DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :2

Topic :-DETERMINANTS

1 **(c)**

Let $A=\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}=\begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega+\omega^2 & \omega^2 & 1 \\ 1+\omega+\omega^2 & 1 & \omega \end{vmatrix}$

$[C_1 \to C_1+C_2+C_3$

$=\begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ o & 1 & \omega \end{vmatrix}=0 \qquad [\because 1+\omega+\omega^2=0]$

2 **(b)**

Applying $C_1 \to C_1+C_2$, we get

$\begin{vmatrix} {}^{10}C_4+{}^{10}C_5 & {}^{10}C_5 & {}^{11}C_m \\ {}^{11}C_6+{}^{11}C_7 & {}^{11}C_7 & {}^{12}C_{m+2} \\ {}^{12}C_8+{}^{12}C_9 & {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix}=0$

$\Rightarrow \begin{vmatrix} {}^{11}C_5 & {}^{10}C_5 & {}^{11}C_m \\ {}^{12}C_7 & {}^{11}C_7 & {}^{12}C_{m+2} \\ {}^{13}C_9 & {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix}=0$

It means either two rows or two columns are identical.

$\therefore\ {}^{11}C_5={}^{11}C_m,\ {}^{12}C_7={}^{12}C_{m+2},\ {}^{13}C_9={}^{13}C_{m+4}$

$\Rightarrow\ m=5$

3 **(b)**

Given, $\begin{vmatrix} 1 & 1 & 0 \\ 2 & 0 & 3 \\ 5 & -6 & x \end{vmatrix}=29$

$\Rightarrow 1(0+18)-1(2x-15)=29$

$\Rightarrow 2x=4 \Rightarrow x=2$

4 **(a)**

Applying $C_1 \to C_1+C_2$, we get

$\begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix}=\begin{vmatrix} 1 & \cos^2 x & 1 \\ 1 & \sin^2 x & 1 \\ 2 & 12 & 2 \end{vmatrix}=0$

5 **(b)**

Since, $|A|=-1$, $|B|=3$

$\therefore |AB|=|A||B|=-3$

Now, $|3AB|=(3)^3(-3)=-81$

7 **(d)**

Applying $C_3 \rightarrow C_3-\alpha C_1+C_2$ to the given determinant, we get

$$\begin{vmatrix} a & b & 0 \\ b & c & 0 \\ 2 & 1 & -2\alpha+1 \end{vmatrix}=(1-2\alpha)(ac-b^2)$$

So, if the determinant is zero, we must have

$(1-2\alpha)(ac-b^2)=0$

$\Rightarrow 1-2\alpha=0$

or $(ac-b^2)=0$

$\Rightarrow \alpha=\frac{1}{2}$ or $ac=b^2$

Which means a, b, c are in GP.

8 **(a)**

We have, $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}=0$

$\Rightarrow (x+9)\begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}=0 \ (R_1 \rightarrow R_1+R_2+R_3)$

$\Rightarrow (x+9)\{1(x^2-12)-1(2x-14)+1(12-7x)\}=0$

$\Rightarrow (x+9)(x^2-9x+14)=0$

$\Rightarrow (x+9)(x-2)(x-7)=0$

$\therefore$ The other two roots are 2 and 7.

9 **(a)**

Let $A \equiv \begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix}=0$

Applying $C_1 \rightarrow C_1+C_2+C_3$

$$=\begin{vmatrix} a+b+c-x & c & b \\ a+b+c-x & b-x & a \\ a+b+c-x & a & c-x \end{vmatrix}$$

$$=(a+b+c-x)\begin{vmatrix} 1 & c & b \\ 1 & b-x & a \\ 1 & a & c-x \end{vmatrix}$$

$\Rightarrow (a+b+c-x)[1\{(b-x)(c-x)-a^2\}-c(c-x-a)+b(a-b+x)]=0$

$\Rightarrow (a+b+c-x)[bc-bx-cx+x^2-a^2-c^2+xc+ac+ab-b^2+bx]=0$

$\Rightarrow (a+b+c-x)[x^2-(a^2+b^2+c^2)+ab+bc+ca]=0$

$\because \ ab+bc+ca=0$ (given)

$\Rightarrow$ either $x=a+b+c$ or $x=(a^2+b^2+c^2)^{1/2}$

10 **(b)**

We have,

$$\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x+1 & 1 & 1 \\ x+1 & x-1 & 1 \\ x+1 & 1 & x-1 \end{vmatrix} = 0 \ \text{[Applying } C_1 \to C_1 + C_2 + C_3\text{]}$$

$$\Rightarrow (x+1) \begin{vmatrix} 1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$$

$$\Rightarrow (x+1) \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-2 & 0 \\ 0 & 0 & x-2 \end{vmatrix} = 0 \ \begin{bmatrix} \text{Applying } R_2 \to R_2 - R_1, \\ R_3 \to R_3 - R_1 \end{bmatrix}$$

$\Rightarrow (x+1)(x-2)^2 = 0$

$\Rightarrow x = -1, 2$

11 **(c)**

$$\text{Let } A = \begin{vmatrix} 1 & 2 & 3 \\ 1^3 & 2^3 & 3^3 \\ 1^5 & 2^5 & 3^5 \end{vmatrix} = 1.2.3 \begin{vmatrix} 1 & 2 & 3 \\ 1^2 & 2^2 & 3^2 \\ 1^4 & 2^4 & 3^4 \end{vmatrix}$$

$$= 6 \begin{vmatrix} 1 & 1 & 1 \\ 1 & 4 & 9 \\ 1 & 16 & 81 \end{vmatrix} = 6 \begin{vmatrix} 1 & 0 & 0 \\ 1 & 3 & 5 \\ 1 & 15 & 65 \end{vmatrix}$$

$[C_2 \to C_2 - C_1, C_3 \to C_3 - C_2]$

$$= 6.3.5 \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 5 & 13 \end{vmatrix} = 90[1(13-5)] = 720 = 6!$$

12 **(c)**

$\because \ |A^3| = |A|^3 = 125$

$$\Rightarrow \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix} = 5$$

$\Rightarrow \alpha^2 - 4 = 5 \ \Rightarrow \ \alpha = \pm 3$

14 **(b)**

Given, angles of a triangle are A, B and C. We know that $A + B + C = \pi$, therefore

$A + B = \pi - C$

$\Rightarrow \cos(A+B) = \cos(\pi - C) = -\cos C$

$\Rightarrow \cos A \cos B - \sin A \sin B = -\cos C$

$\Rightarrow \cos A \cos B + \cos C = \sin A \sin B \quad \ldots\text{(i)}$

$$\text{Let } \Delta = \begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$$

$$= -(1 - \cos^2 A) + \cos C(\cos C + \cos A \cos B) + \cos B(\cos B + \cos A \cos C)$$

$$= -\sin^2 A + \cos C(\sin A \sin B) + \cos B(\sin A \sin C) \ \text{[from Eq.(i)]}$$

$$= -\sin^2 A + \sin A(\sin B \cos C + \cos B \sin C)$$

$$= -\sin^2 A + \sin A \sin(B + C)$$

$= -\sin^2 A + \sin^2 A = 0 \quad [\because \sin(B+C) = \sin(\pi - A) = \sin A]$

15 **(a)**

We have,

$$\Delta = \begin{vmatrix} a^2+x & ab & ac \\ ab & b^3+x & bc \\ ac & bc & c^2+x \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{abc}\begin{vmatrix} a^3+ax & a^2b & a^2c \\ ab^2 & b^3+bx & b^2c \\ ac^2 & bc^2 & c^3+cx \end{vmatrix} \begin{bmatrix}\text{Applying } C_1(a), \\ C_2(b), C_3(c)\end{bmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} a^2+x & a^2 & a^2 \\ b^2 & b^2+x & b^2 \\ c^2 & c^2 & c^2+x \end{vmatrix}$$

$$\Rightarrow \Delta = (a^2+b^2+c^2+x)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+x & b^2 \\ c^2 & c^2 & c^2+x \end{vmatrix}$$

[Applying $R_1 \to R_1 + R_2 + R_3$]

$\Rightarrow \Delta = (a^2+b^2+c^2+x)\{(b^2 x + c^2 x + x^2) - (b^2 x) + (-c^2 x)\}$

$\Rightarrow \Delta = x^2(a^2+b^2+c^2+x)$

$\Rightarrow x^2$ is a factor Δ

16 **(a)**

Given that, $\begin{vmatrix} x+1 & x+2 & x+3 \\ x+2 & x+3 & x+4 \\ x+a & x+b & x+c \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} -1 & -1 & x+3 \\ -1 & -1 & x+4 \\ a-b & b-c & x+c \end{vmatrix} = 0 \quad \begin{pmatrix} C_1 \to C_1 - C_2 \\ C_2 \to C_2 - C_3 \end{pmatrix}$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & -1 \\ -1 & -1 & x+4 \\ a-b & b-c & x+c \end{vmatrix} = 0 \quad (R_1 \to R_1 - R_2)$$

$\Rightarrow (-1)(-b+c+a-b) = 0$

$\Rightarrow 2b - a - c = 0$

$\Rightarrow a + c = 2b$

$\therefore$ a,b,c in AP.

17 **(b)**

Given, $A = \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 0 \\ x & x & 1 \end{vmatrix} \Rightarrow A = 1$

$\therefore A^3 - 4A^2 + 3A + I = (1)^3 - 4(1)^2 + 3(1) + I = I$

18 **(a)**

Let $\Delta = \begin{vmatrix} 1 & x & y \\ 2 & \sin x + 2x & \sin y + 3y \\ 3 & \cos x + 3x & \cos y + 3y \end{vmatrix}$

$$= \begin{vmatrix} 1 & x & y \\ 0 & \sin x & \sin y \\ 0 & \cos x & \cos y \end{vmatrix} \quad \begin{pmatrix} R_2 \to R_2 - 2R_1, \\ R_3 \to R_3 - 3R_1 \end{pmatrix}$$

$= \sin x \ \cos y - \cos x \ \sin y = \sin(x - y)$

19 **(d)**

We have, $\Delta = \frac{1}{abc}\begin{vmatrix} a^3 + ax & a^2b & a^2c \\ ab^2 & b^3 + bx & b^2c \\ c^2a & c^2b & c^2 + xc \end{vmatrix}$

Taking a, b,c common in columns Ist, IInd and IIIrd, we get,

$\Delta = \begin{vmatrix} a^2 + x & a^2 & a^2 \\ b^2 & b^2 + x & b^2 \\ c^2 & c^2 & c^2 + x \end{vmatrix}$

Applying $R_1 \to R_1 + R_2 + R_3$

$= (a^2 + b^2 + c^2 + x)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2 + x & b^2 \\ c^2 & c^2 & c^2 + x \end{vmatrix}$

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$= (a^2 + b^2 + c^2 + x)\begin{vmatrix} 1 & 1 & 0 \\ b^2 & x & 0 \\ c^2 & 0 & x \end{vmatrix}$

$= x(x - b^2)(a^2 + b^2 + c^2 + x)$

Hence, option (d) is correct.

20 **(a)**

Given, $\begin{vmatrix} bc & ca & ab \\ ca & ab & bc \\ ab & bc & ca \end{vmatrix} = 0$

$\Rightarrow (ab)^3 + (bc)^3 + (ca)^3 - 3a^2b^2c^2 = 0$

$\Rightarrow (ab + bc + ca)(a^2b^2 + b^2c^2 + c^2a^2 - ab^2c$

$-bc^2a - ca^2b) = 0$

$\Rightarrow \ ab + bc + ca = 0$

$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	B	A	B	A	D	A	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	B	B	A	A	B	A	D	A