DPP
DAILY PRACTICE PRODUCTS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :4

Topic :-DIFFERENTIAL EQUATIONS

1 **(d)**

Given that, $\frac{dy}{dx}+y=e^{-x}$

It is a linear differential equation, comparing with the standard equation

$\frac{dy}{dx}+Py=Q$

$\Rightarrow P=1, Q=e^{-x}$

$\therefore$ IF $=e^{\int P\,dx}=e^{x}$

$\therefore$ Required solution is

$ye^{x}=\int e^{-x}e^{x}\,dx+c=\int 1\,dx+c$

$\Rightarrow ye^{x}=x+c$

At $x=0, y=0 \;\therefore c=0$

Hence, the required solution is

$ye^{x}=x \Rightarrow y=xe^{-x}$

2 **(c)**

Given, $\frac{dy}{dx}=2\frac{y}{x}$ $\quad(\therefore y=mx)$

$\Rightarrow \int\frac{dy}{y}=2\int\frac{dx}{x}$

$\Rightarrow \log y=2\log x+\log c$

$\Rightarrow y=cx^{2}$

Which represent a parabola of the form

$x^{2}=4ay$

3 **(d)**

Given, $\frac{dy}{dx}+\frac{2}{x}y=x$

$\therefore$ Integrating factor $=e^{\int\frac{2}{x}dx}=x^{2}$

$\therefore$ Required solution is

$y.x^{2}=\int x^{3}\,dx=\frac{x^{4}}{4}+\frac{c}{4}$

$\therefore \quad y=\frac{x^{4}+c}{4x^{2}}$

4 **(a)**

We have,

$$y\frac{dy}{dx}=x-1 \Rightarrow y\,dy=(x-1)dx \Rightarrow \frac{y^2}{2}=\frac{x^2}{2}-x+C$$

For $x=1$, we have $y=1$

$$\therefore \frac{1}{2}=\frac{1}{2}-1+C \Rightarrow C=1$$

Hence, $\frac{y^2}{2}=\frac{x^2}{2}-x+1 \Rightarrow y^2=x^2-2x+2$

5 **(a)**

Equation of line whose slope is equal to y intercept, is

$$y=cx+c=c(x+1)$$

$$\Rightarrow \quad \frac{dy}{dx}=c$$

$$\therefore \quad \frac{dy}{dx}=\frac{y}{x+1}$$

$$\Rightarrow \quad (x+1)\frac{dy}{dx}-y=0$$

6 **(b)**

Given that, $x^2y-x^3\frac{dy}{dx}=y^4\cos x$

i.e., $\quad x^3\frac{dy}{dx}-x^2y=-y^4\cos x$

on dividing by $-y^4x^3$, we get

$$-\frac{1}{y^4}\frac{dy}{dx}+\frac{1}{y^3}.\frac{1}{x}=\frac{1}{x^3}\cos x$$

Put $\frac{1}{y^3}=V$

$$\Rightarrow \quad -\frac{1}{y^4}\frac{dy}{dx}=\frac{1}{3}\frac{dV}{dx}$$

$$\therefore \quad \frac{1}{3}\frac{dV}{dx}+\frac{1}{x}V=\frac{1}{x^3}\cos x$$

$$\Rightarrow \quad \frac{dV}{dx}+\frac{3}{x}V=\frac{3}{x^3}\cos x$$

Which is linear in V.

$$\therefore \quad IF=e^{\int\frac{3}{x}dx}=e^{3\log x}=x^3$$

So, the solution is

$$x^3V=\int x^3.\frac{3}{x^3}\cos x\,dx+c$$

$= 3\sin x + c$

$\Rightarrow \quad \frac{x^3}{y^3} = 3\sin x + c$

Putting $x = 0, y = 1$, we get $c = 0$

Hence, the solution is $x^3 = 3y^3 \sin x$

7 **(d)**

$\because$ Equation of normal at $P(1, 1)$ is

$ay + x = a + 1$ (given)

$\because$ Slope of normal at $(1, 1) = -\frac{1}{a}$

$\therefore$ Slope of tangent at $(1, 1) = a$ ...(i)

Also, given $\frac{dy}{dx} \propto y$

$\Rightarrow \frac{dy}{dx} = ky$

$\left.\frac{dy}{dx}\right|_{(1,1)} = k = a$ [from Eq. (i)]

Then, $\frac{dy}{dx} = ay$

$\Rightarrow \frac{dy}{y} = a\,dx$

$\Rightarrow \text{In}|y| = ax + c$

$\because$ It is passing through (1,1), then $c = -a$

$\Rightarrow \text{In}|y| = a(x - 1)$

$\Rightarrow |y| = e^{a(x-1)}$

8 **(a)**

Given, $\frac{dy}{dx} + 1 = e^{x+y}$

Put $x + y = z$

$\Rightarrow \quad 1 + \frac{dy}{dx} = \frac{dz}{dx}$

$\therefore \quad \frac{dz}{dx} = e^z$

$\Rightarrow \quad \int e^{-z} dz = \int dx$

$\Rightarrow \quad -e^{-z} = x + c$

$\Rightarrow \quad x + e^{-(x+y)} + c = 0$

9 **(a)**

The given equation can be written as

$$\left(\frac{dx}{x} - \frac{dy}{y}\right) + \frac{(x^2dy - y^2dx)}{(x-y)^2} = 0$$

$$\Rightarrow \left(\frac{dx}{x} - \frac{dy}{y}\right) + \frac{\left(\frac{dy}{y^2} - \frac{dx}{x^2}\right)}{\left(\frac{1}{y} - \frac{1}{x}\right)^2} = 0$$

$$\Rightarrow \left(\frac{dx}{x} - \frac{dy}{y}\right) + \frac{\frac{dy}{y^2} - \frac{dx}{x^2}}{\left(\frac{1}{x} - \frac{1}{y}\right)^2} = 0$$

On integrating both sides, we get

$\text{In } |x| - \text{In } |y| - \frac{1}{\left(\frac{1}{x} - \frac{1}{y}\right)} = c$

$\Rightarrow \text{In } \left|\frac{x}{y}\right| - \frac{xy}{(y - x)} = c$

$\Rightarrow \text{In } \left|\frac{x}{y}\right| + \frac{xy}{(x - y)} = c$

10 **(a)**

We have, $e^{dy/dx} = x$

$\Rightarrow \frac{dy}{dx} = \log x$

$\therefore$ Degree is 1.

11 **(a)**

Given differential equation can be rewritten as

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{4}\times 12} = \left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}\times 12}$$

$$\Rightarrow \quad \left[1 + \left(\frac{dy}{dx}\right)^2\right]^9 = \left(\frac{d^2y}{dx^2}\right)^4$$

Here, we see that order of highest derivative is 2 and degree is 4.

12 **(c)**

We have,

$\tan^{-1} x + \tan^{-1} y = C$

Differentiating w.r.t. to x, we get

$\frac{1}{1 + x^2} + \frac{1}{1 + y^2}\frac{dy}{dx} = 0$

$\Rightarrow (1 + x^2)dy + (1 + y^2)dx = 0,$

which is the required differential equation

13 **(d)**

Given that, $\frac{dy}{dx} = 1 + x + y^2 + xy^2$

This can be rewritten as, we get

$\frac{dy}{1 + y^2} = (1 + x)dx$

On integrating both sides, we get

$\int \frac{dy}{1 + y^2} = \int (1 + x)dx$

$\Rightarrow \tan^{-1} y = x + \frac{x^2}{2} + c$

At $x = 0, y = 0$
$\Rightarrow 0 = 0 + 0 + c \Rightarrow c = 0$

$$\therefore \tan^{-1} y = x + \frac{x^2}{2} \Rightarrow y = \tan\left(x + \frac{x^2}{2}\right)$$

15 **(b)**

$$\frac{dy}{dx} = -\left(\frac{\cos x - \sin x}{\sin x + \cos x}\right)$$

$$\Rightarrow dy = -\left(\frac{\cos x - \sin x}{\sin x + \cos x}\right)dx$$

On integrating both sides, we get

$$y = -\log(\sin x + \cos x) + \log c$$

$$\Rightarrow y = \log\left(\frac{c}{\sin x + \cos x}\right)$$

$$\Rightarrow e^y = \frac{c}{\sin x + \cos x}$$

$$\Rightarrow e^y(\sin x + \cos x) = c$$

16 **(a)**

Given, $\frac{x\ dy - y\ dx}{y^2} = dy$

$\Rightarrow \quad d\left(\frac{x}{y}\right) = -dy$

$\Rightarrow \quad \frac{x}{y} = -y + c$ [integrating]

As $\quad y(1) = 1 \Rightarrow c = 2$

$\therefore \quad \frac{x}{y} + y = 2$

Again, for $x = -3$

$$-3 + y^2 = 2y$$

$\Rightarrow \quad (y + 1)(y - 3) = 0$

Also, $\quad y > 0$

$\Rightarrow \quad y = 3$ [neglecting $y = -1$]

17 **(a)**

Given equation is, $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ ...(i)

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Now, Eq. (i) becomes

$$v + x\frac{dv}{dx} = v + \frac{\phi(v)}{\phi'(v)}$$

$$\Rightarrow \frac{\phi'(v)}{\phi(v)}dv = \frac{dx}{x}$$

On integrating both sides, we get

$$\int \frac{\phi'(v)}{\phi(v)}dv = \int \frac{1}{x}dx$$

$\Rightarrow \log \phi(v) = \log x + \log k$

$\Rightarrow \log \phi(v) = \log xk$

$\Rightarrow \phi(v) = kx \Rightarrow \phi\left(\frac{y}{x}\right) = kx \quad \left(\because v = \frac{x}{y}\right)$

18 **(c)**

Given, $\frac{dx}{dy} = x + y + 1 \Rightarrow \frac{dx}{dy} - x = y + 1$

$\therefore$ IF $= e^{\int -1\, dy} = e^{-y}$

$\therefore$ Solution is $x.e^{-y} = \int (y+1)e^{-y} dy$

$\Rightarrow \quad xe^{-y} = -(y+1)e^{-y} + \int e^{-y}\, dy$

$\Rightarrow \quad xe^{-y} = -(y+1)e^{-y} - e^{-y} + c$

$\Rightarrow \quad x = -(y+2) + ce^{y}$

19 **(b)**

Given, $x^2 + y^2 - 2ax = 0$ ...(i)

$\Rightarrow \quad 2x + 2yy' - 2a = 0$

$\Rightarrow \quad a = x + yy'$

On putting the value of a in Eq. (i), we get

$x^2 + y^2 - 2x(x + yy') = 0$

$\Rightarrow \quad y^2 - x^2 = 2xyy'$

20 **(c)**

Given, $y = c_1\cos(x + c_2) + c_3\sin(x + c_4) + c_5e^{x} + c_6$

$y = c_1[\cos x \cos c_2 - \sin x \sin c_2]$

$+ c_3[\sin x \cos c_4 + \cos x \sin c_4] + c_5e^{x} + c_6$

$= \cos x(c_1 \cos c_2 + c_3 \sin c_4) + \sin x(-c_1 \sin c_2 + c_3 \cos c_4) + c_5e^{x} + c_6$

$= A\cos x + B\sin x + Ce^{x} + D$

Where $A = c_1\cos c_2 + c_3\sin c_4$

$B = -c_1\sin c_2 + c_3\cos c_4,\ C = c_5, D = c_6$

Hence, order is 4.

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	D	A	A	B	D	A	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	D	C	B	A	A	C	B	C