

Topic :- CONTINUITY AND DIFFERENTIABILITY

- The set of points where the function $f(x) = x|x|$ is differentiable is
a) $(-\infty, \infty)$ b) $(-\infty, 0) \cup (0, \infty)$ c) $(0, \infty)$ d) $[0, \infty)$
- If $f(x+y) = f(x)f(y)$ for all real x and y , $f(6) = 3$ and $f'(0) = 10$, then $f'(6)$ is
a) 30 b) 13 c) 10 d) 0
- If $f(x) = |x-a|\phi(x)$, where $\phi(x)$ is continuous function, then
a) $f'(a^+) = \phi(a)$ b) $f'(a^-) = \phi(a)$ c) $f'(a^+) = f'(a^-)$ d) None of these
- If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then $f(x)$ is
a) Continuous as well as differentiable for all x
b) Continuous for all x but not differentiable at $x = 0$
c) Neither differentiable nor continuous at $x = 0$
d) Discontinuous everywhere
- If $f(x) = \begin{cases} 3, & x < 0 \\ 2x + 1, & x \geq 0 \end{cases}$, then
a) Both $f(x)$ and $f(|x|)$ are differentiable at $x = 0$
b) $f(x)$ is differentiable but $f(|x|)$ is not differentiable at $x = 0$
c) $f(|x|)$ is differentiable but $f(x)$ is not differentiable at $x = 0$
d) Both $f(x)$ and $f(|x|)$ are not differentiable at $x = 0$
- If $\lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$ exists finitely, then
a) $\lim_{x \rightarrow c} f(x) = f(c)$
b) $\lim_{x \rightarrow c} f'(x) = f'(c)$
c) $\lim_{x \rightarrow c} f(x)$ does not exist
d) $\lim_{x \rightarrow c} f(x)$ may or may not exist
- The number of points at which the function $f(x) = |x - 0.5| + |x - 1| + \tan x$ does not have a derivative in the interval $(0, 2)$, is
a) 1 b) 2 c) 3 d) 4

8. If $f(x) = \begin{cases} \log_{(1-3x)}(1+3x), & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$ is continuous at $x = 0$, then k is equal to
 a) -2 b) 2 c) 1 d) -1
9. Let $f(x)$ be a function differentiable at $x = c$. Then, $\lim_{x \rightarrow c} f(x)$ equals
 a) $f'(c)$ b) $f''(c)$ c) $\frac{1}{f(c)}$ d) None of these
10. If $f(x) = ae^{|x|} + b|x|^2$; $a, b \in R$ and $f(x)$ is differentiable at $x = 0$. Then a and b are
 a) $a = 0, b \in R$ b) $a = 1, b = 2$ c) $b = 0, a \in R$ d) $a = 4, b = 5$
11. Let $f(x) = (x + |x|)|x|$. The, for all x
 a) f and f' are continuous
 b) f is differentiable for some x
 c) f' is not continuous
 d) f'' is continuous
12. If $f(x) = \begin{cases} \frac{x-1}{2x^2-7x+5}, & \text{for } x \neq 1 \\ -\frac{1}{3}, & \text{for } x = 1 \end{cases}$, then $f'(1)$ is equal to
 a) $-\frac{1}{9}$ b) $-\frac{2}{9}$ c) $-\frac{1}{3}$ d) $\frac{1}{3}$
13. Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h \rightarrow 0} \frac{1}{h} f(1+h) = 5$, then $f'(1)$ equals
 a) 6 b) 5 c) 4 d) 3
14. If $f: R \rightarrow R$ is defined by
 $f(x) = \begin{cases} \frac{x+2}{x^2+3x+2}, & \text{if } x \in R - \{-1, -2\} \\ -1, & \text{if } x = -2 \\ 0, & \text{if } x = -1 \end{cases}$, then f is continuous on the set
 a) R b) $R - \{-2\}$ c) $R - \{-1\}$ d) $R - (-1, -2)$
15. Let $f(x) = \frac{(e^x - 1)^2}{\sin(\frac{x}{2}) \log(1 + \frac{x}{4})}$ for $x \neq 0$ and $f(0) = 12$. If f is continuous at $x = 0$, then the value of a is equal to
 a) 1 b) -1 c) 2 d) 3
16. If a function $f(x)$ is given by $f(x) = \frac{x}{1+x} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \dots \infty$ then at $x = 0, f(x)$
 a) Has no limit
 b) Is not continuous
 c) Is continuous but not differentiable
 d) Is differentiable
17. If $f(x)$ is continuous function and $g(x)$ be discontinuous, then
 a) $f(x) + g(x)$ must be continuous
 b) $f(x) + g(x)$ must be discontinuous

- c) $f(x) + g(x)$ for all x
- d) None of these

18. A function $f: R \rightarrow R$ satisfies the equation $f(x + y) = f(x)f(y)$ for all $x, y \in R$ and $f(x) \neq 0$ for all $x \in R$. If $f(x)$ is differentiable at $x = 0$ and $f'(0) = 2$, then $f'(x)$ equals

- a) $f(x)$
- b) $-f(x)$
- c) $2f(x)$
- d) None of these

19. Consider $f(x) = \begin{cases} \frac{x^2}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

- a) $f(x)$ is discontinuous everywhere
- b) $f(x)$ is continuous everywhere
- c) $f'(x)$ exists in $(-1, 1)$
- d) $f'(x)$ exists in $(-2, 2)$

20. If $f(x)$ is continuous at $x = 0$ and $f(0) = 2$, then

$\lim_{x \rightarrow 0} \frac{\int_0^x f(u) du}{x}$ is

- a) 0
- b) 2
- c) $f(2)$
- d) None of these

