DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

**SOLUTIONS**

SUBJECT : MATHS
DPP NO. :3

# Topic :-DIFFERENTITATION

1 **(b)**

Given, $\left(\frac{x^2-y^2}{x^2+y^2}\right)=\sec^{-1}e^a$

$$\Rightarrow \frac{\begin{bmatrix}(x^2+y^2)\left(2x-2y\frac{dy}{dx}\right)\\-(x^2-y^2)\left(2x+2y\frac{dy}{dx}\right)\end{bmatrix}}{(x^2+y^2)^2}=0$$

$$\Rightarrow(2x^3+2xy^2-2x^3+2xy^2)-2y\frac{dy}{dx}$$

$$(x^2+y^2+x^2-y^2)=0$$

$$\Rightarrow 4xy^2-4x^2y\,\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{y}{x}$$

2 **(a)**

$$\because y=\frac{a+bx^{3/2}}{x^{5/4}}$$

On differentiating w.r.t. $x$, we get

$$y'=\frac{\frac{3}{2}bx^{7/4}-\frac{5}{4}(a+bx^{3/2})x^{1/4}}{(x^{5/4})^2}$$

$$\because y'=0 \text{ at } x=5$$

$$\therefore \frac{3}{2}bx^{7/4}-\frac{5}{4}(a+bx^{3/2})x^{1/4}=0, \text{ at } x=5$$

$$\Rightarrow 6bx^{3/2}-5(a+bx^{3/2})=0, \text{ at } x=5$$

$$\Rightarrow bx^{3/2}=5a, \text{ at } x=5 \Rightarrow b(5)^{3/2}=5a$$

$$\Rightarrow \frac{a}{b}=\frac{5^{3/2}}{5}\Rightarrow a:b=\sqrt{5}:1$$

3 **(c)**

Given, $f(x)=be^{ax}+ae^{bx}$

$\Rightarrow f'(x)=abe^{ax}+abe^{bx}$

$\Rightarrow f''(x)=a^2be^{ax}+ab^2e^{bx}$

$\Rightarrow f''(0)=a^2b+ab^2=ab(a+b)$

4 **(b)**

Given, $x^my^n=(x+y)^{m+n}$

$$m\log x + n\log y = (m+n)\log(x+y)$$
$$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{(m+n)}{(x+y)}\left[1 + \frac{dy}{dx}\right]$$
$$\Rightarrow \frac{dy}{dx}\left[\frac{n}{y} - \frac{(m+n)}{(x+y)}\right] = \frac{m+n}{x+y} - \frac{m}{x}$$
$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$
$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1,y=2} = 2$$

5 **(a)**

Since, $f\left(x - \frac{1}{x}\right) = x^3 - \frac{1}{x^3}$

$$f\left(x - \frac{1}{x}\right) = \left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + 1\right)$$

$= \left(x - \frac{1}{x}\right)\ \left[\left(x - \frac{1}{x}\right)^2 + 3\right]$

$$\Rightarrow f(x) = x(x^2 + 3) = x^3 + 3x$$

$\Rightarrow f'(x) = \ 3x^2 + 3$

6 **(b)**

We have,

$x = a\cos t + \frac{b}{2}\cos 2t\,,\ y = a\sin t + \frac{b}{2}\sin 2t$

$\Rightarrow \frac{dy}{dx} = -a\sin t - b\sin 2t\,, \frac{dy}{dt} = a\cos t + b\cos 2t$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{a\cos t + b\cos 2t}{-a\sin t - b\sin 2t}$

$\Rightarrow \frac{dy}{dx} = -\frac{(a\cos t + b\cos 2t)}{(a\sin t + b\sin 2t)}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dt}\right)\cdot\frac{dt}{dx} = -\frac{d}{dt}\frac{(a\cos t + b\cos 2t)}{(a\sin t + b\sin 2t)}\cdot\frac{dt}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} = -\left[\frac{a^2 + 2b^2 + 3ab\cos t}{(a\sin t + b\sin 2t)^3}\right]$

$\therefore \frac{d^2y}{dx^2} = 0 \Rightarrow a^2 + 2b^2 + 3ab\cos t = 0 \Rightarrow \cos t = -\left(\frac{a^2 + 2b^2}{3ab}\right)$

7 **(d)**

$h'(x) = [f(x)^2 + g(x)^2]$

$\Rightarrow\ h''(x) = 2f(x)f'(x) + 2g(x)g'(x)$

$\left[\begin{array}{l} \because g(x) = f'(x) \\ \Rightarrow g'(x) = f''(x) \end{array}\right]$

$\therefore\ h''(x) = 2f(x)g(x) + 2g(x)(-f(x))$

$[\because f''(x) = -f(x)]$

$\Rightarrow h''(x) = 0$

$\Rightarrow h'(x) = C$, a constant for all $x \in R$

$\Rightarrow h(x)=Cx+C_1$
$\Rightarrow h(0)=C_1$ and $h(1)=C+C_1$
$\Rightarrow 2=C_1$ and $8=C+C_1$
$\Rightarrow C_1=2$ and $C=6$
$\therefore h(x)=6x+2$
$\Rightarrow h(2)=6\times 2+2=14$

8 **(b)**

Given, $y=x\log x$

On differentiating w.r.t. $x$, we get

$$\frac{dy}{dx}=\frac{x}{x}+\log x$$
$$\Rightarrow \frac{dy}{dx}=\log e+\log x$$

$\Rightarrow \frac{dy}{dx}=\log(ex)$

9 **(b)**

Given, $f''(x)=-f(x)$

$\Rightarrow g'(x)=-f(x)$ and $f'(x)=g(x)$ ...(i)

Now, $F(x)=\left(f\left(\frac{x}{2}\right)\right)^2+\left(g\left(\frac{x}{2}\right)\right)^2$

$\therefore F'(x)=2\left(f\left(\frac{x}{2}\right)\right).f'\left(\frac{x}{2}\right).\frac{1}{2}$

$+2\left(g\left(\frac{x}{2}\right)\right).g'\left(\frac{x}{2}\right).\frac{1}{2}=0$

[using Eq.(i)]

$\therefore F(x)$ is a constant $\Rightarrow F(10)=F(5)=5$

10 **(a)**

$$y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+\dots\infty}}}$$
$$\Rightarrow y=\sqrt{\sin x+y}$$
$$\Rightarrow y^2=\sin x+y$$

On differentiating w.r.t. $x$, we get

$$2y\frac{dy}{dx}=\cos x+\frac{dy}{dx}$$
$$\Rightarrow \frac{dy}{dx}=\frac{\cos x}{2y-1}$$

11 **(a)**

Given, $y=x^2e^{mx}\Rightarrow\frac{dy}{dx}=2xe^{mx}+mx^2e^{mx}$

$\Rightarrow \frac{d^2y}{dx^2}=2(e^{mx}+mxe^{mx})+m(2xe^{mx}-x^2me^{mx})$

$\Rightarrow \frac{d^2y}{dx^2}=e^{mx}(m^2x^2+4mx+2)$

$\Rightarrow \frac{d^3y}{dx^3}=e^{mx}[m^3x^2+4m^2x+2m+2m^2x+4m]$

$=e^{mx}[m^3x^2+6m^2x+6m]$

12 **(d)**

Given, $x^2+y^2=t-\frac{1}{t}$ and $x^4+y^4=t^2+\frac{1}{t^2}$

$\Rightarrow\ x^4+y^4+2x^2y^2=t^2+\dfrac{1}{t^2}-2$

$\Rightarrow\ x^4+y^4+2x^2y^2=x^4+y^4-2$

$\Rightarrow\ x^2y^2+1=0\Rightarrow y^2=\dfrac{-1}{x^2}$

On differentiating w.r.t. $x$, we get

$2y\ \dfrac{dy}{dx}=\dfrac{2}{x^3}\Rightarrow\dfrac{dy}{dx}=\dfrac{1}{x^3y}$

13 **(d)**

Given that, $y=\cos^{-1}\sqrt{1-t^2}=\ \sin^{-1}t$

and $x=\sin^{-1}(3t-4t^3)=3\sin^{-1}t$

On differentiating both w.r.t. $t$ respectively, we get

$$\frac{dy}{dt}=\frac{1}{\sqrt{1-t^2}}\ \text{and}\ \frac{dx}{dt}=\frac{3}{\sqrt{1-t^2}}$$

$$\therefore\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\left(\frac{1}{\sqrt{1-t^2}}\right)}{3\left(\frac{1}{\sqrt{1-t^2}}\right)}\Rightarrow\frac{dy}{dx}=\frac{1}{3}$$

14 **(c)**

We have,

$f(x)=\sqrt{1-\sin 2x}=\sqrt{(\cos x-\sin x)^2}$

$\Rightarrow f(x)=|\cos x-\sin x|$

$\Rightarrow f(x)=\begin{cases}\cos x-\sin x, & \text{for } 0\le x\le\pi/4\\ -(\cos x-\sin x), & \text{for } \pi/4<x\le\pi/2\end{cases}$

$\therefore f'(x)=\begin{cases}-(\cos x-\sin x), & \text{for } 0<x<\pi/4\\ (\cos x+\sin x), & \text{for } \pi/4<x<\pi/2\end{cases}$

15 **(c)**

Let $u=\sin x$ and $v=\cos x$

On differentiating w.r.t. $x$ respectively, we get

$$\frac{du}{dx}=\cos x\ \text{and}\ \frac{dv}{dx}=-\sin x$$

$$\therefore\ \frac{du}{dv}=\frac{du/dx}{dv/dx}=-\cot x$$

16 **(d)**

Let $y=F\{f(\phi(x))\}$

On differentiating w.r.t. $x$, we get

$$y'=F'[f\{\phi(x)\}]\frac{d}{dx}f\{\phi(x)\}$$

$$= F'[f\{\phi(x)\}]f'\{\phi(x)\}\frac{d}{dx}\phi(x) = F'[f\{\phi(x)\}]f'\{\phi(x)\}\phi'(x)$$

17 **(b)**

Given, $x\sqrt{1+y} = -y\sqrt{1+x}$ ...(i)

On squaring both sides, we get

$$x^2(1+y) = y^2(1+x)$$

$\Rightarrow (x-y)(x+y) + xy(x-y) = 0$

$$\Rightarrow (x-y)(x+y+xy) = 0$$

$x - y \neq 0$ because it does not satisfy the Eq.(i).

$$\therefore\ x + y + xy = 0 \Rightarrow y = -\frac{x}{1+x}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{(1+x)(1) - x(1)}{(1+x)^2} = -\frac{1}{(1+x)^2}$$

18 **(b)**

$$\because \frac{x^2 - y^2}{x^2 + y^2} = \sec^{-1}(e^a)$$

On differentiating w.r.t. $x$, we get

$$\frac{(x^2+y^2)\left(2x - 2y\frac{dy}{dx}\right) - (x^2-y^2)\left(2x + 2y\frac{dy}{dx}\right)}{(x^2+y^2)^2} = 0$$

$$\Rightarrow x(x^2+y^2) - y(x^2+y^2)\frac{dy}{dx}$$

$$= (x^2-y^2)x + y(x^2-y^2)\frac{dy}{dx}$$

$$\Rightarrow (x^2y - y^3 + x^2y + y^3)\frac{dy}{dx} = (x^3 + xy^2 - x^3 + xy^2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy^2}{2x^2y} = \frac{y}{x}$$

19 **(a)**

We have,

$f(x) = \log_a(\log_a x)$

$$\Rightarrow f'(x) = \frac{1}{\log_a x \cdot \log_e a}\frac{d}{dx}(\log_a x)$$

$$\Rightarrow f'(x) = \frac{1}{\log_a x \log_e a} \times \frac{1}{x \log_e a} = \frac{\log_a e}{x \log_e x}$$

20 **(b)**

$$\because y = \log^n x$$

On differentiating w.r.t. $x$, we get

$$x \log x \log^2 x \log^3 x \ldots \log^{n-1} x \log^n x \frac{dy}{dx}$$

$$= \frac{x \log x \log^2 x \log^3 x \ldots \log^{n-1} x \log^n x.1}{x \log x \log^2 x \log^3 x \ldots \log^{n-1} x}$$
$$= \log^n x$$

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | B | A | C | B | A | B | D | B | B | A |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | A | D | D | C | C | D | B | B | A | B |
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