DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :10

Topic :-DIFFERENTIAL EQUATIONS

1 **(a)**

Given equation can be rewritten as

$$\frac{dx}{dy}+\frac{1}{(1+y^2)}x=\frac{e^{\tan^{-1}y}}{(1+y^2)}$$

$\therefore \quad IF=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{1}y}$

$\therefore$ Required solution is

$$xe^{\tan^{1}y}=\int\frac{e^{\tan^{-1}y}e^{\tan^{-1}y}}{1+y^2}dy$$

Put $\quad e^{\tan^{-1}y}=t\Rightarrow e^{\tan^{-1}y}\frac{1}{1+y^2}dy=dt$

$\therefore \quad xe^{\tan^{-1}y}=\int t\,dt=\frac{t^2}{2}+c$

$\Rightarrow \quad 2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$

2 **(b)**

Given, $\frac{dy}{dx}=\log(x+1)$

$\Rightarrow \quad dy=\log(x+1)dx$

$\Rightarrow \quad \int dy=\int\log(x+1)\,dx$

$\Rightarrow \quad y=(x+1)\log|x+1|-x+c$

$\therefore \quad x=0,\ y=3$

$\therefore \quad c=3$

$\therefore \quad y=(x+1)\log|x+1|-x+3$

3 **(d)**

Given equation is

$$\frac{d^2y}{dx^2}=\frac{\log x}{x^2}$$

On integrating, both sides we get

$$\int\frac{d^2y}{dx^2}dx=\int\frac{\log x}{x^2}dx$$

$$\Rightarrow \frac{dy}{dx}=-\frac{\log x}{x}+\int\frac{1}{x^2}dx+c$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\log x}{x} - \frac{1}{x} + c$$

At $x = 1, y = 0$ and $\frac{dy}{dx} = -1 \Rightarrow c = 0$

$$\therefore \frac{dy}{dx} = -\frac{(\log x + 1)}{x}$$

Again on integrating, both sides we get

$$\int \frac{dy}{dx} dx = -\int \frac{\log x + 1}{x} dx + c_1$$

$$y = -\frac{1}{2}(\log x)^2 - \log x + c_1$$

At $x = 1, y = 0$

$\Rightarrow c_1 = 0$

$$\therefore y = -\frac{1}{2}(\log x)^2 - \log x$$

5 **(b)**

Given equation is

$\sin^{-1} x + \sin^{-1} y = c$ ....(i)

On differentiating Eq. (i) w.r.t.x, we get

$$\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\Rightarrow \sqrt{1-x^2}\, dy + \sqrt{1-y^2} dx = 0$$

This is the required differential equation.

6 **(d)**

Given that, $\frac{dy}{dx} = 1 + y^2$

$$\Rightarrow \frac{dy}{1+y^2} = dx$$

On integrating both sides, we get

$$\int \frac{dy}{1+y^2} = \int dx$$

$\Rightarrow \tan^{-1} y = x + c$

At $x = 0, y = 0$, then $c = 0$

At $x = \pi, y = 0$, then $\tan^{-1} 0 = \pi + c \Rightarrow c = -\pi$

$\therefore \tan^{-1} y = x \Rightarrow y = \tan x = \phi(x)$

Therefore, solution becomes $y = \tan x$

But $\tan x$ is not continuous function in $(0, \pi)$

So, $\phi(x)$ is not possible in $(0, \pi)$.

7 **(c)**

Let $p = \frac{dy}{dx}$

$\therefore$ Given differential equation reduces to

$p^2 - xp + y = 0$

Differentiating both sides w.r.t.x, we get

$$2p\frac{dp}{dx}-x\frac{dp}{dx}-p+p=0$$

$$\Rightarrow\frac{dp}{dx}(2p-x)=0$$

$\Rightarrow$ Either $\frac{d^2y}{dx^2}=0$ or $\frac{dy}{dx}=\frac{x}{2}$

$\Rightarrow y=2x-4$ will satisfy.

8 **(c)**

Given, $y=a\sin(5x+c)$

$\Rightarrow \quad \frac{dy}{dx}=5a\cos(5x+c)$

$\Rightarrow \quad \frac{d^2y}{dx^2}=-25a\sin(5x+c)=-25y$

9 **(c)**

Given, $(1-x^2)\frac{dy}{dx}-xy=1$

$$\Rightarrow \frac{dy}{dx}-\frac{x}{1-x^2}y=\frac{1}{1-x^2}$$

This is a linear equation, comparing with the equation

$$\frac{dy}{dx}+Py=Q$$

$$\Rightarrow P=-\frac{x}{1-x^2}, Q=\frac{1}{1-x^2}$$

$$\therefore IF=e^{\int P\,dx}=e^{\int\frac{-x}{1-x^2}dx}$$

$$\Rightarrow \text{IF}=e^{\frac{1}{2}\log(1-x^2)}=\sqrt{1-x^2}$$

10 **(c)**

We have,

Slope $=\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=\frac{1}{2y}\Rightarrow 2\,y\,dy=dx$

Integrating both sides, we get $y^2=x+C$

This passes through (4, 3)

$\therefore 9=4+C\Rightarrow C=5$

So, the equation of the curve is $y^2=x+5$

11 **(a)**

The given differential equation is

$$\frac{dy}{dx}+y\frac{\sin x}{\cos x}=\frac{1}{\cos x}$$

$$\therefore IF=e^{\int\frac{\sin x}{\cos x}dx}=e^{\log\sec x}=\sec x$$

12 **(b)**

Given, $\frac{dy}{dx}+\frac{2x}{1+x^2}.y=\frac{1}{(1+x^2)^2}$

$$\therefore \quad \text{IF}=e^{\int\frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=1+x^2$$

The complete solution is

$$y(1+x^2)=\int (1+x^2).\frac{1}{(1+x^2)^2}\,dx+c$$

$\Rightarrow \quad y(1+x^2)=\tan^{-1}x+c$

13 **(b)**

$\because$ The order of the differential equation is the order of highest derivative in the differential equation.

$\therefore$ The second order differential equation is in option (b) *ie*,

$y'y''+y=\sin x$

14 **(c)**

Given, $\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{y}$

$\Rightarrow \quad \int \frac{y}{\sqrt{1-y^2}}\,dy=\int dx$

$\Rightarrow \quad -\sqrt{1-y^2}=x+c$

$\Rightarrow \quad (x+c)^2+y^2=1$

$\therefore$ Centre $(-c,0)$, radius=1

15 **(c)**

Given, $\frac{dy}{dx}+y=2e^{2x}$

$\therefore$ IF $=e^{\int 1\,dx}=e^x$

$\therefore$ Required solution is

$$ye^x=2\int e^{2x}e^x dx=\frac{2}{3}e^{3x}+c$$

$\Rightarrow \quad y=\frac{2}{3}e^{2x}+ce^{-x}$

16 **(c)**

Given, $y=\sin^{-1}x$

$\Rightarrow \quad \frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$ ...(i)

$\Rightarrow \quad \frac{d^2y}{dx^2}=\frac{0-\frac{1}{2}\cdot\frac{(-2x)}{\sqrt{1-x^2}}}{(\sqrt{1-x^2})^2}$

$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2}=x\frac{dy}{dx}$ ....[from Eq.(i)]

17 **(a)**

Given, $\frac{dy}{dx}=2\cos x-y\cos x\,\text{cosec}\,x$

$\Rightarrow \quad \frac{dy}{dx}+y\cot x=2\cos x$

$\therefore$ IF $=e^{\int \cot x\,dx}=e^{\log(\sin x)}=\sin x$

$\therefore$ Solution is $y\sin x=\int 2\cos x\sin x\,dx+c$

$\Rightarrow \quad y\sin x=\int \sin 2x\,dx+c$

$\Rightarrow \quad y\sin x = \frac{-\cos 2x}{2} + c$

At $\quad x = \frac{\pi}{4}, y = \sqrt{2}$

$\therefore \quad \sqrt{2}\sin\frac{\pi}{4} = \frac{-\cos 2(\pi/4)}{2} + c$

$\Rightarrow \quad c = 1$

$\therefore \quad y\sin x = -\frac{1}{2}\cos 2x + 1$

$\Rightarrow \quad y = -\frac{1}{2}\cdot\frac{\cos 2x}{\sin x} + \operatorname{cosec} x$

$\Rightarrow \quad y = -\frac{1}{2\sin x}(1 - 2\sin^2 x) + \operatorname{cosec} x$

$\Rightarrow \quad y = \frac{1}{2}\operatorname{cosec} x + \sin x$

18 **(b)**

Given, $\frac{\tan^{-1} x}{1 + x^2}dx + \frac{y}{1 + y^2}dy = 0$

$\Rightarrow \quad \frac{(\tan^{-1} y)^2}{2} + \frac{1}{2}\log(1 + y^2) = \frac{c}{2}$ [integrating]

$\Rightarrow \quad (\tan^{-1} x)^2 + \log(1 + y^2) = c$

19 **(d)**

Given, $\frac{dy}{dx} - y\tan x = -2\sin x$

$\therefore \quad \text{IF} = e^{-\int \tan x\, dx} = \cos x$

$\therefore$ Solution is

$y(\cos x) = \int -2\sin x\cos x\, dx + c = -\int \sin 2x\, dx + c$

$\Rightarrow \quad y\cos x = \frac{\cos 2x}{2} + c$

20 **(a)**

Given, $\frac{dy}{dt} - \left(\frac{1}{1 + t}\right)y = \frac{1}{(1 + t)}$ and $y(0) = -1$

$\therefore \quad \text{IF} = e^{\int -\left(\frac{t}{1+t}\right)dt} = e^{-\int \left(1 - \frac{1}{1+t}\right)dt}$

$= e^{-t + \log(1+t)} = e^{-t}(1 + t)$

$\therefore$ Required solution is

$ye^{-t}(1 + t) = \int \frac{1}{1 + t}e^{-t}(1 + t)dt + c$

$= \int e^{-t}\, dt + c$

$\Rightarrow \quad ye^{-t}(1 + t) = -e^{-t} + c$

Since, $\quad y(0) = -1$

$\Rightarrow \quad c = 0$

$\therefore \quad y = -\frac{1}{(1 + t)}$

$\Rightarrow \quad y(1) = -\frac{1}{2}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	D	A	B	D	C	C	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	B	C	C	C	A	B	D	A