DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth **SUBJECT : MATHS**
DATE : **DPP NO. : 3**

Topic :- CONTINUITY AND DIFFERENTIABILITY

1. Let $f(x)=\begin{cases}5^{1/x}, & x<0\\ \lambda[x], & x\geq 0\end{cases}$ and $\lambda\in R$, then at $x=0$
   a) f is discontinuous
   b) f is continuous only, if $\lambda=0$
   c) f is continuous only, whatever λ may be
   d) None of the above

2. If for a continuous function f, $f(0)=f(1)=0$, $f'(1)=2$ and $y(x)=f(e^x)e^{f(x)}$, then $y'(0)$ is equal to
   a) 1
   b) 2
   c) 0
   d) None of these

3. If $f(x)=\begin{cases}ax^2-b, & |x|<1\\ \frac{1}{|x|}, & |x|\geq 1\end{cases}$ is differentiable at $x=1$, then
   a) $a=\frac{1}{2}, b=-\frac{1}{2}$
   b) $a=-\frac{1}{2}, b=-\frac{3}{2}$
   c) $a=b=\frac{1}{2}$
   d) $a=b=-\frac{1}{2}$

4. Let $f(x)=\frac{\sin 4\pi[x]}{1+[x]^2}$, where $[x]$ is the greatest integer less than or equal to x, then
   a) $f(x)$ is not differentiable at some points
   b) $f'(x)$ exists but is different from zero
   c) $f'(x)=0$ for all x
   d) $f'(x)=0$ but f is not a constant function

5. The value of k which makes $f(x)=\begin{cases}\sin(1/k), & x\neq 0\\ k, & x=0\end{cases}$ continuous at $x=0$ is
   a) 8
   b) 1
   c) -1
   d) None of these

6. The function $f(x)=\max[(1-x),(1+x),2]$, $x\in(-\infty,\infty)$ is
   a) Continuous at all points
   b) Differentiable at all points
   c) Differentiable at all points except at $x=1$ and $x=-1$
   d) None of the above

7. Let $f(x)$ be defined for all $x>0$ and be continuous. Let $f(x)$ satisfy $f\left(\frac{x}{y}\right)=f(x)-f(y)$ for all x, y and $f(e)=1$. Then,
   a) $f(x)$ is bounded
   b) $f\left(\frac{1}{x}\right)\to 0$ as $x\to 0$
   c) $xf(x)\to 1$ as $x\to 0$
   d) $f(x)=\ln x$

8. Suppose a function $f(x)$ satisfies the following conditions for all x and y: (i) $f(x+y) = f(x) f(y)$ (ii) $f(x) = 1 + x\, g(x) \log a$, where $a > 1$ and $\lim_{x \to 0} g(x) = 1$. Then, $f'(x)$ is equal to

a) $\log a$ b) $\log a^{f(x)}$ c) $\log (f(x))^a$ d) None of these

9. Let $g(x)$ be the inverse of the function $f(x)$ and $f'(x) = \frac{1}{1+x^3}$. Then, $g'(x)$ is equal to

a) $\frac{1}{1+(g(x))^3}$ b) $\frac{1}{1+(f(x))^3}$ c) $1 + (g(x))^3$ d) $1 + (f(x))^3$

10. If $f(x) = |x^2 - 4x + 3|$, then

a) $f'(1) = -1$ and $f'(3) = 1$
b) $f'(1) = -1$ and $f'(3)$ does not exist
c) $f'(1) = -1$ does not exist and $f'(3) = 1$
d) Both $f'(1)$ and $f'(3)$ do not exist

11. The points of discontinuity of $\tan x$ are

a) $n\pi, n \in I$ b) $2n\pi, n \in I$ c) $(2n+1)\frac{\pi}{2}, n \in I$ d) None of these

12. Let $f(x) = ||x| - 1|$, then points where $f(x)$ is not differentiable, is/(are)

a) $0, \pm 1$ b) ± 1 c) 0 d) 1

13. $f(x) = \begin{cases} 2x, & x < 0 \\ 2x+1, & x \geq 0 \end{cases}$. Then

a) $f(x)$ is continuous at $x = 0$ b) $f(|x|)$ is continuous at $x = 0$ c) $f(x)$ is discontinuous at $x = 0$ d) None of the above

14. Let $f(x) = [x] + \sqrt{x - [x]}$, where $[x]$ denotes the greatest integer function. Then,

a) $f(x)$ is continuous on R^+
b) $f(x)$ is continuous on R
c) $f(x)$ is continuous on $R - Z$
d) None of these

15. The function $f(x) = \frac{1 - \sin x + \cos x}{1 + \sin x + \cos x}$ is not defined at $x = \pi$. The value of $f(\pi)$, so that $f(x)$ is continuous at $x = \pi$, is

a) -1/2 b) ½ c) -1 d) 1

16. Let $f(x) = \begin{cases} (x-1)\sin\frac{1}{x-1}, & \text{if } x \neq 1 \\ 0, & \text{if } x = 1 \end{cases}$. Then, which one of the following is true?

a) f is differentiable at $x = 1$ but not at $x = 0$
b) f is neither differentiable at $x = 0$ nor at $x = 1$
c) f is differentiable at $x = 0$ and at $x = 1$
d) f is differentiable at $x = 0$ but not at $x = 1$

17. If $f(x)=\begin{cases} mx+1, & x\le\frac{\pi}{2} \\ \sin x+n, & x>\frac{\pi}{2}\end{cases}$ is continuous at $x=\frac{\pi}{2}$, then

a) $m=1, n=0$ b) $m=\frac{n\pi}{2}+1$ c) $n=\frac{m\pi}{2}$ d) $m=n=\frac{\pi}{2}$

18. Let f be differentiable for all x. If $f(1)=-2$ and $f'(x)\ge 2$ for $x\in[1,6]$, then

a) $f(6)=5$ b) $f(6)<5$ c) $f(6)<8$ d) $f(6)\ge 8$

19. If $\lim_{x\to a^+} f(x)=l=\lim_{x\to a^-} g(x)$ and $\lim_{x\to a^-} f(x)=m\lim_{x\to a^+} g(x)$, then the function $f(x)\,g(x)$

a) Is not continuous at $x=a$
b) Has a limit when $x\to a$ and it is equal to lm
c) Is continuous at $x=a$
d) Has a limit when $x\to a$ but it is not equal to lm

20. Let $f(x)$ be a function satisfying $f(x+y)=f(x)f(y)$ for all $x,y\in R$ and $f(x)=1+x\,g(x)$ where $\lim_{x\to 0} g(x)=1$. Then, $f'(x)$ is equal to

a) $g'(x)$ b) $g(x)$ c) $f(x)$ d) None of these