rightarrow\frac{dy}{dx}=\frac{2^x-2^{x+y}}{2^{x+y}-2y}\Rightarrow\left(\frac{dy}{dx}\right)_{(1,\,0)}=\frac{2-4}{4-2}=-1$

15 **(a)**

Given, $y=\sec(\tan^{-1}x)$

$$\Rightarrow y= \sec(\sec^{-1}\sqrt{1+x^2})=\sqrt{1+x^2}$$

On differentiating w.r.t. $x$,we get

$$\frac{dy}{dx}=\frac{1}{2\sqrt{1+x^2}}(2x)=\frac{x}{\sqrt{1+x^2}}$$

16 **(b)**

Given, $f(x)=(x-7)^2(x-2)^7$

$\Rightarrow\quad f(\theta)=(\theta-7)^2(\theta-2)^7$

On differentiating w.r.t. $\theta$, we get

$\Rightarrow\quad f'(\theta)=2(\theta-7)(\theta-2)^7+7(\theta-2)^6(\theta-7)^2$

put $\quad f'(\theta)=0$

$\Rightarrow\quad(\theta-7)(\theta-2)^6[2(\theta-2)+7(\theta-7)]=0$

$\Rightarrow\quad 9\theta= 53\Rightarrow\theta=\frac{53}{9}$

17 **(b)**

We have,

$x^2+y^2=t-\frac{1}{t}$ and $x^4+y^4=t^2+\frac{1}{t^2}$

$\Rightarrow(x^2+y^2)^2=t^2+\frac{1}{t^2}-2$

$\Rightarrow(x^2+y^2)^2=x^4+y^4-2$

$\Rightarrow 2x^2y^2=-2$

$\Rightarrow x^2y^2=-1$

$\Rightarrow y^2=-\frac{1}{x^2}\Rightarrow 2y\frac{dy}{dx}=\frac{2}{x^3}\Rightarrow x^3y\frac{dy}{dx}=1$

18 **(b)**

Let $f(x)=|x-1|+|x-5|$

$$\Rightarrow f(x)=\begin{cases}-2x+6, & x<1\\ 4 & 1\le x<5\\ 2x-6 & x\ge 5\end{cases}$$

$$\therefore \frac{d}{dx}(f(x)) = \begin{cases} -2, & x < 1 \\ 0 & 1 < x < 5 \\ 2 & x > 5 \end{cases}$$

Hence, $\left(\frac{d}{dx}(f(x))\right)_{x=3} = 0$

19 **(b)**

$$\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$$
$$\Rightarrow \sin^{-1} x = \frac{\pi}{2} - \sin^{-1} y$$
$$\Rightarrow \sin^{-1} x = \cos^{-1} y$$
$$\Rightarrow y = \sqrt{1 - x^2}$$

On differentiating w.r.t. $x$, we get

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1 - x^2}}(-2x) = -\frac{x}{y}$$

20 **(c)**

$$f[f(f(x))] = f\left[f\left(\frac{1}{1-x}\right)\right] = f\left(\frac{1}{1 - \frac{1}{1-x}}\right)$$

$$\left[\because f(x) = \frac{1}{1-x}\right]$$

$$= f\left(\frac{1-x}{-x}\right) = \frac{1}{1 + \left(\frac{1-x}{x}\right)}$$

$\Rightarrow f[f(f(x))] = x$

$\therefore$ The derivative of composite function is equal to 1.

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | B | D | B | C | C | A | C | B | B | D |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | D | A | C | B | A | B | B | B | B | C |
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