DPP
DAILY PRACTICE PRODUCTS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :10

Topic :-DETERMINANTS

1 **(d)**

Clearly, $x = 0$ satifies the given equation

2 **(c)**

Let $\Delta = \begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$

$= 10!11!12!\begin{vmatrix} 1 & 11 & 11 \times 12 \\ 1 & 12 & 12 \times 13 \\ 1 & 13 & 13 \times 14 \end{vmatrix}$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$

$= 10!11!12!\begin{vmatrix} 1 & 11 & 11 \times 12 \\ 0 & 1 & 24 \\ 0 & 2 & 50 \end{vmatrix}$

$= (10!11!12!)(50 - 48)$

$= 2 \cdot (10!11!12!)$

3 **(c)**

We have, $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$

$\Rightarrow \begin{vmatrix} \sin x + 2\cos x & \cos x & \cos x \\ \sin x + 2\cos x & \sin x & \cos x \\ \sin x + 2\cos x & \cos x & \sin x \end{vmatrix} = 0$

$\Rightarrow (2\cos x + \sin x)\begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix} = 0$

Applying $R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - R_1$

$\Rightarrow (2\cos x + \sin x)\begin{vmatrix} 1 & \cos x & \cos x \\ 0 & \sin x - \cos x & 0 \\ 0 & 0 & \sin x - \cos x \end{vmatrix} = 0$

$\Rightarrow (2\cos x + \sin x)(\sin x - \cos x)^2 = 0$

$\therefore \tan x = -2, 1$ But $\tan x \neq -2$, because it does not lie in the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$.

$\therefore \tan x = 1$

So, $x = \frac{\pi}{4}$

4 **(a)**

$$\begin{vmatrix} (a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1 \\ (b^x+b^{-x})^2 & (b^x-b^{-x})^2 & 1 \\ (c^x+c^{-x})^2 & (c^x-c^{-x})^2 & 1 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$

$$= \begin{vmatrix} 4 & (a^x-a^{-x})^2 & 1 \\ 4 & (b^x-b^{-x})^2 & 1 \\ 4 & (c^x-c^{-x})^2 & 1 \end{vmatrix} = 4\begin{vmatrix} 1 & (a^x-a^{-x})^2 & 1 \\ 1 & (b^x-b^{-x})^2 & 1 \\ 1 & (c^x-c^{-x})^2 & 1 \end{vmatrix} = 0 \quad (\because \text{two columns are identical})$$

5 **(c)**

Given matrix is non-singular, then

$$\begin{vmatrix} \lambda & 1 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & \lambda \end{vmatrix} \neq 0$$

$\Rightarrow \lambda(2\lambda - 0) \neq 0$

$\Rightarrow \lambda \neq 0$

6 **(d)**

$$\text{Let } \Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_3$

$$= \begin{vmatrix} a^2 & b^2 & c^2 \\ 4a & 4b & 4c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

Applying $R_3 \to R_3 - (R_1 - 2R_2)$

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

$\therefore k = 4$

7 **(c)**

Let $f(x) = a_0x^2 + a_1x + a_2$

and $g(x) = b_2x^2 + b_1x + b_2$

Also, $h(x) = c_0x^2 + c_1x + c_2$

$$\text{Then, } \Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ 2a_0x + a_1 & 2b_0x + b_1 & 2c_0x + c_1 \\ 2a_0 & 2b_0 & 2c_0 \end{vmatrix}$$

$$= x\begin{vmatrix} f(x) & g(x) & h(x) \\ 2a_0 & 2b_0 & 2c_0 \\ 2a_0 & 2b_0 & 2c_0 \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ a_1 & b_1 & c_1 \\ 2a_0 & 2b_0 & 2c_0 \end{vmatrix}$$

$= 0 + 2\begin{vmatrix} f(x) & g(x) & h(x) \\ a_1 & b_1 & c_1 \\ a_0 & b_0 & c_0 \end{vmatrix}$

$= 2[(b_1c_0 - b_0c_1)f(x) - (a_1c_0 - a_0c_1)g(x) + (a_1b_0 - a_0b_1)h(x)]$

Hence, degree of $\Delta(x) \leq 2$

8 **(d)**

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$\begin{vmatrix} 2(x+y+z) & y+z & z+x \\ x+y+z & y & z \\ 0 & y-z & z-x \end{vmatrix}$

$= (x+y+z)\begin{vmatrix} 2 & y-z & z+x \\ 1 & y & z \\ 0 & y-z & z-x \end{vmatrix}$

Applying $R_2 \to 2R_2 - R_1$

$= (x+y+z)\begin{vmatrix} 2 & y+z & z+x \\ 0 & y-z & z-x \\ 0 & y-z & z-x \end{vmatrix}$

$= 0$ [$\therefore$ two rows are identical]

9 **(d)**

Given, $f(x) = \begin{vmatrix} 1+a & 1+ax & 1+ax^2 \\ 1+b & 1+bx & 1+bx^2 \\ 1+b & 1+cx & 1+cx^2 \end{vmatrix}$

$\Rightarrow f(x) = \begin{vmatrix} 1+a & a(x-1) & ax(x-1) \\ 1+b & b(x-1) & bx(x-1) \\ 1+b & c(x-1) & cx(x-1) \end{vmatrix}$

$= (x-1)x(x-1)\begin{vmatrix} 1+a & a & a \\ 1+b & b & b \\ 1+c & c & c \end{vmatrix} = 0$

($\therefore$ two columns are same)

10 **(c)**

We have,

$ax^4 + bx^3 + cx^2 + 50x + d = \begin{vmatrix} x^3 - 14x^2 & -x & 3x+\lambda \\ 4x+1 & 3x & x-4 \\ -3 & 4 & 0 \end{vmatrix}$

On differentiating with respect to x, we get

$4ax^3 + 3bx^2 + 2cx + 50 = \begin{vmatrix} 3x^2 - 28x & -1 & 3 \\ 4x+1 & 3x & x-4 \\ -3 & 4 & 0 \end{vmatrix} + \begin{vmatrix} x^3 - 14x^2 & -x & 3x+\lambda \\ 4 & 3 & 1 \\ -3 & 4 & 0 \end{vmatrix}$

Now, put $x = 0$, we get

$50 = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 0 & -4 \\ -3 & 4 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & \lambda \\ 4 & 3 & 1 \\ -3 & 4 & 0 \end{vmatrix}$

$\Rightarrow 50 = 25\lambda$

$\Rightarrow \lambda = 2$

11 **(d)**

We have, $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = Ax-12$

On putting $x=1$ on both sides, we get

$\begin{vmatrix} 2 & 2 & -1 \\ 4 & 3 & 0 \\ 6 & 1 & 1 \end{vmatrix} = A-12$

Applying $C_1 \to C_1 - C_2$

$\Rightarrow \begin{vmatrix} 0 & 2 & -1 \\ 1 & 3 & 0 \\ 5 & 1 & 1 \end{vmatrix} = A-12$

$\Rightarrow -2(1)+(-1)(-14) = A-12$

$\Rightarrow A = 24$

12 **(a)**

We have, $\begin{vmatrix} x+\alpha & \beta & \gamma \\ \gamma & x+\beta & \alpha \\ \alpha & \beta & x+\gamma \end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$

$\Rightarrow \begin{vmatrix} x+\alpha+\beta+\gamma & \beta & \gamma \\ x+\alpha+\beta+\gamma & x+\beta & \alpha \\ x+\alpha+\beta+\gamma & \beta & x+\gamma \end{vmatrix} = 0$

$\Rightarrow (x+\alpha+\beta+\gamma)\begin{vmatrix} 1 & \beta & \gamma \\ 1 & x+\beta & \alpha \\ 1 & \beta & x+\gamma \end{vmatrix} = 0$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow (x+\alpha+\beta+\gamma)\begin{vmatrix} 1 & \beta & \gamma \\ 0 & x & \alpha-\gamma \\ 0 & 0 & x \end{vmatrix} = 0$

$\Rightarrow (x+\alpha+\beta+\gamma)(x^2-0) = 0$

$\Rightarrow x = 0$ or $x = -(\alpha+\beta+\gamma)$

13 **(b)**

We have,

$\Delta = \begin{vmatrix} 1/a & 1 & bc \\ 1/b & 1 & ca \\ 1/c & 1 & ab \end{vmatrix}$

$\Rightarrow \Delta = \frac{1}{abc}\begin{vmatrix} 1 & a & abc \\ 1 & b & abc \\ 1 & c & abc \end{vmatrix}$ Applying $R_1 \to R_1(a)$, $R_2 \to R_2(b)$ and $R_3 \to R_3(c)$

$\Rightarrow \Delta = \frac{abc}{abc}\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$ [Taking abc common from C_3]

$\Rightarrow \Delta = \frac{abc}{abc} \times 0 = 0$

14 **(b)**

We have, $|A| \neq 0$. Therefore, A^{-1} exists

Now, $AB = AC$

$\Rightarrow A^{-1}(AB) = A^{-1}(AC)$

$\Rightarrow (A^{-1}A)B = (A^{-1}A)C \Rightarrow B = C$

16 **(c)**

Applying $C_3 \to C_3 - \omega C_1$, we get

$$\begin{vmatrix} a & b\omega^2 & a\omega \\ b\omega & c & b\omega^2 \\ c\omega^2 & a\omega & c \end{vmatrix} = \begin{vmatrix} a & b\omega^2 & 0 \\ b\omega & c & 0 \\ c\omega^2 & a\omega & 0 \end{vmatrix} = 0$$

17 **(d)**

$$\begin{vmatrix} a+b & a+2b & a+3b \\ a+2b & a+3b & a+4b \\ a+4b & a+5b & a+6b \end{vmatrix}$$

$$= \begin{vmatrix} a+b & a+2b & a+3b \\ b & b & b \\ 2b & 2b & 2b \end{vmatrix} \begin{pmatrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_2 \end{pmatrix}$$

$= 0$ ($\because$ R_2 and R_3 are proportional)

18 **(c)**

Applying $R_1 \to R_1 - (R_2 + R_3)$, we get

$$\begin{vmatrix} 0 & -2z & -2y \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$$

$= 2z(xy + y^2 - yz) - 2y(yz - z^2 - xz)$

$= 2xyz + 2y^2z - 2yz^2 - 2y^2z + 2yz^2 + 2xyz$

$= 4xyz$

19 **(b)**

We have,

$$\frac{d}{dx}(\Delta_1) = \begin{vmatrix} 1 & 0 & 0 \\ a & x & b \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ 0 & 1 & 0 \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ a & x & b \\ 0 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow \frac{d}{dx}(\Delta_1) = \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} = 3\,\Delta_2$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	C	A	C	D	C	D	D	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	A	B	B	B	C	D	C	B	D