

Topic :- CONTINUITY AND DIFFERENTIABILITY

- Which one of the following is not true always?
 - If $f(x)$ is not continuous at $x = a$, then it is not differentiable at $x = a$
 - If $f(x)$ is continuous at $x = a$, then it is differentiable at $x = a$
 - If $f(x)$ and $g(x)$ are differentiable at $x = a$, then $f(x) + g(x)$ is also differentiable at $x = a$
 - If a function $f(x)$ is continuous at $x = a$, then $\lim_{x \rightarrow a} f(x)$ exists
- The value of the derivative of $|x - 1| + |x - 3|$ at $x = 2$ is
 - 2
 - 1
 - 0
 - 2
- On the interval $I = [-2, 2]$, the function $f(x) = \begin{cases} (x + 1)e^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$
 - Is continuous for all $x \in I - \{0\}$
 - Assumes all intermediate values from $f(-2)$ to $f(2)$
 - Has a maximum value equal to $3/e$
 - All the above
- Function $f(x) = \begin{cases} x - 1, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$ is a continuous function
 - For $x = 2$ only
 - For all real values of x such that $x \neq 2$
 - For all real values of x
 - For all integer values of x only
- The function $f(x) = \begin{cases} \frac{\tan x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$, is
 - Continuous but not differentiable at $x = 0$
 - Discontinuous at $x = 0$
 - Continuous and differentiable at $x = 0$
 - Not defined at $x = 0$
- At the point $x = 1$, the function $f(x) = \begin{cases} x^3 - 1, & 1 < x < \infty \\ x - 1, & -\infty < x \leq 1 \end{cases}$
 - Continuous and differentiable
 - Continuous and not differentiable
 - Discontinuous and differentiable
 - Discontinuous and not differentiable

7. If $f(x)$ defined by $f(x) = \begin{cases} \frac{|x^2 - x|}{x^2 - x}, & x \neq 0, 1 \\ 1, & x = 0 \\ -1, & x = 1 \end{cases}$ then $f(x)$ is continuous for all
- x
 - x except at $x = 0$
 - x except at $x = 1$
 - x except at $x = 0$ and $x = 1$
8. The value of derivative of $|x - 1| + |x - 3|$ at $x = 2$, is
- -2
 - 0
 - 2
 - Not defined
9. If $f(x) = \begin{cases} 1 & \text{for } x < 0 \\ 1 + \sin x & \text{for } 0 \leq x \leq \pi/2 \end{cases}$, then at $x = 0$, the derivative $f'(x)$ is
- 1
 - 0
 - Infinite
 - Does not exist
10. Let $g(x) = \frac{(x-1)^n}{\log \cos^m(x-1)}$; $0 < x < 2$, m and n are integers, $m \neq 0$, $n > 0$, and let p be the left hand derivative of $|x - 1|$ at $x = 1$. If $\lim_{x \rightarrow 1^+} g(x) = p$, then
- $n = 1, m = 1$
 - $n = 1, m = -1$
 - $n = 2, m = 2$
 - $n > 2, m = n$
11. The function $f(x) = \frac{2x^2 + 7}{x^3 + 3x^2 - x - 3}$ is discontinuous for
- $x = 1$ only
 - $x = 1$ and $x = -1$ only
 - $x = 1, x = -1, x = -3$ only
 - $x = 1, x = -1, x = -3$ and other values of x
12. If for a function $f(x)$, $f(2) = 3$, $f'(2) = 4$, then $\lim_{x \rightarrow 2} [f(x)]$, where $[\cdot]$ denotes the greatest integer function, is
- 2
 - 3
 - 4
 - Non-existent
13. A function $f(x)$ is defined as follows for real x ,
- $$f(x) = \begin{cases} 1 - x^2, & \text{for } x < 1 \\ 0, & \text{for } x = 1 \\ 1 + x^2, & \text{for } x > 1 \end{cases}$$
- Then,
- $f(x)$, is not continuous at $x = 1$
 - $f(x)$ is continuous but not differentiable at $x = 1$
 - $f(x)$ is both continuous and differentiable at $x = 1$
 - None of the above
14. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \min\{x + 1, |x| + 1\}$. Then, which of the following is true?
- $f(x) \geq 1$ for all $x \in \mathbb{R}$
 - $f(x)$ is not differentiable at $x = 1$
 - $f(x)$ is differentiable everywhere
 - $f(x)$ is not differentiable at $x = 0$
15. If $f(x) = \begin{cases} mx + 1, & x \leq \frac{\pi}{2} \\ \sin x + n, & x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then

a) $m = 1, n = 0$ b) $m = \frac{n\pi}{2} + 1$ c) $n = m\frac{\pi}{2}$ d) $m = n = \frac{\pi}{2}$

16. If $f(x) = \frac{\log_e(1 + x^2 \tan x)}{\sin x^3}, x \neq 0$, is to be continuous at $x = 0$, then $f(0)$ must be defined as

a) 1 b) 0 c) $\frac{1}{2}$ d) -1

17. Let $f(x) = \begin{cases} x^p \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ then $f(x)$ is continuous but not differentiable at $x = 0$, if

a) $0 < p \leq 1$ b) $1 \leq p < \infty$ c) $-\infty < p < 0$ d) $p = 0$

18. The function f defined by

$$f(x) = \begin{cases} \frac{\sin x^2}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases} \text{ is}$$

- a) Continuous and derivable at $x = 0$
- b) Neither continuous nor derivable at $x = 0$
- c) Continuous but not derivable at $x = 0$
- d) None of these

19. A function f on R into itself is continuous at a point a in R , iff for each $\epsilon > 0$, there exists, $\delta > 0$ such that

a) $|f(x) - f(a)| < \epsilon \Rightarrow |x - a| < \delta$ b) $|f(x) - f(a)| > \epsilon \Rightarrow |x - a| > \delta$
 c) $|x - a| > \delta \Rightarrow |f(x) - f(a)| > \epsilon$ d) $|x - a| < \delta \Rightarrow |f(x) - f(a)| < \epsilon$

20. The function $f(x) = x - |x - x^2|, -1 \leq x \leq 1$ is continuous on the interval

a) $[-1, 1]$ b) $(-1, 1)$ c) $[-1, 0) \cup (0, 1]$ d) $(-1, 0) \cup (0, 1)$