DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :9

Topic :-DIFFERENTIAL EQUATIONS

1 **(b)**

Since, $f''(x) = 6(x-1)$

$\Rightarrow \quad f'(x) = 3(x-1)^2 + c \quad$ [integrating] ...(i)

Also, at the point (2,1) the tangent to graph is $y = 3x - 5$

Slope of tangent=3

$\Rightarrow \quad f'(2) = 3$

$3(2-1)^2 + c = 3 \quad$ [from eq. (i)]

$\Rightarrow \quad 3 + c = 3$

$\Rightarrow \quad c = 0$

From Eq. (i),

$f'(x) = 3(x-1)^2$

$\Rightarrow \quad f(x) = (x-1)^2 + k \quad$ [integrating] ...(ii)

Since, it passes through (2,1)

$\therefore \quad 1 = (2-1)^2 + k \Rightarrow k = 0$

Hence, equation of function is

$$f(x) = (x-1)^2$$

2 **(b)**

$\because \log\left(\frac{dy}{dx}\right) = ax + by$

$\Rightarrow \frac{dy}{dx} = e^{ax+by} = e^{ax}e^{by}$

$\Rightarrow e^{-by}dy = e^{ax}dx$

On integrating both sides, we get

$\int e^{-by}dy = \int e^{ax}dx$

$\Rightarrow \frac{e^{-by}}{-b} = \frac{e^{ax}}{a} + c$

3 **(d)**

$y + 4x + 1 = V$ is the suitable substitution

$\because \frac{dy}{dx} = f(ax + by + c)$ is

Solvable for substituting

$ax + by + c = V$

4 **(b)**

Given, $\frac{dy}{dx} = \frac{(1+y^2)x}{y(1+x^2)}$

$\Rightarrow \quad \int \frac{2y}{1+y^2} dy = \int \frac{2x}{1+x^2} dx$

$\Rightarrow \quad \log(1+y^2) = \log(1+x^2) + \log k$

$\Rightarrow \quad (1+y^2) = (1+x^2)k$

This equation represents a family of hyperbola.

5 **(c)**

The equation of the family of circles of radius r is

$(x-a)^2 + (y-b)^2 = r^2$.....(i)

Where a and b are arbitrary constants

$\Rightarrow \quad 2(x-a) + 2(y-b)\frac{dy}{dx} = 0$

$\Rightarrow \quad (x-a) + (y-b)\frac{dy}{dx} = 0 \qquad ...(ii)$

$\Rightarrow \quad 1 + (y-b)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

$\Rightarrow \quad (y-b) = -\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}} \qquad ...(iii)$

From eq. (ii),

$(x-a) = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]\frac{dy}{dx}}{\frac{d^2y}{dx^2}} \qquad ...(iv)$

On putting the value of $(y-b)$ and $(x-a)$, in eq. (i), we get

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]\left(\frac{dy}{dx}\right)^2}{\left(\frac{d^2y}{dx^2}\right)^2} + \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^2}{\left(\frac{d^2y}{dx^2}\right)^2} = r^2$$

$\Rightarrow \quad \left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = r^2\left[\frac{d^2y}{dx^2}\right]^2$

6 **(b)**

Given,

Focus $S = (0,0)$ let $P(x,y)$ be any point on the parabola,

Since, $SP^2 = PM^2$

$\Rightarrow \quad (x-0)^2 + (y-0)^2 = (x+a)^2$

$\Rightarrow \quad y^2 = 2ax + a^2 \qquad ...(i)$

$\Rightarrow \quad 2y\frac{dy}{dx} = 2a \qquad ...(ii)$

From Eqs. (i) and (ii), we get

$$y^2 = 2y\frac{dy}{dx}.x + \left(y\ \frac{dy}{dx}\right)^2$$

$\Rightarrow \quad y^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx} = y^2$

$\Rightarrow \quad -y\left(\frac{dy}{dx}\right)^2 = 2x\ \frac{dy}{dx} - y$

7 **(b)**

Given, $\frac{dx}{x} = \frac{ydy}{1+y^2}$

$\Rightarrow \quad \log x = \frac{1}{2}\log(1+y^2) + \log c$

$\Rightarrow \quad x = c\sqrt{1+y^2}$

But it passes through $(1,0)$, so we get $c = 1$

$\therefore$ Solution is $x^2 - y^2 = 1$

8 **(d)**

Given that, $\frac{dy}{dx} = \frac{x^2}{y+1}$

$\Rightarrow (y+1)dy = x^2 dx$

$\Rightarrow \frac{y^2}{2} + y = \frac{x^2}{3} + c$

This curve passes through the point (3, 2).

$2 + 2 = 9 + c$

$\Rightarrow c = -5$

$\therefore$ Required curve is $\frac{y^2}{2} + y = \frac{x^3}{3} - 5$

9 **(a)**

Given, $\frac{dy}{dx} = \frac{y-1}{x^2+x}$

$\Rightarrow \quad \int \left(\frac{1}{x} - \frac{1}{x+1}\right)dx = \int \frac{1}{y-1}dy$

$\Rightarrow \quad \log x - \log(x+1) = \log(y-1) + \log c$

$\Rightarrow \quad \frac{x}{x+1} = (y-1)c$ ...(i)

Since, this curve passes through $(1,0) c = -\frac{1}{2}$

$\therefore$ From Eq. (i) $2x + (y-1)(x+!) = 0$

10 **(a)**

Given, $\frac{dy}{dt} - \left(\frac{1}{1+t}\right)y = \frac{1}{(1+t)}$ and $y(0) = -1$

$\therefore \quad IF = e^{\int -\left(\frac{t}{1+t}\right)dt} = e^{-\int \left(1-\frac{1}{1+t}\right)dt}$

$e^{-t+\log(1+t)} = e^{-t}(1+t)$

$\therefore$ Required solution is,

$$ye^{-t}(1+t) = \int \frac{1}{1+t}e^{-t}(1+t)dt + c$$

$$= \int e^{-t}\,dt + c$$

$\Rightarrow ye^{-1}(1+t) = -e^{-1} + c$

Since, $y(0) = -1$

$\Rightarrow \quad c = 0$

$\therefore \quad y = -\frac{1}{(1+t)}$

$\Rightarrow \quad y(1) = -\frac{1}{2}$

11 **(a)**

Given curve is $y = x^2$

For this curve there is only one tangent line ie,

x-axis $(y = 0)$

$\therefore \quad \frac{dy}{dx} = 0$

Hence, order is 1.

12 **(c)**

Given, $x^2 + y^2 - 2ay = 0$ ....(i)

$\Rightarrow \quad 2x + 2yy' - 2ay' = 0$

$\Rightarrow \quad \frac{2x + 2yy'}{y'} = 2a$ ...(ii)

$\therefore$ From Eq. (i)

$2a = \frac{x^2 + y^2}{y}$

$\Rightarrow \quad \frac{2x + 2yy'}{y'} = \frac{x^2 + y^2}{y}$ [from Eq. (ii)]

$\Rightarrow \quad (x^2 - y^2)y' = 2xy$

13 **(a)**

$\because \frac{dy}{dx} + 1 = \text{cosec}\,(x + y)$

Let $x + y = t$

and $1 + \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dt}{\text{cosec}\,t} = dx$

$\therefore \int \sin t\, dt = \int dx$

$\Rightarrow -\cos t = x - c$

$\Rightarrow \cos(x + y) + x = c$

14 **(b)**

Given, $\frac{y}{4} dy = -\frac{x}{9} dx$

$\Rightarrow \quad \frac{y^2}{4.2} = -\frac{x^2}{9.2} + \frac{c}{2}$

$\Rightarrow \quad \frac{y^2}{4} + \frac{x^2}{9} = c$

15 **(b)**

The differential equation of the rectangular hyperbola $xy = c^2$ is

$y + x\frac{dy}{dx} = 0 \Rightarrow x\frac{dy}{dx} = -y$

16 **(c)**

Given, $\log\left(\frac{dy}{dx}\right) = 3x + 4y$

$\Rightarrow \quad \frac{dy}{dx} = e^{3x}\, e^{4y}$

$\Rightarrow \quad e^{-4y}\, dy = e^{3x}\, dx$

On integrating both sides, we get

$$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + c$$

At $\quad x = 0, y = 0$

$$-\frac{1}{4} = \frac{1}{3} + c$$

$\Rightarrow \quad c = -\frac{7}{12}$

∴ Solution is

$$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$$

$\Rightarrow \quad 4e^{3x} + 3e^{-4y} = 7$

17 **(b)**

Given differential equation is

$\frac{d^2y}{dx^2} = 2 \Rightarrow \frac{dy}{dx} = 2x + a$

$\Rightarrow y = x^2 + ax + b$

∴ It represents a parabola whose axis is parallel to y-axis.

18 **(b)**

Given, $\quad \frac{dy}{dx} = \left(\frac{y}{x}\right)\left[\log\left(\frac{y}{x}\right) + 1\right]$

Put $\quad \frac{y}{x} = t$

$\Rightarrow \quad y = xt$

$\Rightarrow \quad \frac{dy}{dx} = t + x\frac{dt}{dx}$

$\therefore \quad t + x\frac{dt}{dx} = t(\log t + 1)$

$\Rightarrow \quad \frac{1}{t \log t} dt = \frac{dx}{x}$

$\Rightarrow \quad \log(\log t) = \log x + \log c \qquad$ [integrating]

$\Rightarrow \quad \log\left(\frac{y}{x}\right) = cx$

19 **(a)**

Given, $\frac{dy}{dx} = \frac{-y^2}{x^2 - xy + y^2}$

Put $\quad y = vx$

$\Rightarrow \quad \frac{dy}{dx} = v + x\,\frac{dv}{dx}$

$\therefore \quad v + x\,\frac{dv}{dx} = \frac{-v^2}{v^2 - v + 1}$

$$x\,\frac{dv}{dx} = \frac{-v^3 - v}{v^2 - v + 1}$$

$\Rightarrow \quad \frac{(v^2 - v + 1)}{-v^3 - v} dv = \frac{1}{x} dx$

$\Rightarrow \quad \frac{-(v^2+1)+v}{v(v^2+1)} dv = \frac{1}{x} dx$

$\Rightarrow \quad \int -\frac{1}{v} dv + \int \frac{1}{v^2+1} dv = \int \frac{1}{x} dx$

$\Rightarrow \quad -\log v + \tan^{-1} v = \log x + c$

$\Rightarrow \quad \tan^{-1} v = \log xv + c$

$\Rightarrow \quad \tan^{-1}\left(\frac{y}{x}\right) = \log y + c$

20 **(a)**

Given, $\frac{d^2y}{dx^2}(x^2+1) = 2x\frac{dy}{dx}$

$\Rightarrow \quad \frac{\frac{d^2y}{dx^2}}{\frac{dy}{dx}} = \frac{2x}{x^2+1}$

On integrating both sides, we get

$$\log \frac{dy}{dx} = \log(x^2+1) + \log c$$

$\Rightarrow \quad \frac{dy}{dx} = c(x^2+1) \quad$ ...(i)

As at $x = 0$, $\frac{dy}{dx} = 3$

$\therefore \quad 3 = c(0+1)$

$\Rightarrow \quad c = 3$

$\therefore$ From Eq. (i),

$$\frac{dy}{dx} = 3(x^2+1)$$

$\Rightarrow \quad dy = 3(x^2+1)dx$

Again, integrating both sides, we get

$$y = 3\left(\frac{x^3}{3} + x\right) + c_1$$

At point (0,1)

$$1 = 3(0+0) + c_1 \Rightarrow c_1 = 1$$

$\therefore \quad y = 3\left(\frac{x^3}{3} + x\right) + 1$

$\Rightarrow \quad y = x^3 + 3x + 1$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	D	B	C	B	B	D	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	A	B	B	C	B	B	A	A