# DPP

DAILY PRACTICE PROBLEMS

**CLASS : XIIth**
**DATE :**

**SOLUTIONS**

**SUBJECT : MATHS**
**DPP NO. :10**

## Topic :-DIFFERENTITATION

2 **(a)**

Given, $f(x)=|x|^3=\begin{cases}0, & x=0\\ x^3, & x>0\\ -x^3, & x<0\end{cases}$

Now, $Rf'(0)=\lim_{h\to 0}\frac{f(h)-f(0)}{h}$

$$=\lim_{h\to 0}\frac{h^3-0}{h}=0$$

and $Lf'(0)=\lim_{h\to 0}\frac{f(-h)-f(0)}{-h}$

$$=\lim_{h\to 0}\frac{-h^3-0}{-h}=0$$

$$\because Rf'(0)=Lf'(0)=0$$

$$\therefore f'(0)=0$$

3 **(c)**

We have, $y=\log|x|=\begin{cases}\log x, & x>0\\ \log(-x), & x<0\end{cases}$

$$\therefore \frac{dy}{dx}=\begin{cases}\frac{1}{x}, & x>0\\ \frac{1}{-x}(-1)=\frac{1}{x}, & x<0\end{cases}$$

$\Rightarrow \frac{dy}{dx}=\frac{1}{x}, x\neq 0$

5 **(b)**

We have, $f(x)=x+2$

$\therefore f'(x)=1$ for all $x \Rightarrow f'(x)=1$ for all $x$

6 **(c)**

We have,

$f(x)-\log_x(\log_e x)$

$\Rightarrow f(x)=\frac{\log_e(\log_e x)}{\log_e x}$

$\Rightarrow f'(x)=\frac{\log_e x\times\frac{1}{x\log_e x}-\frac{1}{x}\log_e(\log_e x)}{(\log_e x)^2}$

$\Rightarrow f'(e) = \frac{1}{e} - \frac{1}{e} \times \log(1) = \frac{1}{e}$

7 **(b)**

Given, $\sin(x+y) + \cos(x+y) = \log(x+y)$

On differentiating w.r.t. $x$, we get

$\cos(x+y)\left(1 + \frac{dy}{dx}\right) - \sin(x+y)\left(1 + \frac{dy}{dx}\right)$

$= \frac{1}{(x+y)}\left(1 + \frac{dy}{dx}\right)$

$\Rightarrow 1 + \frac{dy}{dx} = 0 \Rightarrow \frac{d^2y}{dx^2} = 0$

8 **(b)**

$$10^{-x\tan x}\frac{d}{dx}(10^{x\tan x})$$
$$= 10^{-x\tan x}10^{x\tan x}\log 10(\tan x + x\sec^2 x)$$
$$= \log 10(\tan x + x\sec^2 x)$$

9 **(c)**

Given, $y = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!}\ldots$

$\Rightarrow y = e^{-x} \Rightarrow \frac{dy}{dx} = -e^{-x}$

$\therefore \frac{d^2y}{dx^2} = e^{-x} = y$

10 **(b)**

Let $y_1 = \sec^{-1}\frac{1}{2x^2-1}$ and $y_2 = \sqrt{1-x^2}$

$\Rightarrow \frac{dy_1}{dx} = \frac{-2}{\sqrt{1-x^2}}$ and $\frac{dy_2}{dx} = \frac{-x}{\sqrt{1-x^2}}$

$\Rightarrow \frac{dy_1}{dy_2} = \frac{2}{x} \Rightarrow \left(\frac{dy_1}{dy_2}\right)_{x=1/2} = 4$

11 **(d)**

We have,

$y = \cos 2x \cos 3x = \frac{1}{2}[\cos 5x + \cos x]$

$\therefore y_n = \frac{1}{2}\left\{\frac{d^n}{dx^n}(\cos 5x) + \frac{d^n}{dx^n}(\cos x)\right\}$

$\Rightarrow y_n = \frac{1}{2}\left\{5^n\cos\left(\frac{n\pi}{2} + 5x\right) + \cos\left(\frac{n\pi}{2} + x\right)\right\}$

12 **(c)**

Given, $f(x) = \log_{x^2}(\log_e x) = \frac{1}{2}\log_x(\log_e x)$

$$\Rightarrow(x)=\frac{1}{2}\frac{\log_e \log_e x}{\log_e x}$$

$$\Rightarrow f'(x)=\frac{1}{2}\frac{\log_e x\left(\frac{1}{x\log_e x}\right)-\log_e \log_e x\times\frac{1}{x}}{(\log_e x)^{2}}$$

$$\Rightarrow f'(x)=\frac{1}{2}\frac{\frac{1}{x}-\frac{1}{x}\log_e \log_e x}{(\log_e x)^2}$$

At $x=e, f'(e)=\frac{1}{2}\frac{\frac{1}{e}-\frac{1}{e}\log_e 1}{(1)^2}$

$\Rightarrow f'(e)=\frac{1}{2e}$

13 **(d)**

Given, $f(x)=\sin x, g(x)=x^2$

and $h(x)=\log_e x$

Also , $F(x)=(hogof)(x)$

$$\therefore\ (hogof)(x)=(hog)(\sin x)$$

$$\Rightarrow\qquad =h(\sin x^2)$$

$$\Rightarrow F(x)=2\log\sin x$$

On differentiating, we get

$$F'(x)=2\cot x$$

Again differentiating, we get

$$F''(x)=-2\operatorname{cosec}^2 x$$

14 **(c)**

Given, $x=\cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$

and

$y=\sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$

$\Rightarrow=\tan^{-1}t,$

and $y=\tan^{-1}t$

$\therefore\ y=x\Rightarrow\frac{dy}{dx}=1$

15 **(c)**

$$\because y=\tan^{-1}\left(\frac{\log\left(\frac{e}{x^2}\right)}{\log ex^2}\right)+\tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$$

$$=\tan^{-1}\left(\frac{1-\log x^2}{1+\log x^2}\right)+\tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$$

$$=\tan^{-1}(1)-\tan^{-1}(2\log x)+\tan^{-1}(3)+\tan^{-1}(2\log x)$$

$$\therefore y=\tan^{-1}(1)+\tan^{-1}(3)$$

$$\Rightarrow\frac{dy}{dx}=0\Rightarrow\frac{d^2y}{dx^2}=0$$

16 **(b)**

Let $y = \sin^2 \cot^{-1}\left\{\sqrt{\frac{1-x}{1+x}}\right\}$

Put $x = \cos\theta \Rightarrow \theta = \cos^{-1} x$

$$\Rightarrow y = \sin^2 \cot^{-1}\left\{\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right\}$$

$$\Rightarrow y = \sin^2 \cot^{-1}\left\{\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}\right\}$$

$$= \sin^2 \cot^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$\Rightarrow y = \sin^3\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$$

On differentiating w.r.t. θ, we get

$$\frac{dy}{d\theta} = 2\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)\cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right) - \left(-\frac{1}{2}\right)$$

$$\Rightarrow \frac{dy}{d\theta} = -\frac{\sin(\pi - \theta)}{2} = -\frac{\sin\theta}{2} = -\frac{1}{2}\sqrt{1-x^2}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{d\theta}.\frac{d\theta}{dx} = \frac{-1}{2}\sqrt{1-x^2}\frac{d}{dx}(\cos^{-1} x)$$

$$= -\frac{\sqrt{1-x^2}}{2}\left(\frac{-1}{\sqrt{1-x^2}}\right) = \frac{1}{2}$$

17 **(c)**

In the neighbourhood of $x = \sqrt[5]{\frac{\pi}{2}}$, we have

$[x] = 1$

Therefore, in the neighbourhood of $x = \sqrt[5]{\frac{\pi}{2}}$, we have

$f(x) = \sin\left\{\frac{\pi}{2}[x] - x^5\right\} = \sin\left(\frac{\pi}{2} - x^5\right) = \cos x^5$

$\Rightarrow f'(x) = -5x^4 \sin x^5$

$\Rightarrow f'\left(\sqrt[5]{\frac{\pi}{2}}\right) = -5\left(\frac{\pi}{2}\right)^{4/5} \sin\frac{\pi}{2} = -5\left(\frac{\pi}{2}\right)^{4/5}$

18 **(b)**

Since, $h'(x) = 2f(x)f'(x) + 2g(x)g'(x)$

Now, $f'(x) = g(x)$ and $f''(x) = -f(x)$

$\Rightarrow \quad f''(x) = g'(x)$and $f''(x) = -f(x)$

$$\Rightarrow -f(x) = g'(x)$$

Thus, $f'(x) = g(x)$ and $g'(x) = -f(x)$

$$\therefore \quad h'(x) = -2g(x)\, g'(x) + 2g(x)g'(x)$$

$= 0, \forall x$

$\Rightarrow\ h(x) = \text{constant for all } x$

But $h(5) = 11$

Hence, $h(x) = 11$ for all $x$

19 **(d)**

$$f(x) = \begin{vmatrix} x^3 & x^4 & 3x^2 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\Rightarrow \frac{d}{dx} f(x) = \begin{vmatrix} 3x^2 & 4x^3 & 6x \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\Rightarrow \frac{d^2}{dx^2} f(x) = \begin{vmatrix} 6x & 12x^2 & 6 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\Rightarrow \frac{d^3}{dx^3} f(x) = \begin{vmatrix} 6 & 24x & 0 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\Rightarrow \frac{d^4}{dx^4} f(x) = \begin{vmatrix} 0 & 24 & 0 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$$

$$= -24 \begin{vmatrix} 1 & 4 \\ p & p^3 \end{vmatrix} = -24(p^3 - 4p)$$

Hence, $\frac{d^4}{dx^4} f(x)$ is a constant.

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | A | A | C | B | B | C | B | B | C | B |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | D | C | D | C | C | B | C | B | D | B |
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