DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :4

Topic :-DETERMINANTS

1 **(d)**

Applying $R_1 \to R_1 + R_2 + R_3$ and taking common $(a+b+c)$ from R_1,we get

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 0 \\ 2c & 0 & c-a-b \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$,

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & -2b \\ 2c & 0 & -a-b-c \end{vmatrix}$$

$= (c+b+c)[(-b-c-a)(-a-b-c)]$

$= (a+b+c)^3$

2 **(b)**

We know that

$|AB| = |A||B|$

$\Rightarrow AB = 0$

$\Rightarrow |AB| = 0$

$\Rightarrow |A||B| = 0$

$\Rightarrow$either $|A| = 0$ or, $|B| = 0$

3 **(a)**

The given system of equations will have a unique solution, if

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0 \Rightarrow k \neq 0$$

114 **(a)**

$\therefore a_1, a_2, \dots, a_n$ are in GP.

$\Rightarrow a_n, a_n + 2, a_{n+4}, \dots$ are also in GP.

Now, $(a_{n+2})^2 = a_n . a_{n+4}$

$\Rightarrow 2\log(a_{n+2}) = \log a_n + \log a_{n+4}$

Similarly, $2\log(a_{n+8}) = \log a_{n+6} + \log a_{n+10}$

$$\text{Now, } \Delta = \begin{vmatrix} \log a_n & \log a_{n+2} & \log a_{n+4} \\ \log a_{n+6} & \log a_{n+8} & \log a_{n+10} \\ \log a_{n+12} & \log a_{n+14} & \log a_{n+16} \end{vmatrix}$$

Applying $C_2 \to 2C_2 - C_1 - C_3$

$$\begin{vmatrix} \log a_n & 2\log a_{n+2} - \log a_n - \log a_{n+4} & \log a_{n+4} \\ \log a_{n+6} & 2\log a_{n+8} - \log a_{n+6} - \log a_{n+10} & \log a_{n+10} \\ \log a_{n+12} & 2\log a_{n+14} - \log a_{n+12} - \log a_{n+16} & \log a_{n+16} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_n & 0 & \log a_{n+4} \\ \log a_{n+6} & 0 & \log a_{n+10} \\ \log a_{n+12} & 0 & \log a_{n+16} \end{vmatrix} = 0$$

6 **(a)**

We have,

Coefficient of x in $\begin{vmatrix} x & (1+\sin x)^3 & \cos x \\ 1 & \log(1+x) & 2 \\ x^2 & (1+x)^2 & 0 \end{vmatrix}$

$=$ coefficient of x in

$$= \begin{vmatrix} x & \left(1 + x - \frac{x^3}{3!} + \dots\right)^3 & 1 - \frac{x^2}{2!} + \dots \\ 1 & x - \frac{x^2}{2} + \frac{x^3}{3} \dots & 2 \\ x^2 & 1 + 2x + x^2 & 0 \end{vmatrix}$$

$=$ Coefficient of x in $\begin{vmatrix} x & 1 & 1 \\ 1 & x & 2 \\ x^2 & 1 & 0 \end{vmatrix}$

$=$ Coefficient of x in $\left[x(0-2) - (0 - 2x^2) + (1 - x^3)\right] = -2$

9 **(d)**

On putting $x = 0$ in the given equation, we get

$g = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix} = 9$

On differentiating given equation and then put $x = 0$, we get

$f = -5$

10 **(a)**

In ΔABC, given $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$

$\Rightarrow 1(c^2 - ab) - a(c - a) + b(b - c) = 0$

$\Rightarrow a^2 + b^2 + c^2 - ab - bc - ca = 0$

$\Rightarrow 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$

$\Rightarrow (a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc) + (c^2 + a^2 - 2ca) = 0$

$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$

Here, sum of squares of three numbers can be zero, if and only, if $a = b = c$.

$\Rightarrow \Delta\ ABC$ is an equilateral triangle.

$\Rightarrow \angle A = \angle B = \angle C = 60°$

$\therefore \sin^2 A + \sin^2 B + \sin^2 C = \sin^2 60° + \sin^2 60° + \sin^2 60°$

$= \left(\frac{3}{4} + \frac{3}{4} + \frac{3}{4}\right) = \frac{9}{4}$

12 **(d)**

$$\Delta(-x) = \begin{vmatrix} f(-x) + f(x) & 0 & x^4 \\ 3 & f(-x) - f(x) & \cos x \\ x^4 & -2x & f(-x)f(x) \end{vmatrix}$$

$$\begin{vmatrix} f(x) + f(-x) & 0 & x^4 \\ 3 & f(x) - f(-x) & \cos x \\ x^4 & -2x & f(x)f(-x) \end{vmatrix} = -\Delta(x)$$

So, $\Delta(x)$ is an odd function.

$\Rightarrow x^4\Delta(x)$ is an odd function

$\Rightarrow \int_{-2}^{2} x^4\Delta(x)dx = 0$

13 **(c)**

$$\begin{vmatrix} \cos(x-a) & \cos(x+a) & \cos x \\ \sin(x+a) & \sin(x-a) & \sin x \\ \cos a \tan x & \cos a \cot x & \operatorname{cosec} 2x \end{vmatrix}$$

$$\begin{vmatrix} \cos(x-a) + \cos(x-a) & \cos(x+a) & \cos x \\ \sin(x+a) + \sin(x-a) & \sin(x-a) & \sin x \\ \cos a\ (\tan x + \cot x) & \cos a \cot x & \operatorname{cosec} 2x \end{vmatrix}$$

$$= \begin{vmatrix} 2\cos x \cos a & \cos(x+a) & \cos x \\ 2\sin x \cos a & \sin(x-a) & \sin x \\ \cos a\left(\frac{\tan^2 x + 1}{\tan x}\right) & \cos a \cot x & \operatorname{cosec} 2x \end{vmatrix}$$

$$= 2\cos a \begin{vmatrix} \cos x & \cos(x+a) & \cos x \\ \sin x & \sin(x-a) & \sin x \\ \operatorname{cosec} 2x & \cos a \cot x & \operatorname{cosec} 2x \end{vmatrix} = 0$$

[$\because$ two columns are identical]

15 **(a)**

Since $(x-k)$ will be common from each row which vanish by putting $x = k$. Therefore, $(x-k)^r$ will be a factor of $|A|$

16 **(d)**

Putting $x = 0$ in the given determinant equation we get

$$a_0 = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 2 & -3 \\ -3 & 4 & 0 \end{vmatrix}$$

$= 1(0-9) + 3(4+6)$

$= 30 - 9 = 21$

17 **(a)**

Given, $\begin{vmatrix} a & \cot\frac{A}{2} & \lambda \\ b & \cot\frac{B}{2} & \mu \\ c & \cot\frac{C}{2} & \gamma \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} a & \frac{s(s-a)}{\Delta} & \lambda \\ b & \frac{s(s-b)}{\Delta} & \mu \\ c & \frac{s(s-c)}{\Delta} & \gamma \end{vmatrix} = 0$

$\left[\because \cot\frac{A}{2} = \frac{s(s-a)}{\sqrt{(s-a)(s-b)(s-c)}} = \frac{s(s-a)}{\Delta}\right]$

$\Rightarrow \frac{1}{r}\begin{vmatrix} a & s-a & \lambda \\ b & s-b & \mu \\ c & s-c & \gamma \end{vmatrix} = 0$ where $r = \frac{\Delta}{s}$

Applying $C_2 \to C_2 + C_1$

$\Rightarrow \frac{1}{r}\begin{vmatrix} a & s & \lambda \\ b & s & \mu \\ c & s & \gamma \end{vmatrix} = 0$

$\Rightarrow \frac{\Delta}{r^2}\begin{vmatrix} a & 1 & \lambda \\ b & 1 & \mu \\ c & 1 & \gamma \end{vmatrix} = 0$

Applying $R_1 \to R_1 - R_{2,} R_2 \to R_2 - R_3$

$\Rightarrow \frac{\Delta}{r^2}\begin{vmatrix} a-b & 0 & \lambda-\mu \\ b-c & 0 & \mu-\gamma \\ c & 1 & \gamma \end{vmatrix} = 0$

$\Rightarrow \frac{\Delta}{r^2}[(b-c)(\lambda-\mu) - (\mu-\gamma)(a-b)] = 0$

$\Rightarrow b(\lambda-\mu) - c(\lambda-\mu) - a(\mu-\gamma) + b(\mu-\gamma) = 0$

$\Rightarrow -a(\mu-\gamma) + b(\lambda-\mu+\mu-\gamma) - c(\lambda-\mu) = 0$

$\Rightarrow -a(\mu-\gamma) + b(\lambda-\gamma) - c(\lambda-\mu) = 0$

$\Rightarrow a(\mu-\gamma) + b(\gamma-\lambda) + c(\lambda-\mu) = 0$

19 **(d)**

Let $\Delta = \begin{vmatrix} x+2 & x+3 & x+a \\ x+4 & x+5 & x+b \\ x+6 & x+7 & x+c \end{vmatrix}$

Applying $C_2 \to C_2 - C_1$,we get

$\Delta = \begin{vmatrix} x+2 & 1 & x+a \\ x+4 & 1 & x+b \\ x+6 & 1 & x+c \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow \Delta = \begin{vmatrix} x+2 & 1 & x+a \\ 2 & 0 & b-a \\ 4 & 0 & c-a \end{vmatrix}$

$= -1(2c - 2a - 4b + 4a)$

$\Rightarrow \Delta = 2(2b - c - a)$ ...(i)

Since, a,b,c are in AP.

$\therefore b = \dfrac{a+c}{2}$

$\therefore\ \Delta = 2(a + c - c - a)$

$= 0$ [from Eq. (i)]

20 **(a)**

$$\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix}$$

$$= \begin{vmatrix} \cos A\cos P + \sin A\sin P & \cos A\cos Q + \sin A\sin Q & \cos A\cos R + \sin A\sin R \\ \cos B\cos P + \sin B\sin P & \cos B\cos Q + \sin B\sin Q & \cos B\cos R + \sin B\sin R \\ \cos C\cos P + \sin C\sin P & \cos C\cos Q + \sin C\sin Q & \cos C\cos R + \sin C\sin R \end{vmatrix}$$

The determinants can be rewritten as 8 determinants and the value of each of these 8 determinants is zero.

ie, $\cos P\cos Q\cos R \begin{vmatrix} \cos A & \cos A & \cos A \\ \cos B & \cos B & \cos B \\ \cos C & \cos C & \cos C \end{vmatrix} = 0$

Similarly, other determinants can be shown zero.

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	B	A	A	B	A	D	D	D	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	D	C	A	A	D	A	D	D	A