

# DPP

DAILY PRACTICE PROBLEMS

CLASS : XII<sup>th</sup>  
DATE :

**SOLUTIONS**

SUBJECT : MATHS  
DPP NO. :3

## Topic :-DIFFERENTIAL EQUATIONS

1 (a)

It is a linear differential equation of the form of  $\frac{dy}{dx} + Py = Q$ .

$$\Rightarrow P = \sec^2 x, Q = \tan x \sec^2 x$$

$$\therefore \text{IF} = e^{\int P dx} = e^{\int \sec^2 x dx} = e^{\tan x}$$

Solution is  $ye^{\tan x} = \int \tan x e^{\tan x} \sec^2 x dx + c$

$$\Rightarrow ye^{\tan x} = \tan x e^{\tan x} - e^{\tan x} + c$$

$$\Rightarrow y = \tan x - 1 + ce^{-\tan x}$$

2 (b)

We have,

$$y \frac{dy}{dx} + x = a \Rightarrow y dy + x dx = a dx$$

Integrating, we get

$$\frac{y^2}{2} + \frac{x^2}{2} = ax + C \Rightarrow x^2 + y^2 - 2ax + 2C = 0,$$

which represents a set of circles having centre on  $x$ -axis

3 (d)

$\therefore$  Equation of normal at  $(x,y)$  is

$$Y - y = \frac{dx}{dy}(X - x)$$

Put,  $y = 0$

$$\text{Then, } X = x + y \frac{dy}{dx}$$

Given,  $y^2 = 2xX$

$$\Rightarrow y^2 = 2x \left( x + y \frac{dy}{dx} \right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2 - 2x^2}{2xy} = \frac{\left(\frac{y}{x}\right)^2 - 2}{2\left(\frac{y}{x}\right)}$$

Put  $y = vx$ , we get

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\text{Then, } v + x \frac{dv}{dx} = \frac{v^2 - 2}{2v}$$

$$\Rightarrow x \frac{dv}{dx} = -\frac{(2+v^2)}{2v}$$

$$\Rightarrow \frac{2v dv}{(2+v^2)} + \frac{dv}{x} = 0$$

On integrating both sides, we get

$$\ln(2+v^2) + \ln|x| = \ln c$$

$$\Rightarrow \ln(|x|(2+v^2)) = \ln c$$

$$\Rightarrow |x| \left( 2 + \frac{y^2}{x^2} \right) = c$$

∵ It passes through (2, 1), then

$$2 \left( 2 + \frac{1}{4} \right) = c$$

$$\Rightarrow c = \frac{9}{2}$$

$$\text{Then, } |x| \left( 2 + \frac{y^2}{x^2} \right) = \frac{9}{2}$$

$$\Rightarrow 2x^2 + y^2 = \frac{9}{2} |x|$$

$$\Rightarrow 4x^2 + 2y^2 = 9|x|$$

4 (a)

We have,

$$y dx - x dy - 3x^2 y^2 e^{x^3} dx = 0$$

$$\Rightarrow y dx - x dy = 3x^2 y^2 e^{x^3} dx$$

$$\Rightarrow \frac{y dx - x dy}{y^2} = 3x^2 e^{x^3} dx$$

$$\Rightarrow d\left(\frac{x}{y}\right) = d(e^{x^3}) \Rightarrow \frac{x}{y} = e^{x^3} + C$$

5 (c)

$$\text{Given, } \frac{dy}{dx} = \frac{ax+h}{by+k}$$

$$\Rightarrow \int (by+k) dy = - \int (ax+h) dx$$

$$\Rightarrow \frac{by^2}{2} + ky = \frac{ax^2}{2} + hx + c$$

Thus, above equation represents a parabola, if

$$a = 0 \text{ and } b \neq 0$$

$$\text{Or } b = 0 \text{ and } a \neq 0$$

6 (b)

The equations of the ellipses centred at the origin are given by  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , where  $a, b$  are arbitrary constants

Differentiating both sides w.r.t. to  $x$ , we get

$$\frac{2x}{a^2} + \frac{2y}{b^2} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{x}{a^2} + \frac{y}{b^2} \frac{dy}{dx} = 0 \quad \dots(i)$$



Differentiating (i) w.r.t.  $x$ , we get

$$\frac{1}{a^2} + \frac{y_1^2}{b^2} + \frac{y y_2}{b^2} = 0 \quad \dots(\text{ii})$$

Multiplying (ii) by  $x$  and subtracting it from (i), we get

$$\frac{1}{b^2}\{y y_1 - x y_1^2 - x y y_2\} = 0 \Rightarrow x y y_2 + x y_1^2 - y y_1 = 0$$

7 **(b)**

Given equation is  $y = ax^{n+1} + bx^{-n}$

On differentiating with respect to  $x$ , we get

$$\frac{dy}{dx} = a(n+1)x^n - bnx^{-n-1}$$

Again, on differentiating, we get

$$\frac{d^2y}{dx^2} = an(n+1)x^{n-1} + bn(n+1)x^{-n-2}$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} = an(n+1)x^{n+1} + bn(n+1)x^{-n}$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} = n(n+1)(ax^{n+1} + bx^{-n})$$

$$\Rightarrow \frac{x^2 d^2y}{dx^2} = n(n+1)y$$

8 **(b)**

Given,  $y = \text{acos}(x+b)$

$$\Rightarrow \frac{dy}{dx} = -\text{asin}(x+b)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\text{acos}(x+b) = -y$$

$$\Rightarrow \frac{d^2y}{dx^2} + y = 0$$

9 **(a)**

Here,  $\frac{dy}{dt} - \left(\frac{1}{1+t}\right)y = \frac{1}{(1+t)}$  and  $y(0) = -1$

Which represents linear differential equation of first order.

$$\text{IF} = e^{\int -\left(\frac{1}{1+t}\right)dt} = e^{-t+\log|1+t|} = e^{-t}(1+t)$$

$\therefore$  Required solution is

$$y(\text{IF}) = \left[ \int Q(\text{IF})dt \right] + c$$

$$\Rightarrow ye^{-t}(1+t) = \int \frac{1}{1+t} \cdot e^{-t}(1+t)dt + c = \int e^{-t}dt + c$$

$$\Rightarrow ye^{-t}(1+t) = -e^{-t} + c$$

$$\text{Since, } y(0) = -1 \Rightarrow -1 \cdot e^0(1+0) = -e^0 + c$$

$$\Rightarrow c = 0$$

$$\therefore y = -\frac{1}{(1+t)} \text{ and } y(1) = -\frac{1}{2}$$

10 **(b)**

PE

Given,  $\frac{dy}{dx} + \frac{y}{(1-x)\sqrt{x}} = 1 - \sqrt{x}$

$\therefore$  IF =  $e^{\int \frac{1}{(1-x)\sqrt{x}} dx}$

Put  $\sqrt{x} = t$

$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$\therefore$  IF =  $e^{\int \frac{2}{1-t^2} dt}$

$= e^{2 \log \left( \frac{1+t}{1-t} \right)} = \frac{1+t}{1-t} = \frac{1+\sqrt{x}}{1-\sqrt{x}}$

11 (b)

Given,  $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$

Put  $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

$\therefore v + x \frac{dv}{dx} = \frac{x^2 + v^2 x^2}{2xvx}$

$\Rightarrow x \frac{dv}{dx} = \frac{1 - v^2}{2v}$

$\Rightarrow \frac{2v}{1 - v^2} dv = \frac{dx}{x}$

$\Rightarrow -\log(1 - v^2) = \log x + \log c$

$\Rightarrow \log(1 - v^2)^{-1} = \log xc$

$\Rightarrow \left( \frac{x^2 - y^2}{x^2} \right)^{-1} = xc$

$\Rightarrow \frac{x^2}{x^2 - y^2} = xc$

$\Rightarrow x = c(x^2 - y^2)$

12 (c)

$\therefore y = u^n$

$\therefore \frac{dy}{dx} = nu^{n-1} \frac{du}{dx}$

On substituting the values of  $y$  and  $\frac{dy}{dx}$  in the given equation, then

$2x^4 \cdot u^n \cdot nu^{n-1} \frac{du}{dx} + u^{4n} = 4x^6$

$\Rightarrow \frac{du}{dx} = \frac{4x^6 - u^{4n}}{2nx^4 u^{2n-1}}$

Since, it is homogeneous. Then, the degree of  $4x^6 - u^{4n}$  and  $2nx^4 u^{2n-1}$  must be same.

$\therefore 4n = 6$  and  $4 + 2n - 1 = 6$

Then, we get  $n = \frac{3}{2}$

13 (b)

Given equation is  $y = ax \cos \left( \frac{1}{x} + b \right)$  .....(i)

On differentiating Eq. (i), we get

$$y_1 = a \left[ \cos\left(\frac{1}{x} + b\right) - x \sin\left(\frac{1}{x} + b\right) \left(\frac{-1}{x^2}\right) \right]$$

$$\Rightarrow y_1 = a \left[ \cos\left(\frac{1}{x} + b\right) + \frac{1}{x} \sin\left(\frac{1}{x} + b\right) \right] \dots \text{(ii)}$$

Again, on differentiating Eq. (ii), we get

$$y_2 = a \left[ -\sin\left(\frac{1}{x} + b\right) \left(-\frac{1}{x^2}\right) + \frac{1}{x} \cos\left(\frac{1}{x} + b\right) \left(-\frac{1}{x^2}\right) - \frac{1}{x^2} \sin\left(\frac{1}{x} + b\right) \right]$$

$$\Rightarrow y_2 = \frac{-a}{x^3} \cos\left(\frac{1}{x} + b\right) = \frac{-ax}{x^4} \cos\left(\frac{1}{x} + b\right) = \frac{-y}{x^4}$$

$$\Rightarrow x^4 y_2 + y = 0$$

14 **(b)**

Given,  $dy = x \log x \, dx$

$$\Rightarrow y = \frac{x^2}{2} \log x - \int \frac{x}{2} dx \quad [\text{integrating}]$$

$$\Rightarrow y = \frac{x^2}{2} \log x - \frac{x^2}{4} + c$$

15 **(c)**

Given equation is  $y = \sec(\tan^{-1} x)$

On differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = \sec(\tan^{-1} x) \tan(\tan^{-1} x) \cdot \frac{1}{1+x^2} = \frac{xy}{1+x^2} \quad [\because \tan(\tan^{-1} x) = x]$$

$$\Rightarrow (1+x^2) \frac{dy}{dx} = xy$$

16 **(a)**

$$\frac{dy}{dx} \tan y = \sin(x+y) + \sin(x-y)$$

$$\Rightarrow \frac{dy}{dx} \tan y = 2 \sin x \cos y$$

$$\Rightarrow \int \tan y \sec y \, dy = 2 \int \sin x \, dx$$

$$\Rightarrow \sec y + 2 \cos x = c$$

17 **(b)**

Equation of family of parabolas with focus at  $(0, 0)$  and  $x$ -axis as axis is

$$y^2 = 4a(x+a) \dots \text{(i)}$$

On differentiating Eq. (i), we get

$$2yy_1 = 4a, \text{ putting the value of } a \text{ in Eq. (i)}$$

$$\Rightarrow y^2 = 2yy_1 \left( x + \frac{yy_1}{2} \right)$$

$$\Rightarrow y = 2xy_1 + yy_1^2$$

$$\Rightarrow y \left( \frac{dy}{dx} \right)^2 + 2x \frac{dy}{dx} = y$$

18 **(b)**

We have,

$$\frac{dy}{dx} + y = \frac{1+y}{x} \Rightarrow \frac{dy}{dx} + \left(1 - \frac{1}{x}\right)y = \frac{1}{x}$$

$$\therefore \text{I.F.} = e^{\int \left(1 - \frac{1}{x}\right) dx} = e^{x - \log x} = \frac{1}{x} e^x$$

19 (c)

Given,  $y^2 = 4a(x - b)$

$$\Rightarrow 2y \frac{dy}{dx} = 4a$$

$$\Rightarrow 2y \frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = 0$$

$$\Rightarrow y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

20 (a)

The given equation can be rewritten as,  $\frac{d^2y}{dx^2} = -\sin x$ .

On integrating the given equation

$$\int \frac{d^2y}{dx^2} dx = \int -\sin x dx + c$$

$$\Rightarrow \frac{dy}{dx} = -(-\cos x) + c = \cos x + c$$

Again, on integrating, we get

$$\int \frac{dy}{dx} dx = \int \cos x dx + \int c dx + d$$

$$y = \sin x + cx + d$$

PE

**ANSWER-KEY**

Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	D	A	C	B	B	B	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	B	B	C	A	B	B	C	A

PE