DPP
DAILY PRACTICE PRODUCTS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :2

Topic :-DIFFERENTIAL EQUATIONS

1 **(c)**

Given, $\frac{dy}{dx} - \frac{2}{x} y = x^2 e^x$

$\therefore$ IF $= e^{-\int \frac{2}{x} dx} = e^{-\log x^2} = \frac{1}{x^2}$

$\therefore$ Complete solution is $\frac{y}{x^2} = \int \frac{x^2 e^x}{x^2} dx + c$

$\Rightarrow \frac{y}{x^2} = e^x + c$

$\Rightarrow y = x^2(e^x + c)$

When $y = 0, x = 1$, then $c = -e$

$\therefore y = x^2(e^x - e)$

2 **(b)**

Given, $\frac{dy}{dx} + \frac{2x}{1+x^2} . y = \frac{4x^2}{1+x^2}$

$\therefore$ IF $= e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)=(1+x^2)}$

$\therefore$ Complete solution is

$$y.(1+x^2) = \int (1+x^2). \frac{4x^2}{1+x^2} dx$$

$\Rightarrow y(1+x^2) = \frac{4x^3}{3} + c_1$

$\Rightarrow 3y(1+x^2) = 4x^3 + c$

3 **(a)**

Given, $k = PQ =$ length of normal

$\Rightarrow k = y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

$\Rightarrow \frac{k^2}{y^2} = 1 + \left(\frac{dy}{dx}\right)^2$

$\therefore y\frac{dy}{dx} = \pm\sqrt{k^2 - y^2}$

4 **(a)**

We have,

$y_1 y_3 = 3\, y_2^2 \Rightarrow \frac{y_3}{y_2} = 3\frac{y_2}{y_1}$

Integrating both sides, we get

$\log y_2 = 3 \log y_1 + \log c_1$

$\Rightarrow y_2 = c_1 y_1^3 \Rightarrow \frac{y_2}{y_1^3} = c_1 \Rightarrow \frac{d\, y_1}{y_1^3} = c_1$

Integrating both sides w.r.t. x, we get

$-\frac{1}{2y_1^2} = c_1 x + c_2$

$\Rightarrow y_1^2 = \frac{1}{(-2\, c_1)x + (-2\, c_2)}$

$\Rightarrow y_1^2 = \frac{1}{ax + b}$, where $a = -2c_1$, $b = -2\, c_2$

$\Rightarrow y_1 = \frac{1}{\sqrt{ax + b}}$

Integrating both sides w.r.t. x, we get

$y = \frac{2}{a} \sqrt{ax + b} + c_3$

$\Rightarrow \frac{ay - c_3}{2} = \sqrt{ax + b}$

$\Rightarrow ax + b = \left(\frac{ax - c_3}{2}\right)^2$

$\Rightarrow x = \frac{a}{4} y^2 - \frac{c^3}{2} y + \frac{1}{a}\left(\frac{c_3^2}{4} - b\right) \Rightarrow x = A_1 y^2 + A_2 y + A_3,$

where $= A_1 = \frac{a}{4}, A_2 = -\frac{c_3}{2}$ and $A_3 = \frac{1}{a}\left(\frac{c_3^2}{4} - b\right)$

5 **(a)**

Here, $x = A\cos 4t + B \sin 4t$

On differentiating w.r.t.t, we get

$\frac{dx}{dt} = -4A \sin 4t + 4B \cos 4t$

Again, on differentiating w.r.t.t, we get

$\frac{d^2x}{dt^2} = -16A \cos 4t - 16B \sin 4t$

$= -16(A \cos 4t + B \sin 4t)$

$\Rightarrow \frac{d^2x}{dt^2} = -16x$

6 **(a)**

We have,

$y = c_1 + c_2 e^x + c_3 e^{-2x + c_4}$

$\Rightarrow y = c_1 + c_2 e^x + c_3 e^{-2x} \cdot e^{c_4}$

$\Rightarrow y = c_1 + c_2 e^x + c_3{}' e^{-2x}$, where $c_3' = c_3 e^{c_4}$

It is an equation containing three arbitrary constants. So, the associated differential equation is of order 3

7 **(b)**

Equation of parabolas family can be taken as

$x = ay^2 + by + c$

Differentiating $w.r.t.$, y we get

$\frac{dx}{dy} = 2ay + b$

$\Rightarrow \quad \frac{d^2x}{dy^2} = 2a \Rightarrow \frac{d^3x}{dy^3} = 0$

8 **(a)**

Given $\frac{1-y}{y^2}dy + \frac{1+x}{x^2}dx = 0$

$\Rightarrow \quad \int\left(\frac{1}{y^2} - \frac{1}{y}\right)dy + \int\left(\frac{1}{x^2} + \frac{1}{x}\right)dx = 0$

$\Rightarrow \quad \log\left(\frac{x}{y}\right) = \frac{1}{x} + \frac{1}{y} + c$

9 **(b)**

Given, $\frac{dy}{dx} = \frac{\sqrt{x^2+y^2}+y}{x}$

Put $y = vx$

$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore \quad v + x\frac{dv}{dx} = \frac{\sqrt{x^2+v^2x^2}+vx}{x}$

$\Rightarrow \quad \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$

$\Rightarrow \quad \log\left(v + \sqrt{1+v^2}\right) = \log x + \log c$

$\Rightarrow \quad \log\left(\frac{y}{x} + \sqrt{1+\frac{y^2}{x^2}}\right) = \log cx$

$\Rightarrow \quad y + \sqrt{x^2+y^2} = cx^2$

10 **(a)**

Given, $\frac{dy}{dx} = \frac{x\log x^2 + x}{\sin y + y\cos y}$

$\Rightarrow \quad (\sin y + y\cos y)dy = \left(x\log x^2 + x\right)dx$

$\left(\frac{d}{dy}y\sin y\right)dy = \left(\frac{d}{dx}x^2\log x\right)dx$

$\Rightarrow \quad y\sin y = x^2\log x + c$

11 **(d)**

Given, $x(1-x^2)dy + (2x^2y - y - ax^3)dx = 0$

$\Rightarrow \quad \frac{dy}{dx} + \frac{(2x^2-1)}{x(1-x)}y = \frac{ax^3}{(1-x^2)}$

Here, $P = \frac{2x^2-1}{x(1-x^2)}$

12 **(c)**

We have,

$\frac{dy}{dx} + \frac{y}{x} = x^2 \Rightarrow x\frac{dy}{dx} + y = x^3 \Rightarrow \frac{d}{dx}(xy) = x^3$

Integrating, we get

$xy = \frac{x^4}{4} + C \Rightarrow y = \frac{x^3}{4} + C\,x^{-1}$

13 **(c)**

Let $x^2 + y^2 - 2gx = 0$ ....(i)

$\Rightarrow \quad 2x + 2y\frac{dy}{dx} - 2g = 0$

$\Rightarrow \quad 2g = \left(2x + 2y\frac{dy}{dx}\right)$

On putting the value of 2g in eq. (i), we get

$x^2 + y^2 - \left(2x + 2y\,\frac{dy}{dx}\right)x = 0$

$\Rightarrow \quad y^2 = x^2 + 2xy\frac{dy}{dx}$

14 **(d)**

Given differential equation can be written as

$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$

$\Rightarrow \quad (m^2 - 3m + 2)y = 0$

$\Rightarrow \quad (m-1)(m-2)y = 0$

$\Rightarrow \quad m = 1,2$

$\therefore$ Solution is $y = c_1e^x + c_2e^{2x}$

$y' = c_1e^x + 2c_2e^{2x}$

From given condition

$y(0) = 1$

$\Rightarrow c_1 + c_2 = 1$.....(i)

And y'(0)=0

$\Rightarrow c_1 + c_2 = 1$.......(ii)

On solving Eqs. (i) and (ii)we get

$-c_2 = 1$

$\Rightarrow \quad c_2 = -1$

And $c_1 = 2$

$\therefore$ y=$2e^x - e^{2x}$

$\therefore \quad at\ x = \log_e 2$

$y = 2e^{log2} - e^{2\log 2}$

$= 2 \times 2 - 2^2 = 0$

15 **(b)**

The equation of straight line touching the given circle is

$x\cos\theta + y\sin\theta = a$ ...(i)

On differentiating w.r.t.x, regarding θ as a constant

$\Rightarrow \quad \cos\theta + \frac{dy}{dx}\sin\theta = 0$ ...(ii)

From eqs. (i) and (ii), we get

$$\cos\theta = \frac{a\frac{dy}{dx}}{x\frac{dy}{dx}-y} \text{ and } \quad \sin\theta = -\frac{a}{x\frac{dy}{dx}-y}$$

$\therefore \quad \cos^2\theta + \sin^2\theta = 1$

$$\therefore \quad \frac{a^2\left(\frac{dy}{dx}\right)^2 + a^2}{\left(x\frac{dy}{dx}-y\right)^2} = 1$$

$$\Rightarrow \quad \left(y - x\frac{dy}{dx}\right)^2 = a^2\left[1+\left(\frac{dy}{dx}\right)^2\right]$$

16 **(a)**

The given differential equation can be rewritten as

$$\Rightarrow \quad \left(\frac{1}{y^2}-\frac{1}{y}\right)dy = -\left(\frac{1}{x^2}+\frac{1}{x}\right)dx$$

$$\Rightarrow \quad -\frac{1}{y} - \log y = -\left(-\frac{1}{x} + \log x\right) + c \qquad \text{[integrating]}$$

$$\Rightarrow \quad \log\left(\frac{x}{y}\right) = \frac{1}{x} + \frac{1}{y} + c$$

17 **(a)**

We have, $(xy - x^2) = y^2$

$$\Rightarrow \quad y^2\frac{dx}{dy} = xy - x^2$$

$$\Rightarrow \quad \frac{1}{x^2}\frac{dx}{dy} - \frac{1}{x}\cdot\frac{1}{y} = -\frac{1}{y^2}$$

Put $\frac{1}{x} = v \Rightarrow -\frac{1}{x^2}\frac{dx}{dy} = \frac{dv}{dy}$

$\therefore \quad \frac{dv}{dy} + \frac{v}{y} = \frac{1}{y^2}$, which is linear

$$\therefore \quad IF = e^{\int\frac{1}{y}\,dy} = e^{\log y} = y$$

$\therefore$ The solution is $vy = \int\frac{1}{y^2}\,y\,dy + c$

$$\Rightarrow \quad \frac{y}{x} = \log y + c$$

$$\Rightarrow \quad y = x(\log y + c)$$

This passes through the point (-1,1)

$\therefore \quad 1 = -1(\log 1 + c)$

ie., $\quad c = -1$

thus, the equation of the curve is

$y = x(\log y - 1)$

18 **(d)**

Given, $y = 2e^{2x} - e^{-x}$

$\Rightarrow \quad y_1 = 4e^{2x} + e^{-x}$

$\Rightarrow \quad y_2 = 8e^{2x} - e^{-x}$

$\Rightarrow \quad y_2 = 4e^{2x} + e^{-x} + 4e^{2x} - 2e^{-x}$

$\Rightarrow \quad y_2 = y_1 + 2(2e^{2x} - e^{-x})$

$\Rightarrow \quad y_2 = y_1 + 2y$

$\Rightarrow \quad y_2 = y_1 + 2y$

$\Rightarrow \quad y_2 - y_1 - 2y = 0$

19 **(c)**

Given equation is $\frac{dy}{dx} - y = 1 \Rightarrow \frac{dy}{1+y} = dx$

On integrating both sides, we get

$$\int \frac{1}{1+y} dy = \int dx$$

$\Rightarrow \log(1+y) = x + c$

$\Rightarrow 1 + y = e^x \cdot e^c \quad$ ...(i)

At $x = 0, y = -1$

Then $1 - 1 = e^0 \cdot e^c \Rightarrow e^c = 0$

On putting the value of e^c in Eq. (i).

Therefore, solution becomes

$1 + y = e^x \times 0 \Rightarrow y(x) = -1$

20 **(d)**

Let family of circles be

$$(x-\alpha)^2 + (y-2)^2 = 5^2$$

$\Rightarrow \quad x^2 + \alpha^2 - 2\alpha x + y^2 - 21 - 4y = 0 \quad$ ...(i)

$\Rightarrow \quad 2x - 2\alpha + 2y\frac{dy}{dx} - 4\frac{dy}{dx} = 0$

$\Rightarrow \quad \alpha = x + \frac{dy}{dx}(y-2)$

On putting the value of α in Eq. (i), we get

$$\left(x - x - \frac{dy}{dx}(y-2)\right)^2 + (y-2)^2 = 5^2$$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)^2 (y-2)^2 = 25 - (y-2)^2$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	A	A	A	A	B	A	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	C	C	D	B	A	A	D	C	D