DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :6

Topic :-DIFFERENTIAL EQUATIONS

1 **(d)**

We have,

$e^x \cos y \ dx - e^x \sin y \ dy = 0$

$\Rightarrow \cos y \, d(e^x) + e^x d(\cos y) = 0$

$\Rightarrow d(e^x \cos y) = 0 \Rightarrow e^x \cos y = C$ [On integrating]

2 **(d)**

$y = ae^{mx} + be^{-mx}$

On differentiating w.r.t.x, we get

$\frac{dy}{dx} = mae^{mx} - mbe^{-mx}$

Again, on differentiating, we get

$\frac{d^2y}{dx^2} = m^2ae^{mx} + m^2be^{-mx}$

$= m^2(ae^{mx} + be^{-mx}) = m^2y$

$\Rightarrow \frac{d^2y}{dx^2} - m^2y = 0$

3 **(b)**

We have,

$\frac{dy}{dx} + y = e^{-x}$ ...(i)

This is a linear differential equation with I.F. $= e^{\int 1 \cdot dx} = e^x$

Multiplying both sides of (i) by I.F. $= e^x$ and integrating, we get

$y\, e^x = \int e^x \, e^{-x} dx + C \Rightarrow y\, e^x = x + C$

It is given that $y = 0$ when $x = 0$

$\therefore 0 = 0 + C \Rightarrow C = 0$

Hence, $y\, e^x = x \Rightarrow y = xe^{-x}$

4 **(c)**

Given, $\frac{dx}{1+x^2} + \frac{dy}{1+y^2} = 0$

$\Rightarrow \tan^{-1} x + \tan^{-1} y = \tan^{-1} c$ [integrating]

$\Rightarrow \quad \frac{x+y}{1-xy} = c$

$\Rightarrow \quad x + y = c(1 - xy)$

5 **(c)**

Given, IF $= x$

$\therefore \quad e^{\int P\,dx} = x$

$\Rightarrow \quad \int P dx = \log x$

$\Rightarrow \quad P = \frac{d}{dx}\log x = \frac{1}{x}$

6 **(b)**

Given differential equation is

$$\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$\Rightarrow \quad \left(\frac{d^2y}{dx^2}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$$

Hence, order is 2.

7 **(b)**

Given, $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

$\Rightarrow \quad \int \frac{dy}{\sqrt{1-y^2}} + \int \frac{dx}{\sqrt{1-x^2}} = 0$

$\Rightarrow \quad \sin^{-1} x + \sin^{-1} y = c$

8 **(d)**

We have,

$x\,dy - y\,dx = 0$

$\Rightarrow \frac{dy}{y} - \frac{dx}{x} = 0$

$\Rightarrow \log y - \log x = \log C$ [On integrating]

$\Rightarrow \frac{y}{x} = C \Rightarrow y = C\,x$

Clearly, it represents a family of straight lines passing through the origin

9 **(c)**

Let the equation of circle passing through given points is

$x^2 + y^2 - 2fy = a^2$

$\Rightarrow \quad 2x + 2yy_1 - 2fy_1 = 0$ ...(i)

$\Rightarrow \quad x = y_1(f - y)$

$\Rightarrow \quad x = y_1\left(\frac{x^2 + y^2 - a^2}{2y} - y\right)$ [from Eq. (i)]

$\Rightarrow \quad y_1(y^2 - x^2 + a^2) + 2xy = 0$

11 **(a)**

Given, $\frac{dy}{dx}+\left(\frac{x}{y}\right)^2-\left(\frac{x}{y}\right)+1=0$

Put $v=\frac{x}{y}$ $\Rightarrow x=vy$

$\Rightarrow \frac{dx}{dy}=v+y\frac{dv}{dy}$

$\therefore v+y\frac{dv}{dy}+v^2-v+1=0$

$\Rightarrow \frac{dv}{v^2+1}+\frac{dy}{y}=0$

$\Rightarrow \tan^{-1}v+\log y+c=0$ [integrating]

$\Rightarrow \tan^{-1}\frac{x}{y}+\log y+c=0$

12 **(a)**

Given, $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{5/2}=\frac{d^3y}{dx^3}$

$\Rightarrow \left(\frac{d^3y}{dx^3}\right)^2=\left[1+\left(\frac{dy}{dx}\right)^2\right]^5$

Here, order=3, degree=2

13 **(a)**

Given equation is

$x\,dy-y\,dx+x^2e^xdx=0$

$\Rightarrow \frac{x\,dy-y\,dx}{x^2}+e^xdx=0$

$\Rightarrow d\left(\frac{y}{x}\right)+d(e^x)=0$

$\Rightarrow \frac{y}{x}+e^x=c$

14 **(c)**

Given differential equation is

$\frac{dy}{dx}=\frac{x-y+3}{2(x-y)+5}$

Put $x-y=v \Rightarrow \frac{dy}{dx}=1-\frac{dv}{dx}$

$\therefore 1-\frac{dv}{dx}=\frac{v+3}{2v+5}\Rightarrow\frac{dv}{dx}=\frac{v+2}{2v+5}$

$\Rightarrow \int\left(2+\frac{1}{v+2}\right)dv=\int dx$

$\Rightarrow 2v+\log(v+2)=x+c$

$\Rightarrow 2(x-y)+\log(x-y+2)=x+c$

15 **(c)**

The given equation is $Ax^2+By^2=1$

$\Rightarrow 2Ax+2By\frac{dy}{dx}=0$ ...(i)

$\Rightarrow \quad 2A + 2B\left\{\left(\frac{dy}{dx}\right)^2 + y\ \frac{d^2y}{dx^2}\right\} = 0 \quad \text{...(ii)}$

Eliminating A and B from Eqs. (i) and (ii), we get

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 - \frac{y}{x}.\frac{dy}{dx} = 0$$

Here, order =2, degree =1

16 **(a)**

The given equation is

$(y+3)dy = (x+2)dx$

$\Rightarrow \frac{y^2}{2} + 3y = \frac{x^2}{2} + 2x + c$

Since, it passes through (2, 2).

$\therefore 2 + 6 = 2 + 4 + c \Rightarrow c = 2$

$\therefore \frac{y^2}{2} + 3y = \frac{x^2}{2} + 2x + 2$

$\Rightarrow y^2 + 6y = x^2 + 4x + 4$

$\Rightarrow x^2 + 4x - y^2 - 6y + 4 = 0$

17 **(b)**

We have,

$y^2 = 4a(x+a) \quad \text{...(i)}$

Clearly, it is a one parameter family of parabolas

Differentiating (i) w.r.t. to x, we get

$2y\frac{dy}{dx} = 4a \Rightarrow a = \frac{1}{2}y\frac{dy}{dx}$

Substituting this value of a in (i), we get

$y^2 = 2y\frac{dy}{dx}\left(x + \frac{1}{2}y\frac{dy}{dx}\right) \Rightarrow y^2\left(\frac{dy}{dx}\right) + 2xy\frac{dy}{dx} - y^2 = 0$

18 **(b)**

Given differential equation can be rewritten as

$\frac{dy}{dx} + \frac{1}{x\log x}.y = \frac{2}{x}$

$\therefore \quad \text{IF} = e^{\int \frac{1}{x\log x}dx} = e^{\int \frac{1/x}{\log x}dx} = e^{\log\log x} = \log x$

20 **(a)**

Given, $\frac{dy}{dx} + y\tan x = \sec x$

$\therefore \quad IF e^{\int P\,dx} = e^{\int \tan x\ dx} = \sec x$

$\therefore$ Solution is $y\sec x = \int \sec^2 x\,dx + c$

$\Rightarrow \quad y\sec x = \tan x + c$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	D	B	C	C	B	B	D	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	A	C	C	A	B	B	D	A