DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :1

Topic :-DETERMINANTS

1 **(c)**

$$\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$$

On multiplying R_1,R_2,R_3 by a,b,c respectively and divide the whole by abc

$$= \frac{1}{abc}\begin{vmatrix} ab^2c^2 & abc & a(b+c) \\ bc^2a^2 & bca & b(c+a) \\ a^2b^2c & abc & c(a+b) \end{vmatrix}$$

On taking common abc from $C_1$and C_2, we get

$$= \frac{(abc)(abc)}{abc}\begin{vmatrix} bc & 1 & ab+ac \\ ca & 1 & bc+ab \\ ab & 1 & ca+bc \end{vmatrix}$$

Now, $C_1 \rightarrow C_1 + C_3$

$$= abc\begin{vmatrix} ab+bc+ca & 1 & ab+ac \\ ca+bc+ab & 1 & bc+ab \\ ab+bc+ca & 1 & ca+bc \end{vmatrix}$$

$$= (abc)(ab+bc+ca)\begin{vmatrix} 1 & 1 & ab+ac \\ 1 & 1 & bc+ab \\ 1 & 1 & ca+bc \end{vmatrix}$$

$= 0$ [$\because$ two columns are identical]

2 **(b)**

We have, $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \lambda$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$

$\Rightarrow \begin{vmatrix} 1+a & -a & -a \\ 1 & b & 0 \\ 1 & 0 & c \end{vmatrix} = \lambda$

On expanding w.r.t. R_3, we get

$ab + bc + ca + abc = \lambda$ ...(i)

Given $a^{-1} + b^{-1} + c^{-1} = 0$

$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

$\Rightarrow ab + bc + ca = 0$

From Eq. (i), $\lambda = abc$

3 **(c)**

We have,

$$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} x+1 & x+2 & x+a \\ 2x+4 & 2x+6 & 2x+2b \\ x+2 & x+4 & x+c \end{vmatrix} \text{[Applying } R_2 \to 2R_2]$$

$$= \frac{1}{2}\begin{vmatrix} x+1 & x+2 & x+a \\ 0 & 0 & 0 \\ x+2 & x+4 & x+c \end{vmatrix} \text{[Applying } R_2 - (R_1 + R_3)]$$

$= 0$

4 **(b)**

$$\begin{vmatrix} a-b & b-c & c-a \\ x-y & y-z & z-x \\ p-q & p-r & r-p \end{vmatrix} = \begin{vmatrix} 0 & b-c & c-a \\ 0 & y-z & z-x \\ 0 & q-r & r-p \end{vmatrix} = 0 \quad (C_1 \to C_1 + C_2 + C_3)$$

5 **(c)**

$$\begin{vmatrix} a-b+c & -a-b+c & 1 \\ a+b+2c & -a+b+2c & 2 \\ 3c & 3c & 3 \end{vmatrix}$$

$$\begin{vmatrix} 2a & -2a & 0 \\ a+b+2c & -a+b+2c & 2 \\ 3c & 3c & 3 \end{vmatrix}$$

[using $R_1 \to R_1 + R_2 - R_3$]

$= 2a(-3a + 3b + 6c - 6c) + 2a(3a + 3b + 6c - 6c)$

$= 12ab$

6 **(a)**

Ratio of cofactor to its minor of the element -3, which is in the 3rd row and 2nd column $= (-1)^{3+2} = -1$

7 **(d)**

We have,

$$\Delta = \begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} x+1+\omega+\omega^2 & x+\omega+\omega^2+1 & x+1+\omega+\omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$$

[Applying $R_1 \to R_1 + R_2 + R_3$]

$$\Rightarrow \Delta = (x+1+\omega+\omega^2)\begin{vmatrix} 1 & 1 & 1 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$$

$$\Rightarrow \Delta = x\begin{vmatrix} 1 & 0 & 0 \\ \omega & x+\omega^2-\omega & 1-\omega \\ \omega^2 & 1-\omega^2 & x+\omega-\omega^2 \end{vmatrix}$$

$\Rightarrow \Delta = x[(x+\omega^2-\omega)(x+\omega-\omega^2) - (1-\omega)(1-\omega^2)]$

$\therefore \Delta = 0 \Rightarrow x = 0$

9 **(d)**

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$ to the given determinant and expanding it along first now, we get

$\Rightarrow (\sin B - \sin A)(\sin C - \sin A)$

$\times \begin{vmatrix} 1 & 1 \\ 1 + \sin B + \sin A & 1 + \sin C + \sin A \end{vmatrix} = 0$

$\Rightarrow (\sin B - \sin A)(\sin C - \sin A)(\sin C - \sin B) = 0$

$\Rightarrow \sin B = \sin A$ or $\sin C = \sin A$ or $\sin C = \sin B$

$\Rightarrow A = B$ or $B = C$ or $C = A$

$\Rightarrow \Delta ABC$ is isosceles

10 **(c)**

We have, $D_r = \begin{vmatrix} 2^{r-1} & 3^{r-1} & 4^{r-1} \\ x & y & z \\ 2^n - 1 & (3^n - 1)/2 & (4^n - 1)/3 \end{vmatrix}$

$$\Rightarrow \sum_{r=1}^{n} D_r = \begin{vmatrix} \sum_{r=1}^{n} 2^{r-1} & \sum_{r=1}^{n} 3^{r-1} & \sum_{r=1}^{n} 4^{r-1} \\ x & y & z \\ 2^n - 1 & (3^n - 1)/2 & (4^n - 1)/3 \end{vmatrix}$$

$$\Rightarrow \sum_{r=1}^{n} D_r = \begin{vmatrix} 2^n - 1 & (3^n - 1)/2 & (4^n - 1)/3 \\ x & y & z \\ 2^n - 1 & (3^n - 1)/2 & (4^n - 1)/3 \end{vmatrix}$$

$\sum_{r=1}^{n} D_r = 0$ ($\because$ two rows are same)

11 **(b)**

We have,

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 + \sin A & 1 + \sin B & 1 + \sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ \sin A & \sin B & \sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$

$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ \sin A & \sin B & \sin C \\ \sin^2 A & \sin^2 B & \sin^2 C \end{vmatrix} = 0$ Applying $R_3 \to R_3 - R_2$

$\Rightarrow (\sin A - \sin B)(\sin B - \sin C)(\sin C - \sin A) = 0$

$\Rightarrow \sin A = \sin B$ or, $\sin B = \sin C$ or, $\sin C = \sin A$

$\Rightarrow \Delta ABC$ is isosceles

12 **(c)**

We have,

$$\det(A) = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix} = 2(1 + \sin^2\theta)$$

Now,

$0 \le \sin^2\theta \le 1$ for all $\theta \in [0, 2\pi)$

$\Rightarrow 2 \le 2 + 2\sin^2\theta \le 4$ for all $\theta \in [0, 2\pi)$

$\Rightarrow \text{Det}(A) \in [2,4]$

13 **(d)**

Let $\Delta = \begin{vmatrix} 1 & 5 & \pi \\ \log_e e & 5 & \sqrt{5} \\ \log_{10} 10 & 5 & e \end{vmatrix}$

$\Rightarrow \Delta = \begin{vmatrix} 1 & 5 & \pi \\ 1 & 5 & \sqrt{5} \\ 1 & 5 & e \end{vmatrix} = 5\begin{vmatrix} 1 & 1 & \pi \\ 1 & 1 & \sqrt{5} \\ 1 & 1 & e \end{vmatrix}$ $(\because \log_a a = 1)$

$= 0$ ($\because$ two columns are identical)

14 **(a)**

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$f(x) = \begin{vmatrix} 1 + a^2x + x + xb^2 + x + c^2x & (1+b^2)x & (1+c^2)x \\ x + a^2x + 1 + b^2x + x + c^2x & (1+b^2x) & (1+c^2)x \\ x + a^2x + x + b^2x + 1 + c^2x & (1+b^2)x & (1+c^2x) \end{vmatrix}$$

$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$[\because a^2 + b^2 + c^2 + 2 = 0]$

Applying $R_1 \to R_1 - R_3,\ R_2 \to R_2 - R_3$

$$= \begin{vmatrix} 0 & 0 & x-1 \\ 0 & 1-x & x-1 \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix} = 1[0 - (x-1)(1-x)]$$

$= (x-1)^2$

$\Rightarrow f(x)$ is a polynomial of degree 2

15 **(c)**

Since system of equations is consistent.

$$\therefore \begin{vmatrix} 1 & 1 & -1 \\ 2 & -1 & -c \\ -b & 3b & -c \end{vmatrix} = 0$$

$\Rightarrow c + bc - 6b + b + 2c + 3bc = 0$

$\Rightarrow 3c + 4bc - 5b = 0$

$\Rightarrow c = \dfrac{5}{3 + 4b}$

But $c < 1 \Rightarrow \frac{5b}{3+4b} < 1$

$\Rightarrow \dfrac{b-3}{3+4b} < 0$

$\Rightarrow b \in \left(-\frac{3}{4}, 3\right)$

16 **(a)**

Applying $R_2 \to R_2 - R_3$, we get

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & 4 & 4 \\ (2^x - 2^{-x})^2 & (3^x - 3^{-x})^2 & (5^x - 5^{-x})^2 \end{vmatrix}$$

$$= 4\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ (2^x - 2^{-x})^2 & (3^x - 3^{-x})^2 & (5^x - 5^{-x})^2 \end{vmatrix}$$

$= 4 \times 0 = 0$ [$\because$ two rows are identical]

17 **(b)**

We have,

$AA^{-1} = I$

$\Rightarrow \det(AA^{-1}) = \det(I)$

$\Rightarrow \det(A)\det(A^{-1}) = 1$ $\left[\begin{matrix} \because \det(A\,B) = \det(A)\det(B) \\ \text{and, } \det(I) = 1 \end{matrix}\right]$

$\Rightarrow \det(A^{-1}) = \dfrac{1}{\det(A)}$

18 **(b)**

We have,

$$\begin{vmatrix} 1+ax & 1+bx & 1+cx \\ 1+a_1x & 1+b_1x & 1+c_1x \\ 1+a_2x & 1+b_2x & 1+c_2x \end{vmatrix}$$

$$= \begin{vmatrix} 1+ax & (b-a)x & (c-a)x \\ 1+a_1x & (b_1-a_1)x & (c_1-a_1)x \\ 1+a_2x & (b_2-a_2)x & (c_2-a_2)x \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$= x^2\begin{vmatrix} 1+ax & b-a & c-a \\ 1+a_1x & b_1-a_1 & c_1-a_1 \\ 1+a_2x & b_2-a_2 & c_2-a_2 \end{vmatrix}$$

$$= x^2[(1+ax)\{(b_1-a_1)(c_2-a_2)-(b_2-a_2)(c_1-a_1)\}-(1+a_1x)\{(b-a)(c_2-a_2)-(c-a)(b_2-a_2)\}+(1+a_2x)\{(b-a)(c_1-a_1)-(c-a)(b_1-a_1)\}]$$

$= x^2(\lambda\,x + \mu)$, where λ and μ are constants

$= \mu\,x^2 + \lambda\,x^3$

Hence, $A_0 = A_1 = 0$

19 **(b)**

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$$

Applying $R_3 \to R_3 - xR_2$

$$f(x)=\begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ 0 & 0 & a+x \end{vmatrix}=(a+x)(a^2+ax)$$

$\Rightarrow f(x)=a(a+x)^2$

$\therefore f(2x)=a(a+2x)^2$

$\Rightarrow f(2x)-f(x)=ax(2a+3x)$

20 **(c)**

$$\begin{vmatrix} -12 & 0 & \lambda \\ 0 & 2 & -1 \\ 2 & 1 & 15 \end{vmatrix}=-360$$

$\Rightarrow \ -12(30+1)-4\lambda=-360$

$\Rightarrow \ -372+360=4\lambda \Rightarrow \ \lambda=-\frac{12}{4}=-3$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	C	B	C	A	D	D	D	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	D	A	C	A	B	B	B	C