DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :9

Topic :-DETERMINANTS

1 **(c)**

Clearly, the degree of the given determinant is 3. So, there cannot be more that 3 linear factors. Thus, the other factor is a numerical constant. Let it be λ. Then,

$$\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = \lambda(a+b)(b+c)(c+a)$$

Putting $a=0$, $b=1$ and $c=1$ on both sides, we get

$$\begin{vmatrix} 0 & 1 & 1 \\ 1 & -2 & 2 \\ 1 & 2 & -2 \end{vmatrix} = \lambda \times 1 \times 2 \times 1 \Rightarrow 2\lambda \Rightarrow \lambda = 4$$

2 **(b)**

We have,

$$\begin{vmatrix} 1 & \omega^2 & \omega^5 \\ \omega^3 & 1 & \omega^4 \\ \omega^5 & \omega^4 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & \omega^2 \\ 1 & 1 & \omega \\ \omega^2 & \omega & 1 \end{vmatrix}$$

$$= 2 - (\omega^2 - \omega) = 2 - (-1) = 3$$

3 **(b)**

Applying $C_1 \to C_1 + C_2 + C_3$ and taking common $(a+b+c)$ from C_1, we get

$$(a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$(a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix}$$

$$= (a+b+c)\{-(c-b)^2 - (a-b)(a-c)\}$$

$$= -(a+b+c)\{a^2+b^2+c^2-ab-bc-ca\}$$

$$= -\frac{1}{2}(a+b+c)\{2a^2+2b^2+2c^2-2ab-2bc-2ac\}$$

$$= -\frac{1}{2}(a+b+c)\{(a-b)^2+(b-c)^2+(c-a)^2\}$$

Which is always negative.

4 **(c)**

In a ΔABC, we have

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$\Rightarrow b\sin A = a\sin B \; c\sin A = a\sin C$

$$\therefore \begin{vmatrix} a^2 & b\sin A & c\sin A \\ b\sin A & 1 & \cos A \\ c\sin A & \cos A & 1 \end{vmatrix}$$

$$=\begin{vmatrix} a^2 & a\sin B & a\sin C \\ a\sin B & 1 & \cos A \\ a\sin C & \cos A & 1 \end{vmatrix}$$

$$=a^2\begin{vmatrix} 1 & \sin B & \sin C \\ \sin B & 1 & \cos A \\ \sin C & \cos A & 1 \end{vmatrix} \quad \text{Taking } a \text{ common from } R_1 \text{ and } C_1 \text{ both}$$

$$=a^2\{(1-\cos^2 A)-\sin B(\sin B-\cos A\sin C)+\sin C(\sin B\cos A-\sin C)\}$$

$$=a^2\{\sin^2 A-\sin^2 B+2\sin B\sin C\cos A-\sin^2 C\}$$

$$=a^2\{\sin(A+B)\sin(A-B)-\sin^2 C+2\cos A\sin B\sin C\}$$

$$=a^2[\sin C\{\sin(A-B)-\sin C\}+2\cos A\sin B\sin C]$$

$$=a^2[\sin C\{\sin(A-B)-\sin(A+B)\}+2\cos A\sin B\sin C]$$

$$=a^2[\sin C\times -2\cos A\sin B+2\cos A\sin B\sin C]=0$$

5 **(b)**

$$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}+\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}=0$$

$$\Rightarrow\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}+abc\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}=0$$

$$\Rightarrow(1+abc)\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}=0$$

$$\left[\therefore \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}\neq 0\right]$$

$\Rightarrow 1+abc=0$

$\Rightarrow abc=-1$

6 **(b)**

$$\begin{vmatrix} x+\omega^2 & \omega & 1 \\ \omega & \omega^2 & 1+x \\ 1 & x+\omega & \omega^2 \end{vmatrix}=0$$

Applying $C_1\to C_1+C_2+C_3$

$$\begin{vmatrix} x & \omega & 1 \\ x & \omega^2 & 1+x \\ x & x+\omega & \omega^2 \end{vmatrix} = 0 \quad (\therefore 1+\omega+\omega^2=0)$$

$\Rightarrow$ $x = 0$ is one of the values of x which satisfy the above determinant equation.

7 **(a)**

We have,

$$|A| = \begin{bmatrix} 4\,5\,6\,x \\ 5\,6\,7\,y \\ 6\,7\,8\,z \\ x\,y\,z\,0 \end{bmatrix}$$

$$\Rightarrow |A| = \begin{bmatrix} 0\,0\,0 & x-2y+z \\ 5\,6\,7 & y \\ 6\,7\,8 & z \\ x\,y\,z & 0 \end{bmatrix} \quad \begin{matrix} \text{Applying } R_1 \to R_1 \\ -2R_2 + R_3 \end{matrix}$$

$$\Rightarrow |A| = \begin{bmatrix} 0\,0\,0\,0 \\ 5\,6\,7\,y \\ 6\,7\,8\,z \\ x\,y\,z\,0 \end{bmatrix} \quad \begin{bmatrix} \because x,y,z \text{ are in A.P.} \\ \therefore x-2y+z=0 \end{bmatrix}$$

$\Rightarrow |A| = 0$

8 **(a)**

$$\text{Given, } \Delta = \begin{vmatrix} (e^{i\alpha}+e^{-i\alpha})^2 & (e^{i\alpha}-e^{-i\alpha})^2 & 4 \\ (e^{i\beta}+e^{-i\beta})^2 & (e^{i\beta}-e^{-i\beta})^2 & 4 \\ (e^{i\gamma}+e^{-i\gamma})^2 & (e^{i\gamma}-e^{-i\gamma})^2 & 4 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$

$$= \begin{vmatrix} 4 & (e^{i\alpha}-e^{-i\alpha})^2 & 4 \\ 4 & (e^{i\beta}-e^{-i\beta})^2 & 4 \\ 4 & (e^{i\gamma}-e^{-i\gamma})^2 & 4 \end{vmatrix}$$

$= 0$ ($\because$ two columns are same)

Hence, it is independent of α, β and γ.

9 **(b)**

Let A be the first term and R be the common ratio of the GP. Then,

$a = A\,R^{p-1} \Rightarrow \log a = \log A + (p-1)\log R$

$b = A\,R^{q-1} \Rightarrow \log b = \log A + (q-1)\log R$

$c = A\,R^{r-1} \Rightarrow \log c = \log A + (r-1)\log R$

Now,

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} (p-1) & \log R & p & 1 \\ (q-1) & \log R & q & 1 \\ (r-1) & \log R & r & 1 \end{vmatrix}$$

$$= \log R = \begin{vmatrix} p-1 & p & 1 \\ q-1 & q & 1 \\ r-1 & r & 1 \end{vmatrix} \text{ [Applying } C_1 \to C_1 - (\log A)\, C_3]$$

$$= \log R \begin{vmatrix} 0 & p & 1 \\ 0 & q & 1 \\ 0 & r & 1 \end{vmatrix} = 0 \text{ [Applying } C_1 \to C_1 - C_2 + C_3]$$

10 **(c)**

We know that the sum of the products of the elements of a row with the cofactors of the corresponding elements is always equal to the value of the determinant .ie,$|A|$.

11 **(d)**

$\because a,b,c,d,e$ and f are in GP.

$\therefore a = a, b = ar, c = ar^2, d = ar^3, e = ar^4$ and $f = ar^5$

$$\therefore \begin{vmatrix} a^2 & d^2 & x \\ b^2 & e^2 & y \\ c^2 & f^2 & z \end{vmatrix} = \begin{vmatrix} a^2 & a^2r^6 & x \\ a^2r^2 & a^2r^8 & y \\ a^2r^4 & a^2r^{10} & z \end{vmatrix}$$

$$= a^4r^6 \begin{vmatrix} 1 & 1 & x \\ r^2 & r^2 & y \\ r^4 & r^4 & z \end{vmatrix} = 0$$

Thus, the given determinant is independent of x,y and z.

12 **(a)**

$$\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$$

$= 1(1 - \log_y z \log_z y) - \log_x y(\log_y x - \log_z x \log_y z)$

$+ \log_x z\ (\log_z y\ \log_y x - \log\ _z x)$

$= (1 - \log_y y) - \log_x y\ (\log_y x - \log_y x)$

$+ \log_x z(\log_z x - \log_z x)$

$= (1 - 1) - 0 + 0 = 0$

13 **(d)**

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1-x & 1 \\ 1 & 1 & 1+y \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & -x & 0 \\ 1 & 0 & y \end{vmatrix} \begin{bmatrix} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{bmatrix}$$

$= -xy$

14 **(c)**

$$\begin{vmatrix} x & y & z \\ -x & y & z \\ x & -y & z \end{vmatrix} = \begin{vmatrix} x & y & z \\ -x & y & z \\ 0 & 0 & 2z \end{vmatrix} \quad [R_3 \to R_3 + R_2]$$

$= 2z(xy + xy) = 4xyz$

On comparing with $kxyz$, we get $k = 4$

15 **(b)**

Applying $R_1 \to R_1 + R_2 + R_3$ and taking common $(2x + 10)$ from R_1,we get

$$(2x + 10) \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2x & 2 \\ 7 & 6 & 2x \end{vmatrix} = 0$$

$$\Rightarrow (2x + 10) \begin{vmatrix} 1 & 0 & 0 \\ 2 & 2x-2 & 0 \\ 7 & -1 & 2x-7 \end{vmatrix} = 0$$

$[C_3 \to C_3 - C_1 \text{and } C_2 \to C_2 - C_1]$

$\Rightarrow\ (2x+10)(2x-2)(2x-7)=0$

$\Rightarrow x=-5, 1, \frac{7}{2}$

Hence , other roots are 1 and $\frac{7}{2}$ or 1 and 3.5

16 **(b)**

Let $\frac{x^2}{a^2}=X, \frac{y^2}{b^2}=Y$ and $\frac{z^2}{c^2}=Z$

Then the given system of equations becomes

$X+Y-Z=1, X-Y+Z=1, -X+Y+Z=1$

The coefficient matrix is $A=\begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

Clearly, $|A|\neq 0$. So, the given system of equations has a unique solution

17 **(c)**

Applying $R_1 \to R_1+R_2$, we get

$$\begin{vmatrix} 2 & 2 & 1 \\ \cos^2\theta & 1+\cos^2\theta & \cos^2\theta \\ 4\sin 4\theta & 4\sin 4\theta & 1+4\sin 4\theta \end{vmatrix}=0$$

Applying $C_1 \to C_1 - 2C_3, C_2 \to C_2 - 2C_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ -\cos^2\theta & 1-\cos^2\theta & \cos^2\theta \\ -2-4\sin 4\theta & -2-4\sin 4\theta & 1+4\sin 4\theta \end{vmatrix}=0$$

$\Rightarrow [\cos^2\theta(2+4\sin 4\theta)+(1-\cos^2\theta)(2+4\sin 4\theta)]=0$

$\Rightarrow [2\cos^2\theta+4\cos^2\theta\sin 4\theta+2+4\sin 4\theta$
$-2\cos^2\theta-4\cos^2\theta\sin 4\theta]=0$

$\Rightarrow 2+4\sin 4\theta=0$

$\Rightarrow\ \sin 4\theta=-\frac{1}{2}$

18 **(a)**

Given determinant, $\Delta \equiv \begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix}=0$

On splitting the determinant into two determinants, we get

$\Delta \equiv abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}+\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}=0$

$\Rightarrow (1+abc)\left[1(bc^2-cb^2)-a(c^2-b^2)+a^2(c-b)\right]=0$

$\Rightarrow (1+abc)[(a-b)(b-c)(c-a)]=0$

Since a,b,c are different, the second factor cannot be zero.

Hence, $1+abc=0$

19 **(b)**

We have,

$$\begin{vmatrix} a & a^2-bc & 1 \\ b & b^2-ca & 1 \\ c & c^2-ab & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & -bc & 1 \\ b & -ca & 1 \\ c & -ab & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \frac{1}{abc}\begin{vmatrix} a^2 & -abc & a \\ b^2 & -abc & b \\ c^2 & -abc & c \end{vmatrix}$$ Applying $R_1 \to R_1(a)$ $R_2 \to R_2(b), R_3 \to R_3(c)$ in the IInd determinant

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} - \begin{vmatrix} a^2 & 1 & a \\ b^2 & 1 & b \\ c^2 & 1 & c \end{vmatrix}$$

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} - \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$$

20 **(d)**

Given that, $x^a y^b = e^m, x^c y^d = e^n$

and $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}, \Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}, \Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$\Rightarrow a \log x + b \log y = m$

$\Rightarrow c \log x + d \log y = n$

By Cramer's rule

$\log x = \frac{\Delta_1}{\Delta_3}$ and $\log y = \frac{\Delta_2}{\Delta_3}$

$\Rightarrow x = e^{\Delta_1/\Delta_3}$ and $y = e^{\Delta_2/\Delta_3}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	B	C	B	B	A	A	B	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	A	D	C	B	B	C	A	B	D