DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth **SOLUTIONS** **SUBJECT : MATHS**
DATE : **DPP NO. :2**

Topic :-DIFFERENTITATION

1 **(d)**

We have,

$f(x)=\sqrt{(x-1)^2}=|x-1|=\begin{cases} x-1, & \text{if } x\geq 1 \\ -(x-1), & \text{if } x<1 \end{cases}$

$\therefore f'(x)=\begin{cases} 1, & \text{if } x>1 \\ -1, & \text{if } x<1 \end{cases}$

2 **(a)**

$\because u=\sin^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y}{x}\right)$

$\therefore \frac{\partial u}{\partial x}=\frac{1}{\sqrt{\left\{1-\left(\frac{x}{y}\right)^2\right\}}}\frac{1}{y}+\frac{1}{1+\left(\frac{y}{x}\right)^2}.\left(-\frac{y}{x^2}\right)$

$\Rightarrow x\frac{\partial u}{\partial x}=\frac{x}{\sqrt{(y^2-x^2)}}-\frac{xy}{(x^2+y^2)}$ ...(i)

and $\frac{\partial u}{\partial y}=\frac{1}{\sqrt{\left\{1-\left(\frac{x}{y}\right)^2\right\}}}\left(-\frac{x}{y^2}\right)+\frac{1}{1+\left(\frac{y}{x}\right)^2}.\left(\frac{1}{x}\right)$

$\Rightarrow y\frac{\partial u}{\partial y}=-\frac{x}{\sqrt{(y^2-x^2)}}+\frac{xy}{(x^2+y^2)}$ ...(ii)

On adding Eqs. (i)and (ii), we get

$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$

3 **(c)**

On differentiating the given equation partially w.r.t. x and y respectively

$u_x=\frac{y}{x}+\log y,\quad u_y=\log x+\frac{x}{y}$

Now, $u_x u_y-u_x\log x-u_y\log y+\log x\log y$

$=\left(\frac{y}{x}+\log y\right)\left(\log x+\frac{x}{y}\right)-\left(\frac{y}{x}+\log y\log x\right)$

$-\left(\log x+\frac{x}{y}\right)\log y+\log x\log y=1$

4 **(a)**

Here, $x=A\cos 4t+B\sin 4t$

$\Rightarrow \dfrac{dx}{dt} = -4A\sin 4t + 4B\cos 4t$

$\Rightarrow \dfrac{d^2x}{dt^2} = -16\,A\cos 4t - 16\,B\sin 4t$

$\Rightarrow \dfrac{d^2x}{dt^2} = -16x$

5 **(a)**

$$\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$$
$$= \lim_{x\to a}\frac{xf(a)-af(a)-af(x)+af(a)}{x-a}$$
$$= \lim_{x\to a}\frac{f(a)(x-a)-a[f(x)-f(a)]}{x-a}$$
$$= \lim_{x\to a}\frac{f(a)(x-a)}{x-a} - a\lim_{x\to a}\frac{f(x)-f(a)}{(x-a)}$$
$$= f(a) - af'(a)$$

6 **(b)**

$\dfrac{d}{dx}[x^x + x^a + a^x + a^a]$

$= x^x(1+\log x) + ax^{a-1} + a^x\log a + 0$

$= x^x(1+\log x) + ax^{a-1} + a^x\log a$

7 **(d)**

Given, $y = a^x.b^{2x-1}$

$\Rightarrow \log y = x\log a + (2x-1)\log b$

$\Rightarrow \dfrac{1}{y}.\dfrac{dy}{dx} = \log a + 2\log b$

$\Rightarrow \dfrac{dy}{dx} = y\log ab^2$

$\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{dy}{dx}\log ab^2 = y\,(\log ab^2)^2$

9 **(d)**

Differentiating $ax^2 + 2\,h\,xy + by^2 = 1$ w.r.t. x,we get

$2\,ax + 2\,hy + 2\,hx\dfrac{dy}{dx} + 2\,by\dfrac{dy}{dx} = 0$

$\Rightarrow \dfrac{dy}{dx} = -\left(\dfrac{ax+hy}{hx+by}\right)$

$\Rightarrow \dfrac{d^2y}{dx^2} = \left\{\dfrac{(hx+by)\left(a+h\dfrac{dy}{dx}\right)-(ax+hy)\left(h+b\dfrac{dy}{dx}\right)}{(hx+by)^2}\right\}$

$\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{h^2-ab}{(hx+by)^3}$

10 **(d)**

$[hog](x) = h(x^2) = 2\log_e x$

$\Rightarrow (hogof)(x) = hog(\sin x) = 2\log_e \sin x$
$\Rightarrow F(x) = 2\log_e \sin x$
$\Rightarrow F'(x) = 2\cot x$
$\Rightarrow F''(x) = -2\operatorname{cosec}^2 x$

11 **(b)**

Since, $\sin^{-1} x = \frac{\pi}{2} - \sin^{-1} y$

$$\Rightarrow \sin^{-1} x = \cos^{-1} y$$

$\Rightarrow y = \sqrt{1-x^2}$ $(\because \sin^{-1} x = \cos^{-1}\sqrt{1-x^2})$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}(-2x) = -\frac{x}{y}$$

12 **(d)**

$y = \tan^{-1}(\sec x - \tan x)$

$$\frac{dy}{dx} = \frac{d}{dx}\tan^{-1}\left(\frac{1-\sin x}{\cos x}\right)$$

$$\frac{dy}{dx} = \frac{d}{dx}\tan^{-1}\left(\frac{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)}\right)$$

$$= \frac{d}{dx}\tan^{-1}\left(\frac{1-\tan x/2}{1+\tan x/2}\right)$$

$$= \frac{d}{dx}\tan^{-1}\left\{\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right\}$$

$= \frac{d}{dx}\left(\frac{\pi}{4} - \frac{x}{2}\right) = -\frac{1}{2}$

13 **(d)**

We have,

$x = e^t \sin t$ and $y = e^t \cos t$

$\Rightarrow \frac{dx}{dt} = e^t(\sin t + \cos t)$ and $\frac{dy}{dt} = e^t(\cos t - \sin t)$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t - \sin t}{\cos t + \sin t}$$

Now, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{\cos t - \sin t}{\cos t + \sin t}\right) \times \frac{1}{e^t(\sin t + \cos t)}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-(\cos t + \sin t)^2 - (\cos t - \sin t)^2}{(\cos t + \sin t)^2} \times \frac{1}{e^t(\cos t + \sin t)}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-2}{(\cos t + \sin t)^3 e^t} \Rightarrow \left(\frac{d^2y}{dx^2}\right)_{t=\pi} = \frac{-2}{-e^\pi} = \frac{2}{e^\pi}$$

14 **(b)**

Given, $f(x) = 3|2+x|$

$$f(x) = \begin{cases} 3(2+x), & x \geq -2 \\ -3(2+x), & x \leq -2 \end{cases}$$

On differentiating w. r. t, x, we get

$$f'(x) = \begin{cases} 3, & x \geq 2 \\ -3, & x \leq -2 \end{cases}$$

at $x = -3, f'(3) = -3$

15 **(c)**

We know that be Newton's Leibnitz formula

If $I = \int_u^v f(t)dx$,

Then $\frac{dI}{dx} = f(v)\frac{dv}{dx} - f(u)\frac{du}{dx}$

Where u and v are function of x

$$\therefore \frac{dx}{dy} = \frac{1}{\sqrt{1+9y^2}}$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{1+9y^2}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{9y}{\sqrt{1+9y^2}} . \frac{dy}{dx}$$

$$= \frac{9y}{\sqrt{1+9y^2}}\sqrt{1+9y^2} = 9y$$

16 **(a)**

Given, $f(x) = x\tan^{-1} x$

$$\therefore f'(x) = \frac{x}{1+x^2} + \tan^{-1} x$$

$$\Rightarrow f'(1) = \frac{1}{1+1^2} + \tan^{-1} 1 = \frac{1}{2} + \frac{\pi}{4}$$

17 **(b)**

Given ,$g(x) = [f(2f(x)+2)]^2$

$\therefore g'(x) = 2.f(2f(x)+2).f'(2f(x)+2).2f'(x)$

$= 4\, f(2f(x)+2)f'(2f(x)+2)f'(x)$

$\therefore g'(0) = 4f(0)f'(0)f'(0) = -4$

18 **(b)**

Let $f(x) = |x-1| + |x-3|$

$$f(x) = \begin{cases} -(x-1)-(x-3), & x < 1 \\ (x-1)-(x-3), & 1 \leq x < 3 \\ (x-1)+(x-3), & x \geq 3 \end{cases}$$

$$f(x) = \begin{cases} 4-2x, & x < 1 \\ 2, & 1 \leq x < 3 \\ 2x-4, & x \geq 3 \end{cases}$$

At $x = 2$,

$$f(x) = 2 \Rightarrow f'(x) = 0$$

19 **(d)**

We have, $y = \tan^{-1}(\sec x - \tan x)$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\tan^{-1}\left(\frac{1-\sin x}{\cos x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\tan^{-1}\left(\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}\right)$$

$$= \frac{d}{dx}\tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]$$

$$= \frac{d}{dx}\left(\frac{\pi}{4} - \frac{x}{2}\right) = -\frac{1}{2}$$

20 **(a)**

Given that, $x = a\cos^4\theta$ and $y = a\sin^4\theta$

On differentiating w.r.t. θ, we get

$$\frac{dx}{d\theta} = 4\,a\cos^3\theta(-\sin\theta)$$

and $\frac{dy}{d\theta} = 4a\sin^3\theta\cos\theta$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{4a\sin^3\theta\cos\theta}{4\,a\cos^3\theta\sin\theta}$$

$$= -\frac{\sin^2\theta}{\cos^2\theta} = -\tan^2\theta$$

Now, $\left(\frac{dy}{dx}\right)_{\theta=\frac{3\pi}{4}} = -\tan^2\left(\frac{3\pi}{4}\right) = -1$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D		A	C	A	A	D	B	D	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	D	D	B	C	A	B	B	D	A