DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :7

Topic :-DETERMINANTS

1 **(b)**

We have,

$$\Delta = \begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 3 & 5 \\ 4 & 5 & 7 \\ 9 & 7 & 9 \end{vmatrix} \text{ Applying } C_2 \to C_2 - C_1 \text{and } C_3 \to C_3 - C_2$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 3 & 2 \\ 4 & 5 & 2 \\ 9 & 7 & 2 \end{vmatrix} \quad \text{Applying } C_3 \to C_3 - C_2$$

$$\Rightarrow \Delta = 2\begin{vmatrix} 1 & 3 & 1 \\ 4 & 5 & 1 \\ 9 & 7 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = 2\begin{vmatrix} 1 & 3 & 1 \\ 3 & 2 & 0 \\ 8 & 4 & 0 \end{vmatrix} \text{ Applying } R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$$

$$\Rightarrow \Delta = 2 \times -4 = -8$$

2 **(a)**

We have,

$$\begin{vmatrix} x & a & b \\ a & x & b \\ a & b & x \end{vmatrix}$$

$$= \begin{vmatrix} x & a & b \\ a-x & x-a & 0 \\ a-x & b-a & x-b \end{vmatrix} \begin{bmatrix} \text{Applying } R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{bmatrix}$$

$$= (x-a)\begin{vmatrix} x & a & b \\ -1 & 1 & 0 \\ a-x & b-a & x-b \end{vmatrix}$$

$$= (x-\alpha)\begin{vmatrix} x+a+b & a & b \\ 0 & 1 & 0 \\ 0 & b-a & x-b \end{vmatrix} \begin{bmatrix} \text{Applying} \\ C_1 \to C_1 + C_2 + C_3 \end{bmatrix}$$

$$= (x-a)(x+a+b)(x-b) \quad [\text{Expanding along } C_1]$$

3 **(c)**

We have,

$$\Delta = \begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ [x] & [y] & [z]+1 \end{vmatrix} \begin{bmatrix} \text{Applying } R_1 \to R_1 - R_3 \\ R_2 \to R_2 - R_3 \end{bmatrix}$$

$\Rightarrow \Delta = [z] + 1 + [y] + [x] = [x] + [y] + [z] + 1$

Since maximum values of $[x]$,$[y]$ and $[z]$ are 1, 0 and 2 respectively

$\therefore$ Maximum value of $\Delta = 2 + 1 + 0 + 1 = 4$

4 **(c)**

We have,

$$\begin{vmatrix} a & 2b & 2c \\ 3 & b & c \\ 4 & a & b \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a-6 & 0 & 0 \\ 3 & b & c \\ 4 & a & b \end{vmatrix} = 0 \quad \text{Applying } R_1 \to R_1 - 2R_2$$

$\Rightarrow (a-6)(b^2 - ac) = 0 \Rightarrow b^2 = ac \Rightarrow b^3 = abc$

6 **(d)**

We have, $\Delta \equiv \begin{vmatrix} a & b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix} = 0$

$\Rightarrow \Delta \equiv a(a^2 - 0) - b(0 - b^2) = a^3 + b^3$

$\Rightarrow a^3 + b^3 = 0 \Rightarrow \left(\frac{a}{b}\right)^3 = -1$

$\therefore \left(\frac{a}{b}\right)$ is one of the cube roots of -1.

7 **(b)**

We have,

$$\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix} = k \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

Applying $C_1 \leftarrow C_1 + (C_2 + C_3)$ on LHS, we have

$$\Rightarrow \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & b+c & c+a \\ 2(a+b+c) & a+b & b+c \end{vmatrix} = k \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

$$\Rightarrow 2 \begin{vmatrix} a+b+c & c+a & a+b \\ a+b+c & b+c & c+a \\ a+b+c & a+c & b+c \end{vmatrix} = k \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$ on LHS, we have

$$\Rightarrow 2 \begin{vmatrix} a+b+c & -b & -c \\ a+b+c & -a & -b \\ a+b+c & -c & -a \end{vmatrix} = k \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$ on LHS, we have

$$\Rightarrow \begin{vmatrix} a & -b & -c \\ c & -a & -b \\ b & -c & -a \end{vmatrix} = k \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

$$\Rightarrow 2\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} = k\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

$\therefore k = 2$

8 **(b)**

Let $\Delta = \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = abc - (a + b + c) + 2$

$\because \Delta > 0 \Rightarrow abc + 2 > a + b + c$

$\Rightarrow abc + 2 > 3(abc)^{1/3}$

$\left[\because \text{AM} > \text{GM} \Rightarrow \frac{a+b+c}{0} > (abc)^{1/3}\right]$

$\Rightarrow x^3 + 2 > 3x$, where $x = (abc)^{1/3}$

$\Rightarrow x^3 - 3x + 2 > 0 \Rightarrow (x-1)^2(x+2) > 0$

$\Rightarrow x + 2 > 0 \Rightarrow x > -2 \Rightarrow (abc)^{1/3} > -2$

$\Rightarrow abc > -8$

9 **(a)**

Applying $R_3 \to R_3 - R_1(\cos\beta) + R_2(\sin\beta)$

$$\begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ 0 & 0 & 1 + \sin\beta - \cos\beta \end{vmatrix}$$

$= (1 + \sin\beta - \cos\beta)(\cos^2\alpha + \sin^2\alpha)$

$= 1 + \sin\beta - \cos\beta$, which is independent of α

10 **(d)**

Given, $A = B^{-1}AB$

$\Rightarrow BA = AB$

$\therefore \det(B^{-1}AB) = \det(B^{-1}BA) = \det(A)$

11 **(d)**

Given, matrix is singular.

Therefore, $\begin{vmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ \lambda & -3 & 0 \end{vmatrix} = 0$

$\Rightarrow +1(0-6) + \lambda(3) = 0$

$\Rightarrow -6 + 3\lambda = 0$

$\Rightarrow \lambda = 2$

12 **(a)**

We have,

$|A| = \begin{vmatrix} 4 & 5 & 6 & x \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{vmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 4 & 5 & 6 & x \\ 10 & 12 & 14 & 2y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{vmatrix}$ [Applying $R_2 \to 2R_2$]

$\Rightarrow|A| = \begin{vmatrix} 4 & 5 & 6 & x \\ 0 & 0 & 0 & 0 \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{vmatrix}$ [Applying $R_2 \to R_2 - (R_1 + R_3)$]

$\Rightarrow|A| = 0 \quad [\because 2y = x + z]$

13 **(c)**

Putting $r = 1, 2, 3, \ldots, n$ and using the formula

$\sum 1 = n$ and $\sum r = \frac{(n+1)n}{2}$

$\sum (2r-1) = 1 + 3 + 5 + \ldots = n^2$

$\therefore \sum_{r=1}^{n} \Delta_r = \begin{vmatrix} n & n & n \\ n(n+1) & n^2+n+1 & n^2+n \\ n^2 & n^2 & n^2+n+1 \end{vmatrix} = 56$

Applying $C_1 \to C_1 - C_3$, $C_2 \to C_2 - C_3$

$\begin{vmatrix} 0 & 0 & n \\ 0 & 1 & n^2+n \\ -n-1 & -n-1 & n^2+n+1 \end{vmatrix}$

$\Rightarrow n(n+1) = 56$

$\Rightarrow n^2 + n - 56 = 0$

$\Rightarrow (n+8)(n-7) = 0$

$\Rightarrow n = 7 \quad (n \neq -8)$

14 **(a)**

$\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ac \\ 1 & c & c^2 - ab \end{vmatrix} = \begin{vmatrix} 0 & a-b & (a-b)(a+b+c) \\ 0 & b-c & (b-c)(a+b+c) \\ 1 & c & c^2 - ab \end{vmatrix} \quad \begin{pmatrix} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{pmatrix}$

$= (a-b)(b-c)\begin{vmatrix} 0 & 1 & a+b+c \\ 0 & 1 & a+b+c \\ 1 & c & c^2 - ab \end{vmatrix} = 0$ ($\because$ rows R_1 and R_2 are identical)

15 **(c)**

$\because \det(M_r) = r^2 - (r-1)^2 = 2r - 1$

$\therefore \det(M_1) + \det(M_2) + \ldots + \det(M_{2008})$

$= 1 + 3 + 5 + \ldots + 4015$

$= \frac{2008}{2}[2 + (2008-1)2]$

$= 2008(2008) = (2008)^2$

16 **(b)**

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$

$$= \begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega+\omega^2 & \omega^2 & 1 \\ 1+\omega+\omega^2 & 1 & \omega \end{vmatrix} \ (\because 1+\omega+\omega^2 = 0)$$

$$= \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 0 & 1 & \omega \end{vmatrix}$$

$= 0$

17 **(a)**

Given, $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$

$= 1(\omega^{3n} - 1) - \omega^n(\omega^{2n} - \omega^{2n}) + \omega^{2n}(\omega^n - \omega^{4n})$

$= 1(1-1) - 0 + \omega^{2n}(\omega^n - \omega^n) \quad [\because \omega^3 - 1]$

$= 0$

18 **(a)**

Given, $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ a & a-b & a-c \\ a^3 & a^3-b^3 & a^3-c^3 \end{vmatrix}$

$[C_2 \rightarrow C_1 - C_2, C_3 \rightarrow C_1 - C_3]$

$$= (a-b)(a-c) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^3 & a^2+ab+b^2 & a^2+ac+c^2 \end{vmatrix}$$

$= (a-b)(a-c)(c^2 + ac - ab - b^2)$

$= (a-b)(b-c)(c-a)(a+b+c)$

19 **(c)**

We have,

$$\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 2\,(b+c) & 2\,(c+a) & 2\,(a+b) \\ b & c+a & b \\ c & c & a+b \end{vmatrix} \quad \text{Applying } R_1 \rightarrow R_1 + 2R_2 + R_3$$

$$\Rightarrow \Delta = 2\begin{vmatrix} b+c & c+a & a+b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$\Rightarrow \Delta = 2\begin{vmatrix} b+c & c+a & a+b \\ -c & 0 & -a \\ -b & -a & 0 \end{vmatrix} \quad \text{Applying } R_2 \rightarrow R_2 - R_1,\ R_3 \rightarrow R_3 - R_1$$

$\Rightarrow \Delta = 2\{(b+c)(0-a^2) - (c+a)(0-ab) + (a+b)(ac-0)\}$

$\Rightarrow \Delta = 2\{-a^2(b+c) + ab(c+a) + ac(a+b)\}$
$\Rightarrow \Delta = 2(-a^2b - a^2c + abc + a^2b + a^2c + abc)$
$\Rightarrow \Delta = 4\,abc$

20 **(d)**

$$\begin{vmatrix} \log_5 729 & \log_3 5 \\ \log_5 27 & \log_9 25 \end{vmatrix} = \begin{vmatrix} \log_3 3^6 & \log_3 5 \\ \log_5 3^3 & \log_{3^2} 5^2 \end{vmatrix}$$

$$= \begin{vmatrix} 6\log_5 3 & \log_3 5 \\ 3\log_5 3 & \frac{2}{2}\log_3 5 \end{vmatrix}$$

$= 6\log_5 3 \log_3 5 - 3\log_5 3 \log_3 5$

$= 6 - 3 = 3$

And $\begin{vmatrix} \log_3 5 & \log_{27} 5 \\ \log_5 9 & \log_5 9 \end{vmatrix} = \begin{vmatrix} \log_3 5 & \log_{3^3} 5 \\ \log_5 3^2 & \log_5 3^2 \end{vmatrix}$

$$= \begin{vmatrix} \log_3 5 & \frac{1}{3}\log_3 5 \\ 2\log_5 3 & 2\log_5 3 \end{vmatrix}$$

$= 2\log_5 3 \log_3 5 - \frac{2}{3}\log_5 3 \log_3 5$

$= 2 - \frac{2}{3} = \frac{4}{3}$

Now, $\begin{vmatrix} \log_5 729 & \log_3 5 \\ \log_5 27 & \log_9 25 \end{vmatrix} \begin{vmatrix} \log_3 5 & \log_{27} 5 \\ \log_5 9 & \log_5 9 \end{vmatrix} = 3.\frac{4}{3} = 4$

Take option(d),

$\log_3 5 . \log_5 81 = \log_3 81 = \log_3 3^4 = 4$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	C	C	B	D	B	B	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	A	C	A	C	B	A	A	C	D