

Topic :- CONTINUITY AND DIFFERENTIABILITY

1. The function $f(x) = e^{-|x|}$ is
 - a) Continuous everywhere but not differentiable at $x = 0$
 - b) Continuous and differentiable everywhere
 - c) Not continuous at $x = 0$
 - d) None of the above

2. The value of $f(0)$, so that the function

$$f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a+x} - \sqrt{a-x}}$$
 becomes continuous for all x , is given by
 - a) $a^{3/2}$
 - b) $a^{1/2}$
 - c) $-a^{1/2}$
 - d) $-a^{3/2}$

3. The value of k for which the function

$$f(x) = \begin{cases} \frac{1 - \cos 4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$$
 is continuous at $a = 0$, is
 - a) $k = 0$
 - b) $k = 1$
 - c) $k = -1$
 - d) None of these

4. The number of points at which the function $f(x) = (|x - 1| + |x - 2| + \cos x)$ where $x \in [0, 4]$ is not continuous, is
 - a) 1
 - b) 2
 - c) 3
 - d) 0

5. If $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is
 - a) 1
 - b) -1
 - c) 0
 - d) 2

6. Let $f(x)$ be twice differentiable function such that $f''(x) = -f(x)$ and $f'(x) = g(x)$, $h(x) = \{f(x)\}^2 + \{g(x)\}^2$. If $h(5) = 11$, then $h(10)$ is equal to
 - a) 22
 - b) 11
 - c) 0
 - d) None of these

7. if $f(x) = |x|^3$, then $f'(0)$ equals
 - a) 0
 - b) $1/2$
 - c) -1
 - d) $-1/2$

8. Let function $f(x) = \sin^{-1}(\cos x)$, is
 - a) Discontinuous at $x = 0$
 - b) Continuous at $x = 0$

- c) Differentiable at $x = 0$
 d) None of these

9. Let $f(x) = \begin{cases} \frac{x^4 - 5x^2 + 4}{|(x-1)(x-2)|}, & x \neq 1, 2 \\ 6, & x = 1 \\ 12, & x = 2 \end{cases}$ Then, $f(x)$ is continuous on the set

- a) R b) $R - \{1\}$ c) $R - \{2\}$ d) $R - \{1, 2\}$

10. The set of points, where $f(x) = \frac{x}{1 + |x|}$ is differentiable, is

- a) $(-\infty, -1) \cup (-1, \infty)$ b) $(-\infty, \infty)$ c) $(0, \infty)$ d) $(-\infty, 0) \cup (0, \infty)$

11. Given $f(0) = 0$ and $f(x) = \frac{1}{(1 - e^{-1/x})}$ for $x \neq 0$. Then only one of the following statements on $f(x)$ is true. That is $f(x)$, is

- a) Continuous at $x = 0$
 b) Not continuous at $x = 0$
 c) Both continuous and differentiable at $x = 0$
 d) Not defined at $x = 0$

12. Let f and g be differentiable functions satisfying $g'(a) = 2$, $g(a) = b$ and $f \circ g = I$ (identify function). Then, $f'(b)$ is equal to

- a) $1/2$ b) 2 c) $2/3$ d) None of these

13. Let $f(x) = \begin{cases} \frac{\sin \pi x}{5x}, & x \neq 0 \\ k, & x = 0 \end{cases}$, if $f(x)$ is continuous at $x = 0$, then k is equal to

- a) $\frac{\pi}{5}$ b) $\frac{5}{\pi}$ c) 1 d) 0

14. The number of discontinuities of the greatest integer function $f(x) = [x]$, $x \in \left(-\frac{7}{2}, 100\right)$ is equal to

- a) 104 b) 100 c) 102 d) 103

15. For the function $f(x) = \frac{e^{1/x} - 1}{e^{1/x} + 1}$, $x = 0$, which of the following is correct?

- a) $\lim_{x \rightarrow 0} f(x)$ does not exist
 b) $\lim_{x \rightarrow 0} f(x) = 1$
 c) $\lim_{x \rightarrow 0} f(x)$ exists but $f(x)$ is not continuous at $x = 0$
 d) $f(x)$ is continuous at $x = 0$

16. If $f(x) = x^4 + \frac{x^4}{1 + x^4} + \frac{x^4}{(1 + x^4)^2} + \dots$ to ∞ then at $x = 0$, $f(x)$ is

- a) Continuous but not differentiable
 b) Differentiable
 c) Continuous

d) None of these

17. If $f(x) = \begin{cases} 1+x, & 0 \leq x \leq 2 \\ 3-x, & 2 < x \leq 3 \end{cases}$ then the set of points of discontinuity of $g(x) = f \circ f(x)$, is
- a) $\{1, 2\}$ b) $\{0, 1, 2\}$ c) $\{0, 1\}$ d) None of these

18. Let $g(x)$ be the inverse of an invertible function $f(x)$ which is differentiable at $x = c$, then $g'(f(c))$ equals
- a) $f'(c)$ b) $\frac{1}{f'(c)}$ c) $f(c)$ d) None of these

19. If $f(x) = \begin{cases} x^p \cos\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ is differentiable at $x = 0$, then
- a) $p < 0$ b) $0 < p < 1$ c) $p = 1$ d) $p > 1$

20. At $x = 0$, the function $f(x) = |x|$ is
- a) Continuous but not differentiable b) Discontinuous and differentiable
c) Discontinuous and not differentiable d) Continuous and differentiable

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