

Topic :-DIFFERENTIAL EQUATIONS

2 (a)

Given, $(1+x)y \, dx + (1-y)x \, dy = 0$

$$\Rightarrow \frac{(1-y)}{y} \, dy + \frac{(1+x)}{x} \, dx = 0$$

$$\Rightarrow \int \left(\frac{1}{y} - 1 \right) dy + \int \left(\frac{1}{x} + 1 \right) dx = 0$$

$$\Rightarrow \log_e y - y + \log_e x + x = c$$

$$\Rightarrow \log_e(xy) + x - y = c$$

3 (b)

Given, $y^2 = 2c(x + \sqrt{c})$

$$\Rightarrow 2yy_1 = 2c$$

$$\Rightarrow c = yy_1$$

$$\therefore y^2 = 2yy_1(x + \sqrt{yy_1})$$

$$\Rightarrow y^2 - 2yy_1x = \sqrt{yy_1} \cdot 2yy_1$$

$$\Rightarrow (y^2 - 2yy_1x)^2 = 4(yy_1)^3$$

\therefore The degree of above equation is 3 and order is 1.

4 (d)

Given differential equation is

$$\frac{dy}{dx} = \frac{1}{x+y^2}$$

$$\Rightarrow \frac{dx}{dy} - x = y^2$$

Here, $P = -1, Q = y^2$

$$\text{IF} = e^{\int -1 \, dy} = e^{-y}$$

\therefore Solution is

$$\begin{aligned} xe^{-y} &= \int e^{-y} y^2 \, dy \\ &= -e^{-y} y^2 + \int 2e^{-y} y \, dy \end{aligned}$$



$$\begin{aligned}
&= -e^{-y} y^2 + 2[-e^{-y} y + \int e^{-y} dy] + c \\
&= -e^{-y} y^2 + 2[-e^{-y} y - e^{-y}] + c \\
\Rightarrow xe^{-y} &= e^{-y}(-y^2 - 2y - 2) + c \\
\Rightarrow x &= -y^2 - 2y - 2 + ce^y
\end{aligned}$$

5 (a)

Given differential equation can be rewritten as

$$\begin{aligned}
\Rightarrow \frac{dx}{dy} - \frac{x}{y^2} &= 2y \\
\therefore \text{IF} &= e^{-\int \frac{1}{y^2} dy} = e^{1/y}
\end{aligned}$$

6 (c)

It is given that $\frac{dy}{dx} = \frac{x}{y}$

On integration, we get $y^2 - x^2 = C$, which is a rectangular hyperbola

7 (a)

Given differential equation is

$$\begin{aligned}
(1 + y^2) + (x - e^{\tan^{-1} y}) \frac{dy}{dx} &= 0 \\
\Rightarrow (1 + y^2) \frac{dx}{dy} &= -x + e^{\tan^{-1} y} \\
\Rightarrow \frac{dx}{dy} + \frac{x}{1 + y^2} &= \frac{e^{\tan^{-1} y}}{1 + y^2}
\end{aligned}$$

Which is a linear differential equation,

$$\begin{aligned}
\text{Here, } P &= \frac{1}{1 + y^2}, Q = \frac{e^{\tan^{-1} y}}{1 + y^2} \\
\text{IF} &= e^{\int P dy} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}
\end{aligned}$$

\therefore Solution is

$$\begin{aligned}
x \cdot \text{IF} &= \int Q \cdot \text{IF} dy + c \\
xe^{\tan^{-1} y} &= \int \frac{e^{\tan^{-1} y}}{1 + y^2} \cdot e^{\tan^{-1} y} + \frac{c}{2} \\
\Rightarrow xe^{\tan^{-1} y} &= \frac{e^{2\tan^{-1} y}}{2} + \frac{c}{2} \\
\therefore 2xe^{\tan^{-1} y} &= e^{2\tan^{-1} y} + c
\end{aligned}$$

8 (d)

Given differential equation can be rewritten as

$$\begin{aligned}
\frac{ydy}{y+1} &= \frac{e^x dx}{e^x + 1} \\
\Rightarrow \left(1 - \frac{1}{y+1}\right) dy &= \frac{e^x}{e^x + 1} dx \\
\Rightarrow y - \log(y+1) &= \log(e^x + 1) - \log c \quad [\text{integrating}] \\
\Rightarrow y &= \log \frac{(e^x + 1)(y+1)}{c} \\
\Rightarrow (e^x + 1)(y+1) &= ce^y
\end{aligned}$$

9 (d)

The equation of all the straight lines passing through origin is

$$y = mx \\ \Rightarrow \frac{dy}{dx} = m \quad \dots(i)$$

\therefore From Eq. (i), $y = \frac{dy}{dx} x$

10 **(b)**

$$\text{Given, } \frac{dy}{dx} = \sin(x + y)\tan(x + y) - 1$$

$$\text{Put } x + y = z \Rightarrow 1 + \frac{dy}{dx} = \frac{dz}{dx}$$

$$\therefore \frac{dz}{dx} - 1 = \sin z \tan z - 1$$

$$\Rightarrow \int \frac{\cos z}{\sin^2 z} dz = \int dx$$

$$\text{Put } \sin z = t$$

$$\therefore \int \frac{1}{t^2} dt = x - c \Rightarrow -\frac{1}{t} = x - c$$

$$\Rightarrow -\operatorname{cosec} z = x - c$$

$$\Rightarrow x + \operatorname{cosec}(x + y) = c$$

11 **(a)**

$$\text{Given, } \sin^{-1} x + \sin^{-1} y = c$$

$$\Rightarrow \frac{dx}{\sqrt{1-x^2}} + \frac{dy}{\sqrt{1-y^2}} = 0$$

$$\Rightarrow \sqrt{1-y^2} dx + \sqrt{1-x^2} dy = 0$$

12 **(d)**

$$x \frac{dy}{dx} + (1+x)y = x$$

$$\Rightarrow \frac{dy}{dx} + \frac{1+x}{x} y = 1$$

$$\text{IF} = e^{\int \frac{1+x}{x} dx}$$

$$= e^{\int \frac{1}{x} dx + \int dx}$$

$$= e^{\log x + x}$$

$$= xe^x$$

13 **(c)**

We have,

$$(x + 2y^3) \frac{dy}{dx} = y$$

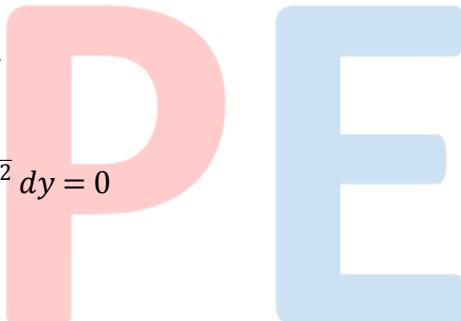
$$\Rightarrow y \frac{dy}{dx} = x + 2y^3 \Rightarrow \frac{dx}{dy} - \frac{x}{y} = 2y^2 \quad \dots(i)$$

This is linear differential equation with

$$\text{I.F.} = e^{\int -\frac{1}{y} dy} = e^{-\log y} = \frac{1}{y}$$

Multiplying (i) by I.F. and integrating, we get

$$\frac{x}{y} = \int 2y dy \Rightarrow \frac{x}{y} = y^2 + C \Rightarrow x = y(y^2 + C)$$



14 (a)

We have,

$$\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^3} \Rightarrow \left(\frac{d^2y}{dx^2}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^3$$

Clearly, it is a second order second degree differential equation

15 (a)

Given equation is $\frac{dy}{dx} = \frac{y+1}{x-1} \Rightarrow \frac{dy}{y+1} = \frac{dx}{x-1}$

On integrating both sides

$$\int \frac{dy}{y+1} = \int \frac{dx}{x-1}$$

$$\Rightarrow \log(y+1) = \log(x-1) + \log c$$

$$\Rightarrow \log(y+1) = \log(x-1)c$$

$$\Rightarrow y+1 = (x-1)c$$

$$\text{At } x=1 \Rightarrow y=-1$$

$$\text{Whereas } y(1)=2.$$

Hence, the above solution is not possible.

16 (a)

Given, $\frac{dy}{dx} = \frac{y(x+y)}{x(x-y)}$

Put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\therefore v + x \frac{dv}{dx} = \frac{vx(x+vx)}{x(x-vx)}$$

$$x \frac{dv}{dx} = \frac{2v^2}{1-v}$$

$$\Rightarrow \frac{1}{2} \left[\frac{1}{v^2} - \frac{1}{v} \right] dv = \frac{dx}{x}$$

$$\Rightarrow \frac{1}{2} \left[-\frac{1}{v} - \log v \right] = \log x + c_1$$

$$\Rightarrow \frac{x}{v} + \log \left(\frac{y}{x} \right) + 2 \log x = -2c$$

$$\Rightarrow \frac{x}{y} + \log(xy) = c \quad [\text{let } c = -2c_1]$$

17 (a)

We have,

$$y^2 dy = x^2 dx$$

$$\text{Integrating we get } y^3 - x^3 = C$$



18 **(b)**

Given, $\frac{d^2y}{dx^2} = e^{-2x}$

$$\Rightarrow \frac{dy}{dx} = -\frac{e^{-2x}}{2} + c_2 \quad [\text{integrating}]$$

$$\Rightarrow y = \frac{e^{-2x}}{4} + c_2x + c_3 \quad [\text{integrating}]$$

But $y = c_1e^{-2x} + c_2x + c_3 \quad [\text{given}]$

$$\therefore c_1 = \frac{1}{4}$$

19 **(d)**

Given, $x\left(\frac{dy}{dx}\right)^2 + 2\sqrt{xy}\frac{dy}{dx} + y = 0$

$$\Rightarrow \left(\sqrt{x}\frac{dy}{dx} + \sqrt{y}\right)^2 = 0$$

$$\Rightarrow \frac{1}{\sqrt{y}}dy + \frac{1}{\sqrt{x}}dx = 0$$

$$\Rightarrow 2\sqrt{y} + 2\sqrt{x} = c_1$$

$$\Rightarrow \sqrt{x} + \sqrt{y} = c$$

20 **(b)**

$$y = mx + \frac{4}{m}$$

$$\therefore \frac{dy}{dx} = m$$

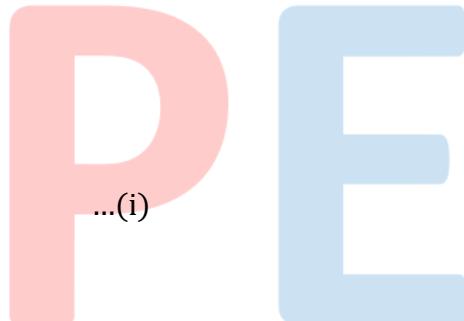
From Eq. (i), we get

$$y = x\left(\frac{dy}{dx}\right) + \frac{4}{\left(\frac{dy}{dx}\right)}$$

$$\Rightarrow y\left(\frac{dy}{dx}\right) = x\left(\frac{dy}{dx}\right)^2 + 4$$

$$\Rightarrow x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} + 4 = 0$$

Which is required differential equations.



ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	B	D	A	C	A	D	D	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	D	C	A	A	A	A	B	D	B

P E