

Topic :- CONTINUITY AND DIFFERENTIABILITY

1. For the function $f(x) = \begin{cases} \frac{x^3 - a^3}{x - a}, & x \neq a \\ b, & x = a \end{cases}$, if $f(x)$ is continuous at $x = a$, then b is equal to
 a) a^2 b) $2a^2$ c) $3a^2$ d) $4a^2$

2. If $y = f(x) = \frac{1}{u^2 + u - 1}$ where $u = \frac{1}{x - 1}$, then the function is discontinuous at $x =$
 a) 1 b) $1/2$ c) 2 d) -2

3. If $f(x) = \text{Min} \{ \tan x, \cot x \}$, then
 a) $f(x)$ is not differentiable at $x = 0, \pi/4, 5\pi/4$
 b) $f(x)$ is continuous at $x = 0, \pi/2, 3\pi/2$
 c) $\int_0^{\pi/2} f(x) dx = \ln \sqrt{2}$
 d) $f(x)$ is periodic with period $\frac{\pi}{2}$

4. If $f(x) = \{ |x| - |x - 1| \}^2$, then $f'(x)$ equals
 a) 0 for all x
 b) $2\{|x| - |x - 1|\}$
 c) $\begin{cases} 0 & \text{for } x < 0 \text{ and for } x > 1 \\ 4(2x - 1) & \text{for } 0 < x < 1 \end{cases}$
 d) $\begin{cases} 0 & \text{for } x < 0 \\ 4(2x - 1) & \text{for } x > 0 \end{cases}$

5. If $f(x) = (x - x_0)\phi(x)$ and $\phi(x)$ is continuous at $x = x_0$, then $f'(x_0)$ is equal to
 a) $\phi'(x_0)$ b) $\phi(x_0)$ c) $x_0\phi(x_0)$ d) None of these

6. The function defined by
 $f(x) = \begin{cases} (x^2 + e^{\frac{1}{2-x}})^{-1} & x \neq 2 \\ k, & x = 2 \end{cases}$ is continuous from right at the point $x = 2$, then k is equal to
 a) 0 b) $\frac{1}{4}$ c) $-\frac{1}{2}$ d) None of these

7. If $f(x) = \begin{cases} \frac{1 - \sin x}{(\pi - 2x)^2} \cdot \frac{\log \sin x}{(\log 1 + \pi^2 - 4\pi x + x^2)}, & x \neq \frac{\pi}{2} \\ k, & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \pi/2$, then $k =$
 a) $-\frac{1}{16}$ b) $-\frac{1}{32}$ c) $-\frac{1}{64}$ d) $-\frac{1}{28}$

8. If $f(x) = \begin{cases} \frac{\sin 5x}{x^2 + 2x}, & x \neq 0 \\ k + \frac{1}{2}, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is
- a) 1 b) -2 c) 2 d) $\frac{1}{2}$
9. Let $f(x) = \begin{cases} x^n \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then, $f(x)$ is continuous but not differentiable at $x = 0$, if
- a) $n \in (0, 1]$ b) $n \in [1, \infty)$ c) $n \in (-\infty, 0)$ d) $n = 0$
10. The function $f(x) = \begin{cases} |x - 3|, & \text{if } x \geq 1 \\ x^2 - \frac{3x}{2} + \frac{13}{4}, & \text{if } x < 1 \end{cases}$ is
- a) Continuous and differentiable at $x = 3$
b) Continuous at $x = 3$, but not differentiable at $x = 3$
c) continuous and differentiable everywhere
d) continuous at $x = 1$, but not differentiable at $x = 1$
11. Let $f(x) = |x|$ and $g(x) = |x^3|$, then
- a) $f(x)$ and $g(x)$ Both are continuous at $x = 0$
b) $f(x)$ and $g(x)$ Both are differentiable at $x = 0$
c) $f(x)$ is differentiable but $g(x)$ is not differentiable at $x = 0$
d) $f(x)$ and $g(x)$ Both are not differentiable at $x = 0$
12. If $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ c, & x = 0 \\ \frac{\sqrt{x + bx^2} - \sqrt{x}}{bx\sqrt{x}}, & x > 0 \end{cases}$ is continuous at $x = 0$, then
- a) $a = -\frac{3}{2}, b = 0, c = \frac{1}{2}$
b) $a = -\frac{3}{2}, b = 1, c = -\frac{1}{2}$
c) $a = -\frac{3}{2}, b \in R - \{0\}, c = \frac{1}{2}$
d) None of these
13. If $f(x) = \begin{cases} \frac{36^x - 9^x - 4^x + 1}{\sqrt{2} - \sqrt{1 + \cos x}}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then k equals
- a) $16\sqrt{2}\log 2\log 3$ b) $16\sqrt{2}\ln 6$ c) $16\sqrt{2}\ln 2\ln 3$ d) None of these
14. Let $[\]$ denotes the greatest integer function and $f(x) = [\tan^2 x]$. Then,
- a) $\lim_{x \rightarrow 0} f(x)$ does not exist b) $f(x)$ is continuous at $x = 0$
c) $f(x)$ is not differentiable at $x = 0$ d) $f(x) = 1$
15. Let a function $f: R \rightarrow R$, where R is the set of real numbers satisfying the equation $f(x + y) = f(x) + f(y), \forall x, y$ if $f(x)$ is continuous at $x = 0$, then
- a) $f(x)$ is discontinuous, $\forall x \in R$ b) $f(x)$ is continuous, $\forall x \in R$
c) $f(x)$ is continuous for $x \in \{1, 2, 3, 4\}$ d) None of the above

16. Let $f(x) = \begin{cases} \sin x, & \text{for } x \geq 0 \\ 1 - \cos x, & \text{for } x \leq 0 \end{cases}$ and $g(x) = e^x$. Then, $(g \circ f)'(0)$ is
- a) 1 b) -1 c) 0 d) None of these

17. The function $f(x) = \begin{cases} (x+1)^{2-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is
- a) Continuous everywhere
b) Discontinuous at only one point
c) Discontinuous at exactly two points
d) None of these

18. If $f(x) = \begin{cases} \frac{\log(1+ax) - \log(1-bx)}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ and $f(x)$ is continuous at $x = 0$, then the value of k is
- a) $a - b$ b) $a + b$ c) $\log a + \log b$ d) None of these

19. The value of $f(0)$, so that the function $f(x) = \frac{(27-2x)^{1/3} - 3}{9 - 3(243 + 5x)^{1/5}}$ ($x \neq 0$) is continuous is given by
- a) $\frac{2}{3}$ b) 6 c) 2 d) 4

20. The function $f: R/\{0\} \rightarrow R$ given by

$$f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$$

Can be made continuous at $x = 0$ by defining $f(0)$ as function

- a) 2 b) -1 c) 0 d) 1

