DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :1

Topic :-DIFFERENTITATION

1 **(c)**

Since, $y=(1+x)(1+x^2)(1+x^4)...(1+x^{2^n})$

$$\Rightarrow (1-x)y=(1-x^2)(1+x^2)(1+x^4)...(1+x^{2^n})$$
$$=(1-x^4)(1+x^4)...(1+x^{2^n})$$
$$\dots\dots\dots\dots\dots$$
$$\dots\dots\dots\dots\dots$$
$$=(1-x^{2^n})(1+x^{2^n})=1-x^{2^{n+1}}$$
$$\therefore\ y=\frac{1-x^{2^{n+1}}}{(1-x)}$$
$$\frac{dy}{dx}=\frac{(1-x)(-2^{n+1}).x^{2n+1-1}-(1-x^{2n+1})(-1)}{(1-x)^2}$$
$$\therefore \left.\frac{dy}{dx}\right|_{x=0}=\frac{(1-0)(-2^{n+1}.0)-(1-0)(-1)}{1}=1$$

2 **(b)**

We have,

$\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$

Putting $x=\sin A$, $y=\sin B$, it reduces to

$\Rightarrow \sin^{-1}x-\sin^{-1}y=2\cot^{-1}(a)$

Differentiating w.r.t. x, we get

$$\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0\Rightarrow \frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$$

3 **(c)**

Let $y=e^{x^3}, z=\log x$

On differentiating w.r.t. x, we get

$\frac{dy}{dx}=e^{x^3}(3x^2)=3x^2e^{x^3}$ and $\frac{dz}{dx}=\frac{1}{x}$

$$\therefore \frac{dy}{dz}=\frac{\frac{dy}{dx}}{\frac{dz}{dx}}=\frac{3x^2e^{x^3}}{\left(\frac{1}{x}\right)}=3x^3e^{x^3}$$

4 **(c)**

Let $y=\sqrt{x^2+16}$ and $z=\frac{x}{x-1}$

On differentiating w.r.t., x, we get

$$\frac{dy}{dx}=\frac{1}{2}(x^2+16)^{-1/2}(2x)$$

and $\frac{dz}{dx}=\frac{x-1-x}{(x-1)^2}=\frac{-1}{(x-1)^2}$

$$\therefore \frac{dy}{dz}=\frac{dy/dx}{dz/dx}=\frac{-x}{\sqrt{x^2+16}}.\frac{1}{\frac{1}{(x-1)^2}}$$

$$\left(\frac{dy}{dz}\right)_{x=3}=\frac{-3(2)^2}{\sqrt{25}}=\frac{-12}{5}$$

5 **(c)**

Given, $x=\log(1+t^2)$ and $y=t-\tan^{-1}t$

$$\frac{dx}{dt}=\frac{1}{1+t^2}.2t$$

and $\frac{dy}{dt}=1-\frac{1}{1+t^2}=\frac{t^2}{1+t^2}$

$\therefore \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{t^2/(1+t^2)}{2t/(1+t^2)}=\frac{t}{2}$ ...(i)

Also, $x=\log(1+t^2)\Rightarrow t^2=e^x-1$ ...(ii)

From Eqs. (i) and (ii), we get

$\frac{dy}{dx}=\frac{\sqrt{e^x-1}}{2}$

6 **(c)**

$$y=\frac{(1-x)(1+x)(1+x^2)(1+x^4)........(1+x^{2n})}{(1-x)}$$

$$=\frac{1-x^{4n}}{1-x}$$

$$\Rightarrow \frac{dy}{dx}=\frac{(1-x)(-4nx^{4n-1})-(1-x^{4n})(-1)}{(1-x)^2}$$

$$=\frac{-4n(1-x)x^{4n-1}+(1-x^{4n})}{(1-x)^2}$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=0}=1$$

7 **(b)**

We have,

$f(x)=(1-x)^n$

$\therefore f'(x)=-n(1-x)^{n-1}, f''(x)=n(n-1)x^{n-2},$

$f'''(x)=-n(n-1)(n-2)x^{n-3}$ and so on

$\Rightarrow f(0)=1, f'(0)=-n, f''(0)=n(n-1),$

$f'''(0)=-n(n-1)(n-2)$ and so on

$$\therefore f(0)+f'(0)+\frac{f''(0)}{2!}+...+\frac{f^n(0)}{n!}$$

$$=1-n+\frac{n(n-1)}{n!}+...+(-1)^n\frac{n(n-1)...3.2.1}{n!}$$

$= {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + ... + (-1)^n\ {}^nC_n = (1-1)^n = 0$

8 **(a)**

$$f(x) = (\log_{\cot x} \tan x)(\log_{\tan x} \cot x)^{-1} + \tan^{-1}\frac{4x}{4-x^2}$$

$$= \frac{\log \tan x}{\log \cot x} \cdot \frac{\log \tan x}{\log \cot x} + \tan^{-1}\left(\frac{4x}{4-x^2}\right)$$

$$= \frac{(\log \tan x)^2}{(-\log \tan x)^2} + \tan^{-1}\left(\frac{4x}{4-x^2}\right)$$

$$= 1 + \tan^{-1}\left(\frac{4x}{4-x^2}\right)$$

$$\therefore f'(x) = \frac{1}{1+\left(\frac{4x}{4-x^2}\right)^2} \cdot \frac{(4-x^2)4 - 4x(-2x)}{(4-x^2)^2}$$

$$= \frac{16 - 4x^2 + 8x^2}{(4-x)^2 + 16x^2} = \frac{4(4+x^2)}{(4-x^2)^2 + (4x)^2}$$

Hence, $f(2) = \frac{4(4+4)}{0+(8)^2} = \frac{32}{64} = \frac{1}{2}$

9 **(c)**

Given, $y = \cos^2\frac{3x}{2} - \sin^2\frac{3x}{2} = 2\cos^2\frac{3x}{2} - 1$

$\Rightarrow \frac{dy}{dx} = 2.2\cos\frac{3x}{2}\left(-\sin\frac{3x}{2}\right)\left(\frac{3}{2}\right)$

$\Rightarrow \frac{dy}{dx} = -6\cos\frac{3x}{2}\sin\frac{3x}{2}$

$\Rightarrow \frac{d^2y}{dx^2} = -6\left[\cos\frac{3x}{2}\left(\cos\frac{3x}{2}\right) \cdot \frac{3}{2} - \sin\frac{3x}{2}\sin\frac{3x}{2} \cdot \frac{3}{2}\right]$

$\Rightarrow \frac{d^2y}{dx^2} = -9\left[\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right] = -9y$

Alternate

$$y = \cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}$$

$\Rightarrow y = \cos 3x$

$\Rightarrow \frac{dy}{dx} = -3\sin 3x$

$\Rightarrow \frac{d^2y}{dx^2} = -9\cos 3x = -9y$

10 **(d)**

Given,

$$f(x) = \log_x(\log_e x) = \frac{\log_e \log_e x}{\log_e x}$$

On differentiating w.r.t. x, we get

$$f'(x) = \frac{\log_e x . \frac{1}{\log_e x} . \frac{1}{x} - \log_e \log_e x . \frac{1}{x}}{(\log_e x)^2}$$

$$\Rightarrow f'(x) = \frac{1 - \log_e \log_e x}{x(\log_e x)^2}$$

$$\Rightarrow f'(e) = \frac{1 - \log_e \log_e e}{e(\log_e e)^2} = \frac{1 - \log_e 1}{e} = \frac{1}{e}$$

11 **(c)**

We have,

$f(x) = \cos^{-1}\left\{\frac{1 + (\log_e x)^2}{1 + (\log_e x)^2}\right\}$

$\Rightarrow f(x) = 2\tan^{-1}(\log_e x)$ $\quad [\because \log_e x > 0 \text{ in the nbd of } x = e]$

$\Rightarrow f'(x) = \frac{2}{1 + (\log_e x)^2} \times \frac{1}{x} \Rightarrow f'(e) = \frac{1}{e}$

12 **(c)**

Given, $f(x) = 1 + nx + \frac{n(n-1)}{2!}x^2$

$+ \frac{n(n-1)(n-2)}{3!}x^3 + \ldots + x^n$

$\Rightarrow f(x) = (1 + x)^n$

$\Rightarrow f'(x) = n(1 + x)^{n-1}$

$\Rightarrow f''(x) = n(n-1)(1 + x)^{n-2}$

$\Rightarrow f''(1) = n(n-1)2^{n-2}$

13 **(c)**

On differentiating w.r.t. x, we get

$$2^x \log 2 + 2^y \log 2 \frac{dy}{dx} = 2^{x+y} \log 2\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow 2^x + 2^y \frac{dy}{dx} = 2^{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow 2^x - 2^{x+y} = \frac{dy}{dx}(2^{x+y} - 2^y)$$

$$\Rightarrow 2^{x-y}\frac{(1 - 2^y)}{(2^x - 1)} = \frac{dy}{dx}$$

14 **(a)**

We have,

$2x^2 - 3xy + y^2 + x + 2y - 8 = 0$

Differentiating w.r.t. to x, we get

$4x - 3\left(x\frac{dy}{dx} + y\right) + 2y\frac{dy}{dx} + 1 + 2\frac{dy}{dx} = 0$

$\Rightarrow 4x - 3y + 1 = \frac{dy}{dx}(3x - 2y - 2)$

$\Rightarrow \frac{dy}{dx} = \frac{3y - 4x - 1}{2y - 3x + 2}$

15 **(c)**

We have, $\sin y + e^{-x \cos y} = e$

Differentiating w.r.t. x, we get

$\cos y \frac{dy}{dx} - e^{-x \cos y}\left(\cos y - x \sin y \frac{dy}{dx}\right) = 0$

Putting $x = 1, y = \pi$, we get

$-\frac{dy}{dx} - e(-1) = 0 \Rightarrow \frac{dy}{dx} = e$

16 **(a)**

$$\frac{d}{dx}\left[\tan^{-1}\left(\frac{a-x}{1+ax}\right)\right] = \frac{d}{dx}[\tan^{-1} a - \tan^{-1} x]$$

$$= 0 - \frac{1}{1+x^2} = -\frac{1}{1+x^2}$$

17 **(b)**

$$\because y = \tan^{-1}\left(\frac{a \cos x - b \sin x}{b \cos x + a \sin x}\right)$$

$$= \tan^{-1}\left(\frac{\frac{a}{b} - \tan x}{1 + \frac{a}{b} \tan x}\right)$$

$$= \tan^{-1}\left[\tan\left(\tan^{-1}\left(\frac{a}{b}\right) - x\right)\right]$$

$$\Rightarrow y = \tan^{-1}\left(\frac{a}{b}\right) - x$$

$$\therefore \frac{dy}{dx} = 0 - 1 = -1$$

18 **(b)**

Given, $y = x^y$

$$\Rightarrow \log y = y \log x$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = y \cdot \frac{1}{x} + \frac{dy}{dx} \cdot \log x$$

$$\Rightarrow \frac{dy}{dx}\left[\frac{1}{y} - \log x\right] = \frac{y}{x}$$

$\Rightarrow \frac{dy}{dx} = \frac{y^2}{x(1 - y \log x)}$

19 **(b)**

$\because x^x y^y z^z = c$

$\Rightarrow \quad x \log x + y \log y + z \log z = \log c$

On differentiating partially w.r.t. x, we get

$x \cdot \frac{1}{x} + \log x + z \cdot \frac{1}{z} \frac{\partial z}{\partial x} + \log z \frac{\partial z}{\partial x} = 0$

$\Rightarrow (1 + \log z) \frac{\partial z}{\partial x} = -(1 + \log x)$

$$\Rightarrow \frac{\partial z}{\partial x} = -\left(\frac{1+\log x}{1+\log z}\right)$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	C	C	C	C	B	A	C	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	C	A	C	A	B	B	B	B