

DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :3

Topic :-DETERMINANTS

1 (c)

$$\text{Given, } f(x) = \begin{vmatrix} 1 & 2(x-1) & 3(x-1)(x-2) \\ x-1 & (x-1)(x-2) & (x-1)(x-2)(x-3) \\ x & x(x-1) & x(x-1)(x-2) \end{vmatrix}$$
$$= (x-1)(x-1)(x-2) \begin{vmatrix} 1 & 2 & 3 \\ x-1 & x-2 & x-3 \\ x & x & x \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 - C_2$, $C_2 \rightarrow C_2 - C_3$

$$= (x-1)^2(x-2) \begin{vmatrix} -1 & -1 & 3 \\ 1 & 1 & x-3 \\ 0 & 0 & x \end{vmatrix}$$
$$= (x-1)^2(x-2)x(-1+1) = 0$$

$$\Rightarrow f(x) = 0$$

$$\therefore f(49) = 0$$

2 (b)

$$\text{Given that, } \begin{vmatrix} 1+ax & 1+bx & 1+cx \\ 1+a_1x & 1+b_1x & 1+c_1x \\ 1+a_2x & 1+b_2x & 1+c_2x \end{vmatrix}$$

$$= A_0 + A_1x + A_2x^2 + A_3x^3$$

On putting $x = 0$ on both sides, we get

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = A_0$$

$$\Rightarrow A_0 = 0$$

4 (d)

We have,

$$\begin{vmatrix} \cos \alpha & \sin \alpha & 0 \\ \cos \beta & \sin \beta & 0 \\ \cos \gamma & \sin \gamma & 0 \end{vmatrix} \begin{vmatrix} \cos \alpha & \sin \alpha & 0 \\ \cos \beta & \sin \beta & 0 \\ \cos \gamma & \sin \gamma & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & \cos(\beta - \alpha) & \cos(\gamma - \alpha) \\ \cos(\alpha - \beta) & 1 & \cos(\gamma - \beta) \\ \cos(\alpha - \gamma) & \cos(\beta - \gamma) & 1 \end{vmatrix}$$

$$\therefore \begin{vmatrix} 1 & \cos(\beta - \alpha) & \cos(\gamma - \alpha) \\ \cos(\alpha + \beta) & 1 & \cos(\gamma - \beta) \\ \cos(\alpha - \gamma) & \cos(\beta - \gamma) & 1 \end{vmatrix} = 0$$

5 (a)

$$\text{Given, } \begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

$$\Rightarrow x(x^2 - 12) - 3(2x - 14) + 7(12 - 7x) = 0$$

$$\Rightarrow x^3 - 67x + 126 = 0$$

$$\Rightarrow (x + 9)(x^2 - 9x + 14) = 0$$

$$\Rightarrow (x + 9)(x - 2)(x - 7) = 0$$

$$\Rightarrow x = -9, 2, 7$$

Hence, the other two roots are 2, 7

6 (c)

From the sine rule, we have

$$\Rightarrow \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k(\text{say}),$$

$$\Rightarrow \sin A = ak, \sin B = bk \text{ and } \sin C = ck$$

$$\begin{aligned} \therefore & \begin{vmatrix} a^2 & b \sin A & c \sin A \\ b \sin A & 1 & \cos(B - C) \\ c \sin A & \cos(B - C) & 1 \end{vmatrix} \\ &= \begin{vmatrix} a^2 & abk & ack \\ abk & 1 & \cos(B - C) \\ ack & \cos(B - C) & 1 \end{vmatrix} \\ &= a^2 \begin{vmatrix} 1 & \sin B & \sin C \\ \sin B & 1 & \cos(B - C) \\ \sin C & \cos(B - C) & 1 \end{vmatrix} \\ &= a^2 \begin{vmatrix} 1 & \sin(A + C) & \sin(A + B) \\ \sin(A + C) & 1 & \cos(B - C) \\ \sin(A + B) & \cos(B - C) & 1 \end{vmatrix} \\ &= a^2 \begin{vmatrix} \sin A & \cos A & 0 \\ \cos C & \sin C & 0 \\ \cos B & \sin B & 0 \end{vmatrix} \begin{vmatrix} \sin A & \cos A & 0 \\ \cos C & \sin C & 0 \\ \cos B & \sin B & 0 \end{vmatrix} = a^2 \times 0 = 0 \end{aligned}$$

7 (b)

$$\text{Given, } D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & y \end{vmatrix} = xy$$

Hence, D is divisible by both x and y .

8 (a)

Taking x common from R_2 and $x(x - 1)$ common from R_3 , we get

$$f(x) = x^2(x-1) \begin{vmatrix} 1 & x & (x+1) \\ 2 & (x-1) & (x+1) \\ 3 & (x-2) & (x+1) \end{vmatrix}$$

$$\Rightarrow f(x) = x^2(x-1)(x+1) \begin{vmatrix} 1 & x & 1 \\ 2 & x-1 & 1 \\ 3 & x-2 & 1 \end{vmatrix}$$

$$= x^2(x^2-1) \begin{vmatrix} 1 & x & 1 \\ 2 & -1 & 0 \\ 3 & -2 & 0 \end{vmatrix} \begin{matrix} [R_2 \rightarrow R_2 - R_1] \\ [R_3 \rightarrow R_3 - R_1] \end{matrix}$$

$$\Rightarrow f(x) = x^2(x^2-1)(-2+2) = 0$$

$$\Rightarrow f(x) = 0 \text{ for all } x$$

$$\therefore f(11) = 0$$

9 **(b)**

Applying $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$, we get

$$\begin{vmatrix} 1 & 4 & 20 \\ 0 & -6 & -15 \\ 0 & 2x-4 & 5x^2-20 \end{vmatrix} = 0$$

$$\Rightarrow 1[-6(5x^2-20) + 15(2x-4)] = 0$$

$$\Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x-2)(x+1) = 0$$

$$\Rightarrow x = -1, 2$$

10 **(b)**

We have,

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 3x-2 & 3 & 3 \\ 3x-2 & 3x-8 & 3 \\ 3x-2 & 3 & 3x-8 \end{vmatrix} = 0 \quad \text{Applying } C_1 \rightarrow C_1 + C_2 + C_3$$

$$\Rightarrow (3x-2) \begin{vmatrix} 1 & 3 & 3 \\ 1 & 3 & 3x-8 \\ 1 & 3 & 3x-8 \end{vmatrix} = 0$$

$$\Rightarrow (3x-2) \begin{vmatrix} 1 & 3 & 3 \\ 0 & 3x-11 & 0 \\ 0 & 0 & 3x-11 \end{vmatrix} = 0 \quad \begin{matrix} \text{Applying } R_2 \rightarrow R_2 - R_1, \\ R_3 \rightarrow R_3 - R_1 \end{matrix}$$

$$\Rightarrow (3x-2)(3x-11)^2 = 0$$

$$\Rightarrow x = 2/3, 11/3$$

11 **(a)**

We have,

$$\begin{vmatrix} \alpha & x & x & x \\ x & \beta & x & x \\ x & x & \gamma & x \\ x & x & x & \delta \end{vmatrix}$$

$$= \begin{vmatrix} \alpha & x-\alpha & x-\alpha & x-\alpha \\ x & -(x-\beta) & 0 & 0 \\ x & 0 & -(x-\gamma) & 0 \\ x & 0 & 0 & -(x-\delta) \end{vmatrix}$$

$$\begin{aligned}
&= \alpha \begin{vmatrix} -(x-\beta) & 0 & 0 \\ 0 & -(x-\gamma) & 0 \\ 0 & 0 & -(x-\delta) \end{vmatrix} - x \begin{vmatrix} x-\alpha & x-\alpha & x-\alpha \\ 0 & -(x-\gamma) & 0 \\ 0 & 0 & -(x-\delta) \end{vmatrix} \\
&+ x \begin{vmatrix} x-\alpha & x-\alpha & x-\alpha \\ -(x-\beta) & 0 & 0 \\ 0 & 0 & -(x-\delta) \end{vmatrix} - x \begin{vmatrix} x-\alpha & x-\alpha & x-\alpha \\ -(x-\beta) & 0 & 0 \\ 0 & -(x-\gamma) & 0 \end{vmatrix} \\
&= -\alpha(x-\beta)(x-\gamma)(x-\delta) - (x-\alpha)(x-\gamma)(x-\delta) \\
&-x(x-\alpha)(x-\beta)(x-\delta) - x(x-\alpha)(x-\beta)(x-\gamma) \\
&= -\alpha(x-\beta)(x-\gamma)(x-\delta) + x(x-\beta)(x-\gamma)(x-\delta) \\
&-x(x-\beta)(x-\gamma)(x-\delta) - x(x-\alpha)(x-\gamma)(x-\delta) \\
&-x(x-\alpha)(x-\beta)(x-\delta) - x(x-\alpha)(x-\beta)(x-\gamma) \\
&= (x-\beta)(x-\gamma)(x-\delta)(x-\alpha) - x[(x-\alpha)(x-\beta)(x-\gamma) + (x-\beta)(x-\gamma)(x-\delta) + (x-\gamma)(x-\delta)(x-\alpha) + (x-\alpha)(x-\beta)(x-\delta)] \\
&= f(x) - xf'(x), \text{ where, } f(x) = (x-\alpha)(x-\beta)(x-\gamma)(x-\delta)
\end{aligned}$$

12 (b)

$$\text{Given } \begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$$

$$\begin{aligned}
&\Rightarrow c^2 - ab - a(c-a) + b(b-c) = 0 \\
&\Rightarrow a^2 + b^2 + c^2 - ab - bc - ca = 0 \\
&\Rightarrow \frac{1}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2] = 0 \\
&\Rightarrow a = b = c
\end{aligned}$$

So, ΔABC is equilateral triangle.

$$\therefore \angle A = 60^\circ, \angle B = 60^\circ, \angle C = 60^\circ$$

$$\begin{aligned}
\sin^2 A + \sin^2 B + \sin^2 C &= \sin^2 60^\circ + \sin^2 60^\circ + \sin^2 60^\circ \\
&= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 \\
&= 3 \times \frac{3}{4} = \frac{9}{4}
\end{aligned}$$

14 (a)

$$\text{Given that, } \Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$

$$\begin{aligned}
&= \begin{vmatrix} 1 + \omega^n + \omega^{2n} & \omega^n & \omega^{2n} \\ 1 + \omega^n + \omega^{2n} & 1 & \omega^n \\ 1 + \omega^n + \omega^{2n} & \omega^{2n} & 1 \end{vmatrix} \\
&= \begin{vmatrix} 0 & \omega^n & \omega^{2n} \\ 0 & 1 & \omega^n \\ 0 & \omega^{2n} & 1 \end{vmatrix}
\end{aligned}$$

$$\begin{aligned}
&(\because \text{ If } n \text{ multiple of } 3, \text{ then } 1 + \omega^n + \omega^{2n} = 0) \\
&= 0
\end{aligned}$$

15 (b)

$$\begin{vmatrix} x & 3 & 6 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix} = \begin{vmatrix} 2 & x & 7 \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix} = \begin{vmatrix} 4 & 5 & x \\ 5 & x & 4 \\ x & 4 & 5 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x+9 & x+9 & x+9 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix} = \begin{vmatrix} 9+x & x+9 & 9+x \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix}$$

$$= \begin{vmatrix} 9+x & 9+x & 9+x \\ 5 & x & 4 \\ x & 4 & 5 \end{vmatrix} = 0$$

$$\Rightarrow (x+9) \begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & x-3 \\ 6 & x-6 & -3 \end{vmatrix} = (9+x) \begin{vmatrix} 1 & 0 & 0 \\ x & 7-x & 2-x \\ 7 & -5 & x-7 \end{vmatrix}$$

$$= (9+x) \begin{vmatrix} 1 & 0 & 0 \\ 5 & x-5 & -1 \\ x & 4-x & 5-x \end{vmatrix}$$

$$= 0$$

$$\Rightarrow x+9=0 \Rightarrow x=-9$$

16 (b)

The given system of equations will have a unique solution, if

$$\begin{vmatrix} k & 2 & -1 \\ 0 & k-1 & -2 \\ 0 & 0 & k+2 \end{vmatrix} \neq 0 \Rightarrow k(k-1)(k+2) \neq 0 \Rightarrow k \neq 0, 1, -2$$

18 (d)

$$\begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_2$

$$\Rightarrow \begin{vmatrix} a & b-y & c-z \\ -x & y & 0 \\ 0 & -y & z \end{vmatrix} = 0$$

$$\Rightarrow a(yz) + x(bz - yz + cy - yz) = 0$$

$$\Rightarrow ayz + bzax + cyx = 2xyz$$

$$\Rightarrow \frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$$

19 (a)

Given,

$$\begin{vmatrix} 1 & \cos(\alpha - \beta) & \cos \alpha \\ \cos(\alpha - \beta) & 1 & \cos \beta \\ \cos \alpha & \cos \beta & 1 \end{vmatrix} \text{ is symmetric determinant.}$$

\therefore Its value is

$$1 + 2 \cos(\alpha - \beta) \cos \alpha \cos \beta$$

$$- \cos^2 \alpha - \cos^2 \beta - \cos^2(\alpha - \beta)$$

$$= 1 - \cos^2 \alpha - \cos^2 \beta - \cos(\alpha - \beta)$$

$$[\cos(\alpha - \beta) - 2 \cos \alpha \cos \beta]$$

$$\begin{aligned}
&= 1 - \cos^2 \alpha - \cos^2 \beta - \cos(\alpha - \beta) \\
&[\cos(\alpha - \beta) - \cos(\alpha + \beta) - \cos(\alpha - \beta)] \\
&= 1 - \cos^2 \alpha - \cos^2 \beta + \cos(\alpha - \beta) \cos(\alpha + \beta) \\
&= 1 - \cos^2 \alpha - \cos^2 \beta + \cos^2 \alpha \cos^2 \beta - \sin^2 \alpha \sin^2 \beta \\
&= 1 - \cos^2 \alpha - \cos^2 \beta(1 - \cos^2 \alpha) - \sin^2 \alpha \sin^2 \beta \\
&= (1 - \cos^2 \alpha)(1 - \cos^2 \beta) - \sin^2 \alpha \sin^2 \beta \\
&= \sin^2 \alpha \sin^2 \beta - \sin^2 \alpha \sin^2 \beta = 0
\end{aligned}$$

20 (d)

We have,

$$\begin{aligned}
\Delta &= \begin{vmatrix} 2 \sin A \cos A & \sin C & \sin B \\ \sin C & 2 \sin B \cos B & \sin A \\ \sin B & \sin A & 2 \sin C \cos C \end{vmatrix} \\
\Rightarrow \Delta &= \begin{vmatrix} 2ka \cos A & kc & kb \\ kc & 2kb \cos B & ka \\ kb & ka & 2kc \cos C \end{vmatrix} \quad [\text{Using: Sine rule}] \\
\Rightarrow \Delta &= k^3 \begin{vmatrix} 2a \cos A & c & b \\ c & 2b \cos B & a \\ b & a & 2c \cos C \end{vmatrix} \\
\Rightarrow \Delta &= k^3 \begin{vmatrix} a \cos A + a \cos A & a \cos B + b \cos A & c \cos A + a \cos C \\ a \cos B + b \cos A & b \cos B + b \cos B & b \cos C + c \cos B \\ c \cos A + a \cos C & b \cos C + c \cos B & c \cos C + c \cos C \end{vmatrix} \\
\Rightarrow \Delta &= k^3 \begin{vmatrix} \cos A & a & 0 \\ \cos B & b & 0 \\ \cos C & c & 0 \end{vmatrix} \begin{vmatrix} a & \cos A & 0 \\ b & \cos B & 0 \\ c & \cos C & 0 \end{vmatrix} \\
\Rightarrow \Delta &= k^3 \times 0 \times 0 = 0
\end{aligned}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	C	D	A	C	B	A	B	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	A	A	B	B	B	D	A	D

PE