DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :5

Topic :-DETERMINANTS

1 **(b)**

We have, $\Delta(x)=\begin{vmatrix} x^n & \sin x & \cos x \\ n! & sin\frac{n\pi}{2} & \cos\frac{n\pi}{2} \\ a & a^2 & a^3 \end{vmatrix}$

$$\frac{d^n}{dx^n}[\Delta(x)]=\begin{vmatrix} \frac{d^n}{dx^n}x^n & \frac{d^n}{dx^n}\sin x & \frac{d^n}{dx^n}\cos x \\ n! & \sin\left(\frac{n\pi}{2}\right) & \cos\left(\frac{n\pi}{2}\right) \\ a & a^2 & a^3 \end{vmatrix}$$

($\because$ Differentiation of R_2 and R_3 are zero)

$$\begin{vmatrix} n! & \sin\left(x+\frac{n\pi}{2}\right) & \cos\left(x+\frac{n\pi}{2}\right) \\ n! & \sin\left(\frac{n\pi}{2}\right) & \cos\left(\frac{n\pi}{2}\right) \\ a & a^2 & a^3 \end{vmatrix}$$

$$\Rightarrow[\Delta^n(x)]_{x=0}=\begin{vmatrix} n! & \sin\left(0+\frac{n\pi}{2}\right) & \cos\left(0+\frac{n\pi}{2}\right) \\ n! & \sin\left(\frac{n\pi}{2}\right) & \cos\left(\frac{n\pi}{2}\right) \\ a & a^2 & a^3 \end{vmatrix}$$

$$=\begin{vmatrix} n! & \sin\left(\frac{n\pi}{2}\right) & \cos\left(\frac{n\pi}{2}\right) \\ n! & \sin\left(\frac{n\pi}{2}\right) & \cos\left(\frac{n\pi}{2}\right) \\ a & a^2 & a^3 \end{vmatrix}$$

$=0$ ($\because R_1$ and R_2 are identical)

2 **(a)**

Let, $\Delta=\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$

$=1(1-\log_z y\ \ \log_y z)-\log_x y(\log_y x-\log_y z\ \ \log_z x)+\log_x z\,(\log_y x\ \ \log_z y-\log_z x$

$=(1-\log_z z)-\log_x y(\log_y x-\log_y z\ \ \log_z x)+\log_x z\,(\log_y x\log_z y-\log_z x)$

$= (1-1) - (1 - \log_x y \log_y x) + (\log_x z \log_z x - 1) = 0$ (Since, $\log_x y \log_y x = 1$)
$= 0 - (1-1) + (1-1) = 0$

3 **(b)**
Given determinant is

$$\Delta = \begin{vmatrix} 15! & 16! & 17! \\ 16! & 17! & 18! \\ 17! & 18! & 19! \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_2$

$$\Delta = \begin{vmatrix} 15! & 15 \times 15! & 16 \times 16! \\ 16! & 16 \times 16! & 17 \times 17! \\ 17! & 17 \times 17! & 18 \times 18! \end{vmatrix}$$

$$= (15!)(16!)(17!) \begin{vmatrix} 1 & 15 & 16 \times 16 \\ 1 & 16 & 17 \times 17 \\ 1 & 17 & 18 \times 18 \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$= (15!)(16!)(17!) \begin{vmatrix} 0 & -1 & -33 \\ 0 & -1 & -35 \\ 1 & 17 & 18 \times 18 \end{vmatrix}$$

$= 2 \times (15!)(16!)(17!)$

4 **(b)**
We have,

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 5 & 9 \\ 0 & 3 & 9 & 19 \end{vmatrix} \begin{bmatrix} \text{Applying } R_2 \to R_2 - R_1, \\ R_3 \to R_3 - R_1, R_4 \to R_4 - R_1 \end{bmatrix}$$

$$= \begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 9 \\ 3 & 9 & 19 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 3 & 10 \end{vmatrix} \begin{bmatrix} \text{Applying } R_2 \to R_2 - 2R_1, \\ R_3 \to R_3 - 3R_1 \end{bmatrix}$$

$= (10 - 9) = 1$

5 **(c)**
The homogenous linear system of equations is consistent *ie*, possesses trivial solution, if $\Delta \equiv$

$$\begin{vmatrix} 2 & 3 & 5 \\ 1 & k & 5 \\ k & -12 & -14 \end{vmatrix} \neq 0$$

$\Rightarrow 2(-14k + 60) - 3(-14 - 5k) + 5(-12 - k^2) \neq 0$
$\Rightarrow 5k^2 + 13k - 102 \neq 0$
$\Rightarrow (5k - 17)(k + 6) \neq 0$
$\Rightarrow k \neq -6, \frac{17}{5}$

6 **(c)**

We have,

$$\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ca & c\,b & a^2+b^2 \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} a(b^2+c^2) & a^2b & a^2c \\ ab^2 & b(c^2+a^2) & b^2c \\ c^2a & c^2b & c(a^2+b^2) \end{vmatrix}$$

[Applying $R_1 \to R_1(a), R_2 \leftrightarrow R_2(b), R_3 \leftrightarrow R_3(c)$]

$$= \frac{1}{abc}abc\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & -2\,c^2 & -2\,b^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$ Applying $R_1 \to R_1 - (R_2 + R_3)$

$= 4\,a^2b^2c^2$

$\therefore ka^2b^2c^2 = 4a^2b^2c^2 \Rightarrow k = 4$

7 **(a)**

We have,

$$\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$

$$= \begin{vmatrix} 2(x+y+z) & x+y+z & x+y+z \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$ Applying $R_1 \to R_1 + R_2 + R_3$

$$= (x+y+z)\begin{vmatrix} 2 & 1 & 1 \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 0 & 1 & 1 \\ 0 & z & x \\ x-z & y & z \end{vmatrix}$$ Applying $C_1 \to C_1 - C_2 - C_3$

Hence, the repeating factor is $(z - x)$

8 **(d)**

$$\begin{vmatrix} 4+x^2 & -6 & -2 \\ -6 & 9+x^2 & 3 \\ -2 & 3 & 1+x^2 \end{vmatrix}$$

$= (4+x^2)[(1+x^2)(9+x^2) - 9]$

$+6[-6(1+x^2)+6] - 2[-18 + 2(9+x^2)]$

$= (4+x^2)(10x^2+x^4) - 36x^2 - 4x^2$

$= 40x^2 + 4x^4 + 10x^4 + x^6 - 40x^2$

$= x^4(x^2+14)$

Which is not divisible by x^5.

9 **(d)**

Since, for $x=0$, the determinant reduces to the determinant of a skew-symmetric matrix of odd order which is always zero. Hence, $x=0$ is the solution of the given equation.

10 **(c)**

$$\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$=-2\begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} \; [R_1 \to R_1-(R_2+R_3)]$$

$$=-2\begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & a^2 & 0 \\ c^2 & 0 & a^2 \end{vmatrix} \; \begin{pmatrix} R_2 \to R_2-R_1 \\ R_3 \to R_3-R_1 \end{pmatrix}$$

$$=-2[-c^2(b^2a^2-0)+b^2(0-a^2c^2)]$$

$$=-2[-2a^2b^2c^2]=4a^2b^2c^2$$

11 **(c)**

We have, $\begin{vmatrix} p & b & c \\ p+a & q+b & 2c \\ a & b & r \end{vmatrix}=0$

$$\Rightarrow \begin{vmatrix} p & b & c \\ p & b & c \\ a & b & r \end{vmatrix}+\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix}=0$$

$$\Rightarrow 0+\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix}=0$$

$$\Rightarrow p(qr-bc)-b(ar-ac)-c(ab-aq)=0$$

$$\Rightarrow -pqr+pbc+bar+acq=0$$

On simplifying, we get

$$\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}=2$$

12 **(d)**

Let $\Delta=\begin{vmatrix} a+b+2c & a & b \\ c & 2a+b+c & b \\ c & a & a+2b+c \end{vmatrix}$

Applying $C_1 \to C_1+C_2+C_3$

$$=\begin{vmatrix} 2(a+b+c) & a & b \\ 2(a+b+c) & 2a+b+c & b \\ 2(a+b+c) & a & a+2b+c \end{vmatrix}$$

$$=2(a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & 2a+b+c & b \\ 1 & a & a+2b+c \end{vmatrix}$$

$$=2(a+b+c)\begin{vmatrix} 0 & -(a+b+c) & 0 \\ 0 & (a+b+c) & -(a+b+c) \\ 1 & a & a+2b+c \end{vmatrix} \; \begin{pmatrix} R_1 \to R_1-R_2 \\ R_2 \to R_2-R_3 \end{pmatrix}$$

$$=2(a+b+c)^3\begin{vmatrix} 0 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & a & a+2b+c \end{vmatrix}$$

$= 2(a+b+c)^3$

13 **(c)**

Since, $-1 \le x < 0$

$\therefore [x] = -1$

Also, $0 \le y < 1 \Rightarrow [y] = 0$

and $1 \le z < 2 \Rightarrow [z] = 1$

$\therefore$ Given determinant becomes

$$\begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & 0 & 2 \end{vmatrix} = 1 = [z]$$

14 **(b)**

For singular matrix,

$$\begin{bmatrix} -x & x & 2 \\ 2 & x & -x \\ x & -2 & -x \end{bmatrix} = 0$$

Applying $C_2 \to C_2 + C_1$, $C_3 \to C_3 + C_1$

$$\Rightarrow \begin{vmatrix} -x & 0 & 2-x \\ 2 & 2+x & 2-x \\ x & x-2 & 0 \end{vmatrix} = 0$$

$$\Rightarrow (2-x) \begin{vmatrix} -x & 0 & 1 \\ 2 & 2+x & 1 \\ x & x-2 & 0 \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$

$$\Rightarrow (2-x) \begin{vmatrix} -x & 0 & 1 \\ 2+x & 2+x & 0 \\ x & x-2 & 0 \end{vmatrix} = 0$$

$$\Rightarrow (2-x)(2+x) \begin{vmatrix} -x & 0 & 1 \\ 1 & 1 & 0 \\ x & x-2 & 0 \end{vmatrix} = 0$$

$\Rightarrow (2-x)(2+x)(x-2-x) = 0$

$\Rightarrow x = 2, -2$

$\therefore$ Given matrix is non $-$ singular for all x other than 2 and -2.

16 **(c)**

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^n \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ a & -b & c \end{vmatrix}^1$$

$$= \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^n \begin{vmatrix} a+1 & a-1 & a \\ b+1 & b-1 & -b \\ c-1 & c+1 & c \end{vmatrix}$$

$$== \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n+1} \begin{vmatrix} a+1 & a & a-1 \\ b+1 & -b & b-1 \\ c-1 & c & c+1 \end{vmatrix}$$

$C_2 \leftrightarrow C_3$

$= (1 + (-1)^{n+2})\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}$

This is equal to zero only, if $n+2$ is odd *ie*, n is an odd integer.

17 **(d)**

Given that, $\begin{vmatrix} \frac{1}{a} & a^2 & bc \\ \frac{1}{b} & b^2 & ca \\ \frac{1}{c} & c^2 & ab \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} 1 & a^3 & abc \\ 1 & b^3 & abc \\ 1 & c^3 & abc \end{vmatrix}$

$= \frac{abc}{abc}\begin{vmatrix} 1 & a^3 & 1 \\ 1 & b^3 & 1 \\ 1 & c^3 & 1 \end{vmatrix}$

$= 0$

($\because$ columns C_1 and C_3 are same)

18 **(b)**

Given that, $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix}=0$

$\Rightarrow \begin{vmatrix} x & -6 & -1 \\ 5 & -5x & -5 \\ -3 & 2x & x+2 \end{vmatrix} = 0 \ (R_2 \to R_2 - R_3)$

$\Rightarrow 5\begin{vmatrix} x & -6 & -1 \\ 1 & -x & -1 \\ -3 & 2x & x+2 \end{vmatrix} = 0$

$\Rightarrow x(-x^2 - 2x + 2x) - 1(-6x - 12 + 2x) - 3(6 - x) = 0$

$\Rightarrow -x^3 + 7x - 6 = 0$

$\Rightarrow x^3 - 7x + 6 = 0$

$\Rightarrow (x-1)(x-2)(x+3) = 0$

$\Rightarrow x = 1, 2, -3$

$\therefore$ Option (b) is correct.

19 **(a)**

$\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$

Applying $R_2 \to R_2 - R_3$

$= \begin{vmatrix} a^2 & b^2 & c^2 \\ 4a & 4b & 4c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$

$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$

Applying $R_3 \to R_3 - (R_1 - 2R_2)$

$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	B	B	C	C	A	D	D	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	D	C	B	B	C	D	B	A	D