DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :s6

Topic :-DETERMINANTS

1 **(a)**

Given, $A=\begin{bmatrix}1 & 2\\ 3 & 5\end{bmatrix}$

$\Rightarrow |A|=5-6=-1$

$\therefore |A^{2009}-5A^{2008}|=|A^{2008}||A-5I|$

$=(-1)^{2008}\left|\begin{bmatrix}1 & 2\\ 3 & 5\end{bmatrix}-\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}\right|$

$=\begin{vmatrix}-4 & 2\\ 3 & 0\end{vmatrix}=-62$ **(b)**

$f(1)=\begin{vmatrix}-2 & -16 & -78\\ -4 & -48 & -496\\ 1 & 2 & 3\end{vmatrix}=2928$

$f(3)=\begin{vmatrix}0 & 0 & 0\\ -2 & -32 & -392\\ 1 & 2 & 3\end{vmatrix}=0$

and $f(5)=\begin{vmatrix}2 & 32 & 294\\ 0 & 0 & 0\\ 1 & 2 & 3\end{vmatrix}=0$

$\therefore\ f(1).f(3)+f(3).f(5)+f(5).f(1)$

$=f(1).0+0+f(1).0=0=f(3)$ or $f\ (5)$

3 **(d)**

$\Delta=(x+a+b+c)\begin{vmatrix}1 & a & b+c\\ 1 & b & c+a\\ 1 & c & a+b\end{vmatrix}$ $[C_1\to C_1+C_2+C_3]$

$=(x+a+b+c)(a+b+c)\begin{vmatrix}1 & 1 & b+c\\ 1 & 1 & c+b\\ 1 & 1 & a+b\end{vmatrix}$

$=0$ $[C_2\to C_2+C_3]$

Hence, x may have any value.

4 **(c)**

It has a non-zero solution, if $\begin{vmatrix}1 & k & -1\\ 3 & -k & -1\\ 1 & -3 & 1\end{vmatrix}=0$

$\Rightarrow 1(-k-3)-k(3+1)-1(-9+k)=0$

$\Rightarrow -6k+6=0$

$\Rightarrow k=1$

5 **(a)**

Given, $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix}=0$

$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}+\begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix}=0$

$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}+xyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}=0$

$\Rightarrow (1+xyz)\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}=0$

$\Rightarrow 1+xyz=0$

$\Rightarrow \ xyz=-1$

6 **(a)**

$\begin{vmatrix} [e] & [\pi] & [\pi^2-6] \\ [\pi] & [\pi^2-6] & [e] \\ [\pi^2-6] & [e] & [\pi] \end{vmatrix}$

$=\begin{vmatrix} 2 & 3 & 3 \\ 3 & 3 & 2 \\ 3 & 2 & 3 \end{vmatrix}$

$=2(9-4)-3(9-6)+3(6-9)$

$=10-9-9$

$=-8$

7 **(b)**

We have,

$\Delta=\begin{vmatrix} a & b & a\,x+b \\ b & c & b\,x+c \\ a\,x+b & b\,x+c & 0 \end{vmatrix}$

$\Rightarrow \Delta=\begin{vmatrix} a & b & a\,x+b \\ b & c & b\,x+c \\ 0 & 0 & -\left(a\,x^2+2\,b\,x+c\right) \end{vmatrix}$, $\begin{bmatrix} \text{Applying } R_3 \to R_3 - x \\ R_1 - R_2 \end{bmatrix}$

$\Rightarrow \Delta=(b^2-ac)(ac^2+2\,b\,x+c)$

$\therefore \Delta=0$

$\Rightarrow b^2=ac$ or, $a\,x^2+2\,b\,x+c=0$

$\Rightarrow a,b,c$ are in G.P. or, x is a root of the equation

$ax^2+2b\,x+c=0$

8 **(d)**

All statements are false.

9 **(b)**

Applying $C_3 \to C_3-C_1$,we get

$$\Delta = \begin{vmatrix} 1 & \alpha & \alpha^2 - 1 \\ \cos(p-d)\,a & \cos pa & 0 \\ \sin(p-d)\,a & \sin pa & 0 \end{vmatrix}$$

$= (\alpha^2 - 1)\{\sin pa \cos(p-d)\,a - \cos pa \sin(p-d)\,a\}$

$= (\alpha^2 - 1)\sin\{-(p-d)a + pa\}$

$\Rightarrow \Delta = (\alpha^2 - 1)\sin da$

Which is independent of p.

10 **(c)**

Given, $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$ and taking common $(3a-x)$ from C_1, we get

$(3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 1 & a+x & a-x \\ 1 & a-x & a+x \end{vmatrix} = 0$

$\Rightarrow (3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 0 & 2x & 0 \\ 0 & 0 & 2x \end{vmatrix} = 0 \begin{bmatrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{bmatrix}$

$\Rightarrow (3a-x)(4x^2) = 0$

$\Rightarrow x = 3a, 0$

11 **(a)**

Since, the given equations are consistent.

$\therefore \begin{vmatrix} 2 & 3 & 1 \\ 3 & 1 & -2 \\ a & 2 & -b \end{vmatrix} = 0$

$\Rightarrow 2(-b+4) - 3(-3b+2a) + 1(6-a) = 0$

$\Rightarrow -2b + 8 + 9b - 6a + 6 - a = 0$

$\Rightarrow 7b - 7a = -14$

$\Rightarrow a - b = 2$

12 **(d)**

Given, $\Delta(x) = \begin{vmatrix} 1 & \cos x & 1 - \cos x \\ 1 + \sin x & \cos x & 1 + \sin x - \cos x \\ \sin x & \sin x & 1 \end{vmatrix}$

Applying $C_3 \to C_3 + C_2 - C_1$

$= \begin{vmatrix} 1 & \cos x & 0 \\ 1 + \sin x & \cos x & 0 \\ \sin x & \sin x & 1 \end{vmatrix}$

$= \cos x - \cos x\,(1 + \sin x)$

$= -\cos x \sin x$

$= -\frac{1}{2}\sin 2x$

$\therefore \int_0^{\pi/2} \Delta x\, dx = -\frac{1}{2}\int_0^{\pi/2} \sin 2x\, dx$

$= -\frac{1}{2}\left[-\frac{\cos 2x}{2}\right]_0^{\pi/2} = -\frac{1}{2}$

13 **(c)**

For the non-trivial solution, we must have

$$\begin{vmatrix} 1 & a & a \\ b & 1 & b \\ c & c & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1-a & 0 & a \\ b-1 & 1-b & b \\ 0 & c-1 & 1 \end{vmatrix} = 0 \begin{bmatrix} \text{Applying } C_1 \to C_1 - C_2; \\ C_2 \to C_2 - C_3 \end{bmatrix}$$

$\Rightarrow (1-a)[(1-b)-b(c-1)]+a(b-1)(c-1)=0$

$$\Rightarrow \frac{1}{c-1}+\frac{b}{b-1}+\frac{a}{a-1}=0$$

$$\Rightarrow \left(\frac{1}{c-1}+1\right)+\frac{b}{b-1}+\frac{a}{a-1}=1$$

$$\Rightarrow \frac{c}{c-1}+\frac{b}{b-1}+\frac{a}{a-1}=1$$

$$\Rightarrow \frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=-1$$

14 **(d)**

Given system equations are

$3x-2y+z=0$

$\lambda x-14y+15z=0$ and $x+2y-3z=0$

The system of equations has infinitely many (non-trivial solutions, if $\Delta = 0$.

$$\Rightarrow \Delta = \begin{vmatrix} 3 & -2 & 1 \\ \lambda & -14 & 15 \\ 1 & 2 & -3 \end{vmatrix} = 0$$

$\Rightarrow 3(42-30)-\lambda(6-2)+1(-30+14)=0$

$\Rightarrow 36-4\lambda-16=0$

$\Rightarrow \lambda=5$

16 **(c)**

Since, $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

$\Rightarrow \sin x(\sin^2 x-\cos^2 x)-\cos x(\cos x \sin x-\cos^2 x)$
$+\cos x(\cos^2 x-\sin x\cos x)=0$

$\Rightarrow \sin x(\sin^2 x-\cos^2 x)-2\cos^2 x(\sin x-\cos x)=0$

$\Rightarrow (\sin x-\cos x)[\sin x(\sin x+\cos x)-2\cos^2 x]=0$

$\Rightarrow (\sin x-\cos x)[(\sin^2 x-\cos^2 x)$
$+(\sin x\cos x-\cos^2 x)]=0$

$\Rightarrow (\sin x-\cos x)^2[\sin x+\cos x+\cos x]=0$

$\Rightarrow (\sin x-\cos x)^2(\sin x+2\cos x)=0$

$\Rightarrow$ Either $(\sin x-\cos x)^2=0$

or $\sin x+2\cos x)=0$

$\Rightarrow$Either $\tan x=1$ or $\tan x=-2$

$\Rightarrow$ Either $x = \frac{\pi}{4}$ or $\tan x = -2$

As $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$, $\tan x \in [-1, 1]$

Hence, real solution is only $x = \frac{\pi}{4}$

17 **(a)**

Applying $R_1 \to R_1 + R_3 - 2R_2$, we get

$$\Delta = \begin{vmatrix} 0\ 0\ 0 & x+z-zy \\ 4\ 5\ 6 & y \\ 5\ 6\ 7 & z \\ x\ y\ z & 0 \end{vmatrix}$$

$$= -(x+z-2y)\begin{vmatrix} 4 & 5 & 6 \\ 5 & 6 & 7 \\ x & y & z \end{vmatrix} \quad [\text{Expanding along } R_1]$$

$$= -(x+z-2y)\begin{vmatrix} 0 & -1 & 6 \\ 0 & -1 & 7 \\ x-2y+z & y-z & z \end{vmatrix} \quad \begin{bmatrix} \text{Applying } C_1 \to C_1 + C_3 \\ -2C_2 \text{ and } C_2 \to C_2 - C_3 \end{bmatrix}$$

$$= -(x+z-2y)^2\begin{vmatrix} -1 & 6 \\ -1 & 7 \end{vmatrix} = (x-2y+z)^2$$

19 **(c)**

We have, $a = 1 + 2 + 4 + 8 + \ldots$ upto n terms

$$= 1\left(\frac{2^n - 1}{2 - 1}\right) = 2^n - 1$$

$b = 1 + 3 + 9 + \ldots$ upto n terms$=\frac{3^n - 1}{2}$

and $c = 1 + 5 + 25 + \ldots$ upto n terms$=\frac{5^n - 1}{4}$

$$\therefore \begin{vmatrix} a & 2b & 4c \\ 2 & 2 & 2 \\ 2^n & 3^n & 5^n \end{vmatrix} = 2\begin{vmatrix} 2^n - 1 & 3^n - 1 & 5^n - 1 \\ 1 & 1 & 1 \\ 2^n & 3^n & 5^n \end{vmatrix}$$

$$= 2\begin{vmatrix} 2^n & 3^n & 5^n \\ 1 & 1 & 1 \\ 2^n & 3^n & 5^n \end{vmatrix} \quad [R_1 \to R_1 + R_2]$$

$= 2 \times 0 = 0$ [$\therefore$ two rows are identical]

20 **(d)**

$$\text{Let } \Delta = \begin{vmatrix} c & 1 & 0 \\ 1 & c & 1 \\ 6 & 1 & c \end{vmatrix} = c(c^2 - 1) - 1(c - 6)$$

$= 8\cos^3\theta - 4\cos\theta + 6$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	D	C	A	A	B	D	B	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	D	C	D	D	C	A	C	C	D