DPP
DAILY PRACTICE PRODUCTS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :2

Topic :-DIFFERENTIAL EQUATIONS

1 **(b)**

We have,

$y\frac{dy}{dx}=\lambda y^2 \Rightarrow \frac{dy}{dx}=\lambda y$

$\Rightarrow \frac{1}{y}dy=\lambda\, dx \Rightarrow \log y = \lambda x+\log C \Rightarrow y=Ce^{\lambda x}$

2 **(b)**

We have,

$$\frac{dy}{dx}+\frac{1+\cos 2y}{1-\cos 2x}=0$$

Given, $\frac{dy}{dx}=-\frac{1+\cos 2y}{1-\cos 2x}=-\frac{2\cos^2 y}{2\sin^2 x}$

$\Rightarrow \quad \int \sec^2 y\, dy=-\int \operatorname{cosec}^2 x\, dx$

$\Rightarrow \tan y = \cot x + c.$

3 **(a)**

Given differential equation can be rewritten as

$$\frac{dy}{dx}+\frac{y}{\sqrt{x}}=e^{-2\sqrt{x}}$$

Here, $P=\frac{1}{\sqrt{x}}, Q=e^{-2\sqrt{x}}$

$\therefore \quad \text{IF}=e^{\int \frac{1}{\sqrt{x}}dx}=e^{2\sqrt{x}}$

$\therefore$ Solution is

$$ye^{2x}=\int e^{2\sqrt{x}}e^{-2\sqrt{x}}\,dx=\int 1\,dx$$

$\Rightarrow \quad ye^{2\sqrt{x}}=x+c$

4 **(a)**

Given, $\left(1+\frac{1}{y}\right)dy=-e^x\left(\cos^2 x-\sin 2x\right)dx$

On integrating both sides, we get

$$y + \log y = -e^x \cos^2 x + \int e^x \sin 2x\, dx - \int e^x \sin 2x\, dx + c$$

$\Rightarrow \quad y + \log y = -e^x \cos^2 x + c$

At x=0, y=1

$1 + 0 = -e^0 \cos 0 + c \Rightarrow c = 2$

$\therefore$ Required solution is

$y + \log y = -e^x \cos^2 x + 2$

5 **(d)**

Given, $\frac{dy}{dx} = (4x + y + 1)^2$

Put $4x + y + 1 = v$

$\Rightarrow \quad \frac{dy}{dx} = \frac{dv}{dx} - 4$

$\therefore \quad \frac{dv}{dx} - 4 = v^2$

$\Rightarrow \quad \frac{dv}{v^2 + 4} = dx$

$\Rightarrow \quad \frac{1}{2}\tan^{-1}\left(\frac{v}{2}\right) = x + c$ [integrating]

$\Rightarrow \quad \tan^{-1}\left(\frac{4x + y + 1}{2}\right) = 2x + c$

$\Rightarrow \quad 4x + y + 1 = 2\tan(2x + c)$

6 **(c)**

Given differential equation is

$x\frac{dy}{dx} + y \log x = xe^x x^{-\frac{1}{2}\log x}$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x}\log x = e^x x^{-\frac{1}{2}\log x}$

Here, $P = \frac{1}{x}\log x$ and $Q = e^x x^{-\frac{1}{2}\log x}$

$\therefore \text{IF} = e^{\int \frac{\log x}{x} dx} = e^{\frac{(\log x)^2}{2}} = \left(\sqrt{e}\right)^{(\log x)^2}$

7 **(a)**

Let us assume the equation of parabola whose axis is parallel to y-axis and touch x-axis.

$y = ax^2 + bx + c$ ...(i)

and $b^2 = 4ac$ ($\because$ curve touches x-axis)

$\because$ There are two arbitrary constant.

$\therefore$ Order of this equation is 2.

8 **(a)**

Here, $y = A\cos\omega t + B\sin\omega t$ ....(i)

On differentiating w.r.t.t, we get

$\frac{dy}{dt} = -\omega A \sin\omega t + \omega B \cos\omega t$

Again, on differentiating w.r.t.t, we get

$\frac{d^2y}{dt^2} = -\omega^2 A\cos\omega t - \omega^2 B \sin\omega t$

$\Rightarrow \frac{d^2y}{dt^2} = -\omega^2(A\cos\omega t - B\sin\omega t)$

$\therefore y_2 = -\omega^2 y$ [from Eq. (i)]

9 **(a)**

Given, $\frac{dy}{dx}\tan y = \sin(x+y) + \sin(x-y)$

$\Rightarrow \frac{dy}{dx}\tan y = 2\sin x\cos y$

$\Rightarrow \tan y\sec y\, dy = 2\sin x\, dx$

$\Rightarrow \sec y = -2\cos x + c$ [integrating]

$\Rightarrow \sec y + 2\cos x = c$

10 **(a)**

Putting $x = \tan A$, and $y = \tan B$ in the given relation, we get

$\cos A + \cos B = \lambda(\sin A - \sin B)$

$\Rightarrow \tan\left(\frac{A-B}{2}\right) = \frac{1}{\lambda}$

$\Rightarrow \tan^{-1}x - \tan^{-1}y = 2\tan^{-1}\left(\frac{1}{\lambda}\right)$

Differentiating w.r.t. to x, we get

$\frac{1}{1+x^2} - \frac{1}{1+y^2}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

Clearly, it is a differential equation of degree 1

11 **(b)**

Given, $\frac{dy}{dx} - y\tan x = e^x\sec x$

$\therefore$ $\text{IF} = e^{-\int \tan x\, dx} = e^{-\log\sec x} = \frac{1}{\sec x}$

$\therefore$ Complete solution is

$\Rightarrow y.\frac{1}{\sec x} = \int e^x\sec x.\frac{1}{\sec x}\, dx$

$\Rightarrow \frac{y}{\sec x} = e^x + c$

$\Rightarrow y\cos x = e^x + c$

12 **(c)**

$x = 1 + \frac{dy}{dx} + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + \ldots.$

$\Rightarrow x = e^{\frac{dy}{dx}} \Rightarrow \frac{dy}{dx} = \log_e x$

$\Rightarrow$ Degree of differential equation is 1.

13 **(a)**

Given, $\frac{dy}{y+1} = \frac{-\cos x}{2+\sin x}dx$

$\Rightarrow \int\frac{dy}{y+1} = -\frac{\int\cos x}{2+\sin x}dx$

$\Rightarrow \log(y+1) = -\log(2+\sin x) + \log c$

When $x = 0, y = 1$

$\Rightarrow \quad c = 4$

$\therefore \quad y + 1 = \frac{4}{2 + \sin x}$

At $x = \frac{\pi}{2}, \quad y + 1 = \frac{4}{2 + 1}$

$\Rightarrow \quad y = \frac{1}{3}$

14 **(c)**

Given, $\cos y \frac{dy}{dx} = e^{x + \sin y} + x^2 e^{\sin y}$

$\Rightarrow \quad \cos y \frac{dy}{dx} = e^{\sin y}(e^x + x^2)dx$

$\Rightarrow \quad \int \frac{\cos y}{e^{\sin y}} dy = \int (e^x + x^2)dx$

Put $\sin y = t$ in LHS$\Rightarrow \cos y \, dy = dt$

$\therefore \quad \int \frac{dt}{e^t} = \int (e^x + x^2)dx$

$\Rightarrow \quad -e^{-t} = e^x + \frac{x^3}{3} - c$

$\Rightarrow \quad e^x + e^{\sin y} + \frac{x^3}{3} = c$

15 **(a)**

The given differential equation can be written as

$\frac{y\,dx - x\,dy}{y^2} + 3x^2 e^{x^3} dx = 0 \Rightarrow d\left(\frac{x}{y}\right) + d\left(e^{x^3}\right) = 0 \Rightarrow \frac{x}{y} + e^{x^3} = C$

16 **(a)**

Given that, $\frac{dy}{dx} = \frac{2x - y}{x + 2y}$ ....(i)

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore v + x\frac{dv}{dx} = \frac{2 - v}{1 + 2v}$

$\Rightarrow x\frac{dv}{dx} = \frac{2 - v - v(1 + 2v)}{1 + 2v}$

$\Rightarrow \int \frac{1 + 2v}{2(1 - v - v^2)} dv = \int \frac{1}{x} dx$

$\Rightarrow \log k - \frac{1}{2}\log(1 - v - v^2) = \log x$

$\Rightarrow 2\log k - \log(1 - v - v^2) = 2\log x$

$\Rightarrow \log c = \log[x^2(1 - v - v^2)]$

$\Rightarrow c = x^2\left(1 - \frac{y}{x} - \frac{y^2}{x^2}\right)$

$\Rightarrow x^2 - xy - y^2 = c$

17 **(a)**

Given $y + x^2 = \frac{dy}{dx} \Rightarrow \frac{dy}{dx} - y = x^2$

This is the linear differential equation of the form

$$\frac{dy}{dx}+Py=Q$$
$\Rightarrow P=-1, Q=x^2$

$\therefore$ IF $=e^{\int P\,dx}=e^{\int -1\,dx}=e^{-x}$

Hence, required solution is

$$ye^{-x}=\int x^2e^{-x}\,dx$$
$$y\cdot e^{-x}=-x^2e^{-x}-2xe^{-x}-2e^{-x}+c$$
$$\Rightarrow y+x^2+2x+2=ce^x$$

18 **(b)**

Given, $\frac{x\,dy-y\,dx}{x}=-\left(\cos^2\frac{y}{x}\right)dx$

$\Rightarrow \quad \sec^2\left(\frac{y}{x}\right)\left(\frac{x\,dy-y\,dx}{x^2}\right)=-\frac{dx}{x}$

$\Rightarrow \quad \sec^2\left(\frac{y}{x}\right)d\left(\frac{y}{x}\right)=-\frac{dx}{x}$

$\Rightarrow \quad \tan\frac{y}{x}=-\log x+c$ [integrating]

When $x=1,\ y=\frac{\pi}{4}\ \Rightarrow c=1$

$\therefore \quad \tan\left(\frac{y}{x}\right)=1-\log x \quad \Rightarrow x=e^{1-\tan\left(\frac{y}{x}\right)}$

19 **(d)**

Given, $\frac{dy}{1+y+y^2}=(1+x)dx$

$\Rightarrow \quad \int\frac{dy}{\left(y+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=\int(1+x)dx$

$\Rightarrow \quad \frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\left(\frac{y+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)=x+\frac{x^2}{2}+\frac{c}{2}$

$\Rightarrow \quad 4\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right)=\sqrt{3}(2x+x^2)+c$

20 **(c)**

Given equation is

$$\frac{dy}{dx}=2^y\cdot 2^{-x}\Rightarrow 2^{-y}dy=2^{-x}dx$$

On integrating both sides, we get

$$\frac{2^{-y}}{\log 2}(-1)=\frac{2^{-x}}{\log 2}(-1)+c_1$$
$$\Rightarrow -\frac{2^{-y}}{\log 2}=-\frac{2^{-x}}{\log 2}+c_1$$
$$\Rightarrow -2^{-y}=-2^{-x}+c_1\log 2$$
$$\therefore \frac{1}{2^x}-\frac{1}{2^y}=c_1\log 2=c$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	A	A	D	C	A	A	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	A	C	A	A	A	B	D	C