DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :5

Topic :-DIFFERENTIAL EQUATIONS

2 **(a)**

Given, $(1+x)y\,dx+(1-y)x\,dy=0$

$\Rightarrow \quad \frac{(1-y)}{y}dy+\frac{(1+x)}{x}dx=0$

$\Rightarrow \quad \int\left(\frac{1}{y}-1\right)dy+\int\left(\frac{1}{x}+1\right)dx=0$

$\Rightarrow \quad \log_e y - y+\log_e x + x = c$

$\Rightarrow \quad \log_e(xy) + x - y = c$

3 **(b)**

Given, $\quad y^2=2c(x+\sqrt{c})$

$\Rightarrow \quad 2yy_1=2c$

$\Rightarrow \quad c=yy_1$

$\therefore \quad y^2=2yy_1(x+\sqrt{yy_1})$

$\Rightarrow \quad y^2 - 2yy_1x=\sqrt{yy_1}.2yy_1$

$\Rightarrow \quad (y^2-2yy_1x)^2=4(yy_1)^3$

$\therefore$ The degree of above equation is 3 and order is 1.

4 **(d)**

Given differential equation is

$\frac{dy}{dx}=\frac{1}{x+y^2}$

$\Rightarrow \quad \frac{dx}{dy} - x=y^2$

Here, $\quad P=-1,\ Q=y^2$

$\text{IF}=e^{\int -1\,dy}=e^{-y}$

$\therefore$ Solution is

$xe^{-y}=\int e^{-y}\ y^2\ \ dy$

$=-e^{-y}y^2+\int 2e^{-y}\,y\,dy$

$$= -e^{-y}y^2 + 2[-e^{-y}y + \int e^{-y}\ dy] + c$$
$$= -e^{-y}y^2 + 2[-e^{-y}y - e^{-y}] + c$$
$$\Rightarrow \quad xe^{-y} = e^{-y}(-y^2 - 2y - 2) + c$$
$$\Rightarrow \quad x = -y^2 - 2y - 2 + ce^y$$

5 **(a)**

Given differential equation can be rewritten as

$$\Rightarrow \quad \frac{dx}{dy} - \frac{x}{y^2} = 2y$$
$$\therefore \quad \text{IF} = e^{-\int \frac{1}{y^2} dy} = e^{1/y}$$

6 **(c)**

It is given that $\frac{dy}{dx} = \frac{x}{y}$

On integration, we get $y^2 - x^2 = C$, which is a rectangular hyperbola

7 **(a)**

Given differential equation is

$$(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$$
$$\Rightarrow \quad (1 + y^2)\frac{dx}{dy} = -x + e^{\tan^{-1} y}$$
$$\Rightarrow \quad \frac{dx}{dy} + \frac{x}{1 + y^2} = \frac{e^{\tan^{-1} y}}{1 + y^2}$$

Which is a linear differential equation,

Here, $P = \frac{1}{1 + y^2}$, $Q = \frac{e^{\tan^{-1} y}}{1 + y^2}$

$$\text{IF} = e^{\int P\,dy} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$$

$\therefore$ Solution is

$$x.\text{IF} = \int Q.\text{IF}\ dy + c$$
$$xe^{\tan^{-1} y} = \int \frac{e^{\tan^{-1} y}}{1 + y^2}.e^{\tan^{-1} y} + \frac{c}{2}$$
$$\Rightarrow \quad xe^{\tan^{-1} y} = \frac{e^{2\tan^{-1} y}}{2} + \frac{c}{2}$$
$$\therefore \quad 2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + c$$

8 **(d)**

Given differential equation can be rewritten as

$$\frac{y\,dy}{y + 1} = \frac{e^x dx}{e^x + 1}$$
$$\Rightarrow \quad \left(1 - \frac{1}{y + 1}\right)dy = \frac{e^x}{e^x + 1}dx$$
$$\Rightarrow \quad y - \log(y + 1) = \log(e^x + 1) - \log c \quad \text{[integrating]}$$
$$\Rightarrow \quad y = \log\frac{(e^x + 1)(y + 1)}{c}$$
$$\Rightarrow \quad (e^x + 1)(y + 1) = ce^y$$

9 **(d)**

The equation of all the straight lines passing through origin is

$$y = mx$$

$\Rightarrow \quad \frac{dy}{dx} = m \quad$ ...(i)

$\therefore$ From Eq. (i), $\quad y = \frac{dy}{dx}x$

10 **(b)**

Given, $\frac{dy}{dx} = \sin(x+y)\tan(x+y) - 1$

Put $\quad x + y = z \Rightarrow 1 + \frac{dy}{dx} = \frac{dz}{dx}$

$\therefore \quad \frac{dz}{dx} - 1 = \sin z \tan z - 1$

$\Rightarrow \quad \int \frac{\cos z}{\sin^2 z} dz = \int dx$

Put $\quad \sin z = t$

$\therefore \quad \int \frac{1}{t^2} dt = x - c \Rightarrow -\frac{1}{t} = x - c$

$\Rightarrow \quad -\operatorname{cosec} z = x - c$

$\Rightarrow \quad x + \operatorname{cosec}(x+y) = c$

11 **(a)**

Given, $\quad \sin^{-1} x + \sin^{-1} y = c$

$\Rightarrow \quad \frac{dx}{\sqrt{1-x^2}} + \frac{dy}{\sqrt{1-y^2}} = 0$

$\Rightarrow \quad \sqrt{1-y^2}\,dx + \sqrt{1-x^2}\,dy = 0$

12 **(d)**

$x\frac{dy}{dx} + (1+x)y = x$

$\Rightarrow \quad \frac{dy}{dx} + \frac{1+x}{x}y = 1$

$\text{IF} = e^{\int \frac{1+x}{x} dx}$

$= e^{\int \frac{dx}{x} + \int dx}$

$= e^{\log x + x}$

$= xe^x$

13 **(c)**

We have,

$(x + 2y^3)\frac{dy}{dx} = y$

$\Rightarrow y\frac{dy}{dx} = x + 2y^3 \Rightarrow \frac{dx}{dy} - \frac{x}{y} = 2y^2 \quad$ ...(i)

This is linear differential equation with

$\text{I.F.} = e^{\int -\frac{1}{y}dy} = e^{-\log y} = \frac{1}{y}$

Multiplying (i) by I.F. and integrating, we get

$\frac{x}{y} = \int 2y\,dy \Rightarrow \frac{x}{y} = y^2 + C \Rightarrow x = y(y^2 + C)$

14 **(a)**
We have,

$$\frac{d^2y}{dx^2}=\sqrt{1+\left(\frac{dy}{dx}\right)^3}\Rightarrow\left(\frac{d^2y}{dx^2}\right)^2=1+\left(\frac{dy}{dx}\right)^3$$

Clearly, it is a second order second degree differential equation
15 **(a)**
Given equation is $\frac{dy}{dx}=\frac{y+1}{x-1}\Rightarrow\frac{dy}{y+1}=\frac{dx}{x-1}$
On integrating both sides

$$\int\frac{dy}{y+1}=\int\frac{dx}{x-1}$$

$\Rightarrow\log(y+1)=\log(x-1)+\log c$
$\Rightarrow\log(y+1)=\log(x-1)c$
$\Rightarrow y+1=(x-1)c$
At $x=1\Rightarrow y=-1$
Whereas $y(1)=2$.
Hence, the above solution is not possible.
16 **(a)**
Given, $\frac{dy}{dx}=\frac{y(x+y)}{x(x-y)}$

Put $y=vx$

$\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore v+x\frac{dv}{dx}=\frac{vx(x+vx)}{x(x-vx)}$

$$x\frac{dv}{dx}=\frac{2v^2}{1-v}$$

$\Rightarrow \frac{1}{2}\left[\frac{1}{v^2}-\frac{1}{v}\right]dv=\frac{dx}{x}$

$\Rightarrow \frac{1}{2}\left[-\frac{1}{v}-\log v\right]=\log x+c_1$

$\Rightarrow \frac{x}{v}+\log\left(\frac{y}{x}\right)+2\log x=-2c$

$\Rightarrow \frac{x}{y}+\log(xy)=c$ [let $c=-2c_1$]

17 **(a)**
We have,
$y^2dy=x^2dx$
Integrating we get $y^3-x^3=C$

18 **(b)**

Given, $\frac{d^2y}{dx^2} = e^{-2x}$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{e^{-2x}}{2} + c_2$ [integrating]

$\Rightarrow \quad y = \frac{e^{-2x}}{4} + c_2x + c_3$ [integrating]

But $\quad y = c_1e^{-2x} + c_2x + c_3$ [given]

$\therefore \quad c_1 = \frac{1}{4}$

19 **(d)**

Given, $x\left(\frac{dy}{dx}\right)^2 + 2\sqrt{xy}\frac{dy}{dx} + y = 0$

$\Rightarrow \quad \left(\sqrt{x}\ \frac{dy}{dx} + \sqrt{y}\right)^2 = 0$

$\Rightarrow \quad \frac{1}{\sqrt{y}}dy + \frac{1}{\sqrt{x}}dx = 0$

$\Rightarrow \quad 2\sqrt{y} + 2\sqrt{x} = c_1$

$\Rightarrow \quad \sqrt{x} + \sqrt{y} = c$

20 **(b)**

$y = mx + \frac{4}{m}$ ...(i)

$\therefore \quad \frac{dy}{dx} = m$

From Eq. (i), we get

$y = x\left(\frac{dy}{dx}\right) + \frac{4}{\left(\frac{dy}{dx}\right)}$

$\Rightarrow \quad y\left(\frac{dy}{dx}\right) = x\left(\frac{dy}{dx}\right)^2 + 4$

$\Rightarrow \quad x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} + 4 = 0$

Which is required differential equations.

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	B	D	A	C	A	D	D	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	D	C	A	A	A	A	B	D	B