DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :8

Topic :-DETERMINANTS

1 **(c)**

Given, $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$

Applying $R_1 \to R_1 + R_2 + R_3$

$\Rightarrow (a+b+c-x)\begin{vmatrix} 1 & 1 & 1 \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$

$\Rightarrow (a+b+c-x)\begin{vmatrix} 1 & 0 & 0 \\ c & b-x-c & a-c \\ b & a-b & c-x-b \end{vmatrix} = 0$

$\Rightarrow (a+b+c-x)[1(b-x-c)(c-x-b)-(a-c)(a-b)] = 0$

$\Rightarrow (a+b+c-x)\left[bc-xb-b^2-xc+x^2+bx-c^2+cx+bc-\left(a^2-ab-ac+bc\right)\right] = 0$

$\Rightarrow (a+b+c-x)[x^2-a^2-b^2-c^2+ab+bc+ca] = 0$

$\Rightarrow x = a+b+c$ or $x^2 = a^2+b^2+c^2+ab+bc+ca$

$\Rightarrow x = 0$ or $x^2 = a^2+b^2+c^2+\frac{1}{2}(a^2+b^2+c^2)$

$\Rightarrow x = 0$ or $x = \pm\sqrt{\frac{3}{2}(a^2+b^2+c^2)}$

2 **(d)**

We have,

$\Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc \times 0$

3 **(a)**

Applying $R_1 \to R_1 + R_3$,we get

$\begin{vmatrix} 1-i & \omega^2+\omega & \omega^2-1 \\ 1-i & -1 & \omega^2-1 \\ -i & -1+\omega-i & -1 \end{vmatrix}=0$

[$\therefore \omega^2 + \omega = -1$, so $R_1$and R_2become identical]

4 **(a)**

$$\sum_{n=1}^{N} U_n = \begin{vmatrix} \sum n & 1 & 5 \\ \sum n^2 & 2N+1 & 2N+1 \\ \sum n^3 & 3N^2 & 3N \end{vmatrix}$$

$$= \begin{vmatrix} \frac{N(N+1)}{2} & 1 & 5 \\ \frac{N(N+1)(2N+1)}{6} & 2N+1 & 2N+1 \\ \left(\frac{N(N+1)}{2}\right)^2 & 3N^2 & 3N \end{vmatrix}$$

$$= \frac{N(N+1)}{12} \begin{vmatrix} 6 & 1 & 5 \\ 4N+2 & 2N+1 & 2N+1 \\ 3N(N+1) & 3N^2 & 3N \end{vmatrix}$$

Applying $C_3 \to C_3 + C_2$

$$= \frac{N(N+1)}{12} \begin{vmatrix} 6 & 1 & 6 \\ 4N+2 & 2N+1 & 4N+2 \\ 3N(N+1) & 3N^2 & 3N(N+1) \end{vmatrix}$$

$= 0$ ($\because$ two columns are identical)

5 **(c)**

$$\begin{bmatrix} 215 & 342 & 511 \\ 6 & 7 & 8 \\ 36 & 49 & 54 \end{bmatrix}$$

$= 215(378 - 392) - 342(324 - 288) + 511(294 - 252)$

$= -3010 - 12312 + 21462 = 6140$

Which is exactly divisible by 20

6 **(a)**

$\det(A^{-1}\text{adj } A) = \det(A^{-1}) \det(\text{adj } A)$

$= (\det A)^{-1}(\det A)^{3-1} = \det A$

7 **(d)**

$$A = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$$

$= 1(1 + \sin^2\theta) - \sin\theta(-\sin\theta + \sin\theta) + 1(\sin^2\theta + 1)$

$= 2(1 + \sin^2\theta)$

Since, the maximum and minimum value of $\sin^2\theta$ is 1 and 0.

$\therefore |A| \in [2,4]$

8 **(d)**

Since, the first column consists of sum of two terms, second column consists of sum of three terms and third column consists of sum four terms.

$\therefore n = 2 \times 3 \times 4 = 24$

9 **(c)**

Given ,a_1,a_2, a_3,... $\in$ GP

$\Rightarrow \log a_1, \log a_2, ... \in$ AP

$\Rightarrow \log a_n, \log a_{n+1}, \log a_{n+2}, ... \in$ AP

$\Rightarrow \log a_{n+1} = \frac{\log a_n + \log a_{n+2}}{2}$ ...(i)

Similarly, $\log a_{n+4} = \frac{\log a_{n+3} + \log a_{n+5}}{2}$ ...(ii) and $\log a_{n+7} = \frac{\log a_{n+6} + \log a_{n+8}}{2}$ ...(ii)

Given, $\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$

Applying $C_2 \to C_2 - \frac{C_1 + C_3}{2}$

$$\Delta = \begin{vmatrix} \log a_n & 0 & \log a_{n+2} \\ \log a_{n+3} & 0 & \log a_{n+5} \\ \log a_{n+6} & 0 & \log a_{n+8} \end{vmatrix} = 0$$

10 **(a)**

$$\begin{vmatrix} 1 & \omega & -\omega^2/2 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = -\frac{1}{2}\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & 1 & -2 \\ 1 & -1 & 0 \end{vmatrix}$$

$$= -\frac{1}{2}\begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ 0 & 1 & -2 \\ 0 & -1 & 0 \end{vmatrix} \quad (C_1 \to C_1 + C_2 + C_3)$$

$$= -\frac{1}{2}\begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & 1 & -2 \\ 0 & -1 & 0 \end{vmatrix} \quad (\because 1 + \omega + \omega^2 = 0)$$

$= 0$

11 **(a)**

$$\begin{vmatrix} \log e & \log e^2 & \log e^3 \\ \log e^2 & \log e^3 & \log e^4 \\ \log e^3 & \log e^4 & \log e^5 \end{vmatrix} = \begin{vmatrix} \log e & 2\log e & 3\log e \\ 2\log e & 3\log e & 4\log e \\ 3\log e & 4\log e & 5\log e \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{vmatrix}$$

(Using $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_2$)

$= 0$[$\therefore$ two columns are identical]

12 **(b)**

$$\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & \sqrt{10} \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix}$$

$$= \begin{vmatrix} \sqrt{13} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{16} & 5 & \sqrt{10} \\ \sqrt{65} & \sqrt{15} & 5 \end{vmatrix} + \begin{vmatrix} \sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15} & 5 & \sqrt{10} \\ 3 & \sqrt{15} & 5 \end{vmatrix}$$

$$= \sqrt{13}.\sqrt{5}.\sqrt{5}\begin{vmatrix} 1 & 2 & 3 \\ \sqrt{2} & \sqrt{5} & \sqrt{2} \\ \sqrt{5} & \sqrt{3} & \sqrt{5} \end{vmatrix}$$

$$+ \sqrt{3}.\sqrt{5}.\sqrt{5}\begin{vmatrix} 1 & 2 & 3 \\ \sqrt{5} & \sqrt{5} & \sqrt{2} \\ \sqrt{3} & \sqrt{3} & \sqrt{5} \end{vmatrix}$$

$$= 0 + 5\sqrt{3}\begin{vmatrix} -1 & 2 & 1 \\ 0 & \sqrt{5} & \sqrt{2} \\ 0 & \sqrt{3} & \sqrt{5} \end{vmatrix} = 5\sqrt{3}\,(\sqrt{6} - 5)$$

14 **(d)**

We can write $\Delta = \Delta_1 + y_1\Delta_2$, where

$$\Delta_1 = \begin{vmatrix} 1 & 1 + x_1y_2 & 1 + x_1y_3 \\ 1 & 1 + x_2y_2 & 1 + x_2y_3 \\ 1 & 1 + x_3y_2 & 1 + x_3y_3 \end{vmatrix}$$

$$\text{and } \Delta_2 = \begin{vmatrix} x_1 & 1 + x_1y_2 & 1 + x_1y_3 \\ x_2 & 1 + x_2y_2 & 1 + x_2y_3 \\ x_3 & 1 + x_3y_2 & 1 + x_3y_3 \end{vmatrix}$$

In Δ_1, use $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$ so that,

$$\Delta_1 = \begin{vmatrix} 1 & x_1y_2 & x_1y_3 \\ 1 & x_2y_2 & x_2y_3 \\ 1 & x_3y_2 & x_3y_3 \end{vmatrix} = 0 \quad (\because C_2 \text{ and } C_3 \text{ are proportional})$$

In Δ_2, $C_2 \to C_2 - y_2C_1$ and $C_3 \to C_3 - y_3C_1$ to get

$$\Delta_2 = \begin{vmatrix} x_1 & 1 & 1 \\ x_2 & 1 & 1 \\ x_3 & 1 & 1 \end{vmatrix} = 0 \quad (\because C_2 \text{ and } C_3 \text{ are identical})$$

$\therefore\ \Delta = 0$

16 **(c)**

$$\text{Let } \Delta = \begin{vmatrix} b^2 - ab & b - c & bc - ac \\ ab - a^2 & a - b & b^2 - ab \\ bc - ac & c - a & ab - a^2 \end{vmatrix}$$

$$= (b - a)(b - a)\begin{vmatrix} b & b - c & c \\ a & a - b & b \\ c & c - a & a \end{vmatrix}$$

$$= (a - b)^2\begin{vmatrix} b & b & c \\ a & a & b \\ c & c & a \end{vmatrix} \quad (C_2 \to C_2 + C_3)$$

$= 0$ ($\because$ two columns are same)

18 **(d)**

$$\begin{vmatrix} x + 1 & \omega & \omega^2 \\ \omega & x + \omega^2 & 1 \\ \omega^2 & 1 & x + \omega \end{vmatrix}$$

$$= \begin{vmatrix} x + 1 + \omega + \omega^2 & \omega & \omega^2 \\ x + 1 + \omega + \omega^2 & x + \omega^2 & 1 \\ x + 1 + \omega + \omega^2 & 1 & x + \omega \end{vmatrix} \quad C_1 \to C_1 + C_2 + C_3)$$

$$= x\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & x + \omega^2 & 1 \\ 1 & 1 & x + \omega \end{vmatrix} \quad (\because 1 + \omega + \omega^2 = 0$$

$= x[1\{(x + \omega^2)(x + \omega) - 1\} + \omega\{1 - (x + \omega)\} + \omega^2\{1 - (x + \omega^2)\}]$

$= x[(x^2 + \omega x + \omega^2 x + \omega^3 - 1 + \omega - \omega x - \omega^2 + \omega^2 - \omega^2 x - \omega^4]$

$= x^3 \ (\because \omega^3 = 1)$

20 **(b)**

Given , $f(x) = \begin{vmatrix} x & 1 + \sin x & \cos x \\ 1 & \log(1 + x) & 2 \\ x^2 & 1 + x^2 & 0 \end{vmatrix}$

$= x\{-2(1 + x^2)\} - (1 + \sin x)(-2x^2) + \cos x\{1 + x^2 - x^2 \log(1 + x)\}$

$= -2x - 2x^3 + 2x^2 + 2x^2 \sin x + \cos x\{1 + x^2 - x^2 \log(1 + x)\}$

$\therefore$ Coefficient of x in $f(x) = -2$.

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	D	A	A	C	A	D	D	C	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	A	D	C	C	C	D	B	B