DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth **SOLUTIONS** **SUBJECT : MATHS**
DATE : **DPP NO. :2**

Topic :-DIFFERENTIAL EQUATIONS

2 **(c)**

We have,

$\frac{dy}{dx}=\left(\frac{y}{x}\right)^{1/3}$

$\Rightarrow y^{-1/3}dy=x^{-1/3}dx$

$\Rightarrow \int y^{-1/3}dy=\int x^{-1/3}dx$

$\Rightarrow \frac{3}{2}y^{2/3}=\frac{3}{2}x^{2/3}+C$

$\Rightarrow y^{2/3}=x^{2/3}+C'$, where $C'=2C$

$\Rightarrow y^{2/3}-x^{2/3}+C'$

3 **(c)**

Given, $\frac{dy}{dx}=\frac{y\sin\left(\frac{y}{x}\right)-x}{x\sin\left(\frac{y}{x}\right)}=\frac{\frac{y}{x}\sin\left(\frac{y}{x}\right)-1}{\sin\left(\frac{y}{x}\right)}$

Put $\frac{y}{x}=u$

$\Rightarrow \frac{dy}{dx}=x\frac{du}{dx}+u$

$\therefore x\frac{du}{dx}+u=\frac{u\sin u-1}{\sin u}$

$\Rightarrow -\sin u\,du=\frac{1}{x}dx$

$\Rightarrow \cos u=\log x+c$ [integrating]

$\Rightarrow \cos\left(\frac{y}{x}\right)=\log x+c$

$\therefore y(1)=\frac{\pi}{2}$

$\therefore \cos\frac{\pi}{2}=\log 1+c$

$\Rightarrow c=0$

Thus, $\cos\left(\frac{y}{x}\right)=\log x$

4 **(b)**

Given, $\frac{dy}{dx}-\frac{y}{x}=\frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$

$\Rightarrow \quad \dfrac{\phi'\left(\frac{y}{x}\right)\left(\frac{x\ dy - y\ dx}{x^2}\right)}{\phi\left(\frac{y}{x}\right)} = \frac{1}{x}\,dx$

$\Rightarrow \quad \int \dfrac{\phi'\left(\frac{y}{x}\right)d\left(\frac{y}{x}\right)}{\phi\left(\frac{y}{x}\right)} = \int \frac{1}{x}\,dx + \log k$

$\Rightarrow \quad \log\phi\left(\frac{y}{x}\right) = \log x + \log k$

$\Rightarrow \quad \phi\left(\frac{y}{x}\right) = kx$

5 **(b)**

Given, $\dfrac{dy}{dx} = \dfrac{xy}{x^2 + y^2}$

Put $y = vx$

$\Rightarrow \quad \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$\therefore \quad v + x\dfrac{dv}{dx} = \dfrac{x^2 v}{x^2(1 + v^2)}$

$\Rightarrow \quad \int \dfrac{1 + v^2}{v^3}\,dv = -\int \dfrac{dx}{x}$

$\Rightarrow \quad -\dfrac{1}{2v^2} + \log v = -\log x + \log c$

$\Rightarrow \quad -\dfrac{1}{2}\cdot\dfrac{x^2}{y^2} + \log|y| = \log c$

$\therefore \quad y(1) = 1,\ -\dfrac{1}{2} = \log c$

$\therefore \quad -\dfrac{1}{2}\cdot\dfrac{x^2}{y^2} + \log|y| = -\dfrac{1}{2}$

$\Rightarrow \quad \log_e|y| + \dfrac{1}{2} = \dfrac{x^2}{2y^2}$

Again, when $x = x_0, y = e$

$1 + \dfrac{1}{2} = \dfrac{x_0^2}{2e^2} \Rightarrow x_0 = \sqrt{3}e$

6 **(d)**

Given, $3^{-y}dy = 3^x dx$

$\Rightarrow \quad \int 3^{-y}dy = \int 3^x dx$

$\Rightarrow \quad \dfrac{-3^{-y}}{\log 3} = \dfrac{3^x}{\log 3} + k$

$\Rightarrow \quad 3^x + 3^{-y} = c$, where $c = -k\log 3$

7 **(a)**

$(x - h)^2 + (y - k)^2 = r^2$, here only one arbitrary constant r. So, order of differential equation = 1.

9 **(b)**

Given differential equation can be rewritten as

$\dfrac{y}{(1 + y^2)}dy = \dfrac{dx}{x(1 + x^2)}$

$\Rightarrow \quad \dfrac{1}{2}\int \dfrac{2y}{(1 + y^2)}dy = \dfrac{1}{2}\int \dfrac{2x}{x^2(1 + x^2)}dx$

$\Rightarrow \quad \dfrac{1}{2}\int \dfrac{2y}{(1 + y^2)}dy = \dfrac{1}{2}\int \dfrac{dt}{t(1 + t)}$

[put $x^2 = t$ in RHS integral]

$\Rightarrow \quad \frac{1}{2}\int \frac{2y\,dy}{1+y^2} = \frac{1}{2}\int \left(\frac{1}{t} - \frac{1}{1+t}\right)dt$

$\Rightarrow \quad \frac{1}{2}\log(1+y^2) = \frac{1}{2}[\log t - \log(1+t)] + \frac{1}{2}\log c$

$\Rightarrow \quad \log(1+y^2) = \log x^2 - \log(1+x^2) + \log c$

$\Rightarrow \quad \log(1+y^2)(1+x^2) = \log cx^2$

$\Rightarrow \quad (1+y^2)(1+x^2) = cx^2$

10 **(b)**

Given, $\frac{dy}{dx} = \frac{2x-y}{x+2y}$

Put $\quad y = vx$

$\Rightarrow \quad \frac{dy}{dx} = v + \frac{xdv}{dx}$

$\therefore \quad v + x\frac{dv}{dx} = \frac{2-v}{1+2v}$

$\Rightarrow \quad \frac{xdv}{dx} = \frac{2-v-v(1+2v)}{1+2v}$

$\Rightarrow \quad \int \frac{1+2v}{2(1-v-v^2)}dv = \int \frac{1}{x}dx$

$\Rightarrow \quad \log k - \frac{1}{2}\log(1-v-v^2) = \log x$

$\Rightarrow \quad \log c = \log[x^2(1-v-v^2)]$ [put $k^2 = c$]

$\Rightarrow \quad x^2 - xy - y^2 = c$ [put $v = \frac{y}{x}$]

11 **(c)**

$y^2 = 2c(x + c^{2/3})$

$\Rightarrow \quad 2y\frac{dy}{dx} = 2c \Rightarrow c = y\frac{dy}{dx}$

$\therefore \quad y^2 = 2y\frac{dy}{dx}\left(x + \left(y\frac{dy}{dx}\right)^{2/3}\right)$

$\Rightarrow \quad \left(\frac{y}{2\frac{dy}{dx}} - x\right) = \left(y\frac{dy}{dx}\right)^{2/3}$

$\Rightarrow \quad \left(y - 2x\frac{dy}{dx}\right)^3 = \left(2\frac{dy}{dx}\right)^3\left(y\frac{dy}{dx}\right)^2$

$\Rightarrow \quad \left(y - 2x\frac{dy}{dx}\right)^3 = 8y^2\left(\frac{dy}{dx}\right)^5$

Here, order=1, degree=5

12 **(a)**

Given equation is $\frac{dx}{x} + \frac{dy}{y} = 0$

On integrating, we get

$\int \frac{dx}{x} + \int \frac{dy}{y} = 0$

$\Rightarrow \log x + \log y = \log c$

$\Rightarrow \log(xy) + \log c \Rightarrow xy = c$

13 **(a)**

Given, $\quad y = \left(x + \sqrt{1+x^2}\right)^n$

$\Rightarrow \quad \frac{dy}{dx} = n\left[x + \sqrt{1+x^2}\right]^{n-1}\left(1 + \frac{x}{\sqrt{x^2+1}}\right)$

$= \frac{n\left[x+\sqrt{1+x^2}\right]^n}{\sqrt{1+x^2}}$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)^2(1+x^2) = n^2y^2$

Again, differentiating, we get

$2\,\frac{dy}{dx}\cdot\frac{d^2y}{dx^2}(1+x^2) + 2x\left(\frac{dy}{dx}\right)^2 = 2n^2y\,\frac{dy}{dx}$

$\Rightarrow \quad \frac{d^2y}{dx^2}(1+x^2) + x\frac{dy}{dx} = n^2y$ [divide by $2\frac{dy}{dx}$]

14 **(c)**

$y = (c_1+c_2)\cos(x+c_3) - c_4e^{x+c_5}$

$y_1 = -(c_1+c_2)\sin(x+c_3) - c_4e^{x+c_5}$

$y_2 = -(c_1+c_2)\cos(x+c_3) - c_4e^{x+c_5} = -y - 2c_4e^{x+c_5}$

$y_3 = -y_1 - 2c_4e^{x+c_5}$

$y_3 = -y_1 + y_2 - y$

$\therefore$ Differential equation is

$y_3 - y_2 + y_1 - y = 0$

Which is order 3

15 **(c)**

The given equation is

$y = ae^{bx}$

$\Rightarrow \quad \frac{dy}{dx} = abe^{bx}$ ...(i)

$\Rightarrow \quad \frac{d^2y}{dx^2} = ab^2e^{bx}$ ...(ii)

$\Rightarrow \quad ae^{bx}\,\frac{d^2y}{dx^2} = a^2b^2e^{2bx}$

$\Rightarrow \quad y\,\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ [from eq. (ii)]

16 **(d)**

Let $ax + by = 1$, where $a \neq 0$

$\Rightarrow \quad a\frac{dx}{dy} + b = 0$

$\Rightarrow \quad a\frac{d^2x}{dy^2} = 0$

$\Rightarrow \quad \frac{d^2x}{dy^2} = 0$

17 **(a)**

We have, $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$

Putting $x = \sin A$, $y = \sin B$, we get

$\cos A + \cos B = a(\sin A - \sin B)$

$\Rightarrow \quad \cot\frac{A-B}{2} = a$

$\Rightarrow \quad A - B = 2\cot^{-1} a$

$\Rightarrow \quad \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$

On differentiating w.r.t.x, we get

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$\Rightarrow \quad \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

Clearly, it is differential equation of the first order and first degree.

18 **(b)**

Given differential equation is

$$\frac{dy}{dx} = e^{y+x} + e^{y-x}$$

$\Rightarrow \quad \int e^{-y} dy = \int (e^x - e^{-x}) dx$

$\Rightarrow \quad -e^{-y} = e^x - e^{-x} - c$

$\Rightarrow \quad e^{-y} = e^{-x} - e^{-x} + c$

19 **(a)**

Given, $\frac{dy}{dx} + \frac{1}{x}.y = 3x$

$\therefore \quad \text{IF} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$

20 **(a)**

Given, $\frac{\sec^2 x}{\tan x} dx = -\frac{\sec^2 y}{\tan y} dy$

$\Rightarrow \quad \int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} dy$

Put $\quad \tan x = u$

$\Rightarrow \quad \sec^2 x \, dx = du$

And $\quad \tan y = v$

$\Rightarrow \quad \sec^2 y \, dy = dv$

$\therefore \quad \int \frac{du}{u} = -\int \frac{dv}{v}$

$\Rightarrow \quad \log u = -\log v + \log c \Rightarrow uv = c$

$\therefore \quad \tan x.\tan y = c$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	C	B	B	D	A	B	B	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	A	C	C	D	A	B	A	A