

Topic :- CONTINUITY AND DIFFERENTIABILITY

- Let $[x]$ denotes the greatest integer less than or equal to x and $f(x) = [\tan^2 x]$. Then,
 - $\lim_{x \rightarrow 0} f(x)$ does not exist
 - $f(x)$ is continuous at $x = 0$
 - $f(x)$ is not differentiable at $x = 0$
 - $f'(0) = 1$
- The value of $f(0)$ so that $\frac{(-e^x + 2^x)}{x}$ may be continuous at $x = 0$ is
 - $\log\left(\frac{1}{2}\right)$
 - 0
 - 4
 - $-1 + \log 2$
- Let $f(x)$ be an even function. Then $f'(x)$
 - Is an even function
 - Is an odd function
 - May be even or odd
 - None of these
- If $f(x) = \begin{cases} [\cos \pi x], & x < 1 \\ |x - 2|, & 2 > x \geq 1 \end{cases}$, then $f(x)$ is
 - Discontinuous and non-differentiable at $x = -1$ and $x = 1$
 - Continuous and differentiable at $x = 0$
 - Discontinuous at $x = 1/2$
 - Continuous but not differentiable at $x = 2$
- If $f(x) = \begin{cases} \frac{|x+2|}{\tan^{-1}(x+2)}, & x \neq -2 \\ 2, & x = -2 \end{cases}$, then $f(x)$ is
 - Continuous at $x = -2$
 - Not continuous $x = -2$
 - Differentiable at $x = -2$
 - Continuous but not derivable at $x = -2$
- If $f(x) = |\log |x||$, then
 - $f(x)$ is continuous and differentiable for all x in its domain
 - $f(x)$ is continuous for all x in its domain but not differentiable at $x = \pm 1$
 - $f(x)$ is neither continuous nor differentiable at $x = \pm 1$
 - None of the above
- If $f'(a) = 2$ and $f(a) = 4$, then $\lim_{x \rightarrow a} \frac{xf(a) - af(x)}{x - a}$ equals
 - $2a - 4$
 - $4 - 2a$
 - $2a + 4$
 - None of these

8. If $f(x) = x(\sqrt{x} + \sqrt{x+1})$, then
 a) $f(x)$ is continuous but not differentiable at $x = 0$ b) $f(x)$ is differentiable at $x = 0$
 c) $f(x)$ is not differentiable at $x = 0$ d) None of the above

9. If $f(x) = \begin{cases} ax^2 + b, & b \neq 0, x \leq 1 \\ x^2b + ax + c, & x > 1 \end{cases}$, then, $f(x)$ is continuous and differentiable at $x = 1$, if
 a) $c = 0, a = 2b$ b) $a = b, c \in R$ c) $a = b, c = 0$ d) $a = b, c \neq 0$

10. For the function $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ x^2 - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$ which one of the following is incorrect?
 a) Continuous at $x = 1$ b) Derivable at $x = 1$ c) Continuous at $x = 3$ d) Derivable at $x = 3$

11. If $f: R \rightarrow R$ is defined by

$$f(x) = \begin{cases} \frac{2 \sin x - \sin 2x}{2x \cos x}, & \text{if } x \neq 0, \\ a, & \text{if } x = 0 \end{cases}$$

Then the value of a so that f is continuous at 0 is

- a) 2 b) 1 c) -1 d) 0

12. $f(x) = x + |x|$ is continuous for
 a) $x \in (-\infty, \infty)$ b) $x \in (-\infty, \infty) - \{0\}$ c) Only $x > 0$ d) No value of x

13. If the function

$$f(x) = \begin{cases} \{1 + |\sin x|\}^{\frac{a}{|\sin x|}}, & -\frac{\pi}{6} < x < 0 \\ b, & x = 0 \\ e^{\frac{\tan 2x}{\tan 3x}}, & 0 < x < \frac{\pi}{6} \end{cases}$$

Is continuous at $x = 0$

- a) $a = \log_e b, b = \frac{2}{3}$ b) $b = \log_e a, a = \frac{2}{3}$ c) $a = \log_e b, b = 2$ d) None of these

14. If $f(x) = x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \dots + \frac{x^2}{(1+x^2)^n} + \dots$, then at $x = 0, f(x)$

- a) Has no limit
 b) Is discontinuous
 c) Is continuous but not differentiable
 d) Is differentiable

15. Let $f(x) = \begin{cases} 1, & \forall x < 0 \\ 1 + \sin x, & \forall 0 \leq x \leq \pi/2 \end{cases}$, then what is the value of $f'(x)$ at $x = 0$?

- a) 1 b) -1 c) ∞ d) Does not exist

16. The function $f(x) = x - |x - x^2|$ is

- a) Continuous at $x = 1$ b) Discontinuous at $x = 1$
 c) Not defined at $x = 1$ d) None of the above

17. If $f(x + y + z) = f(x).f(y).f(z)$ for all x,y,z and $f(2) = 4, f'(0) = 3$, then $f'(2)$ equals
 a) 12 b) 9 c) 16 d) 6
18. If $f(x) = |\log_e |x||$, then $f'(x)$ equals
 a) $\frac{1}{|x|}, x \neq 0$
 b) $\frac{1}{x}$ for $|x| > 1$ and $\frac{-1}{x}$ for $|x| < 1$
 c) $\frac{-1}{x}$ for $|x| > 1$ and $\frac{1}{x}$ for $|x| < 1$
 d) $\frac{1}{x}$ for $|x| > 0$ and $-\frac{1}{x}$ for $x < 0$
19. If the function $f(x) = \begin{cases} \frac{1 - \cos x}{x^2}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is
 a) 1 b) 0 c) $\frac{1}{2}$ d) -1
20. Function $f(x) = |x - 1| + |x - 2|, x \in R$ is
 a) Differentiable everywhere in R
 b) Except $x = 1$ and $x = 2$ differentiable everywhere in R
 c) Not continuous at $x = 1$ and $x = 2$ d) Increasing in R

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