DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. : 9

Topic :- CONTINUITY AND DIFFERENTIABILITY

1 **(c)**

LHL $= \lim_{x \to a^-} \frac{x^3 - a^3}{x - a} = \lim_{h \to 0} \frac{(a-h)^3 - a^3}{a - h - a}$

$= \lim_{h \to 0} \frac{(a-h-a)\{(a-h)^2 + a^3 + a(a-h)\}}{-h} = 3a^2$

Since, $f(x)$ is continuous at $x = a$

$\therefore$ LHL $= f(a)$

$\Rightarrow$ $3a^2 = b$

3 **(a)**

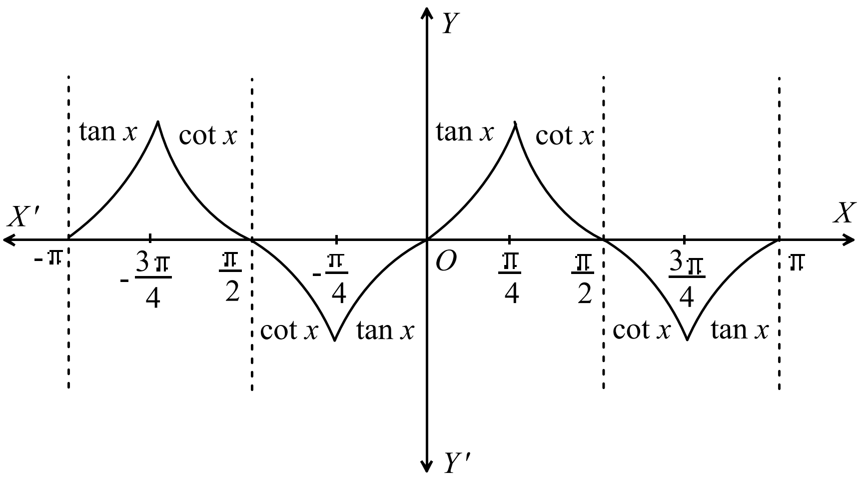
We have,

$$f(x) = \begin{cases} \tan x, & 0 \le x \le \pi/4 \\ \cot x, & -\pi/4 \le x \le \pi/2 \\ \tan x, & \pi/2 < x \le 3\pi/4 \\ \cot x, & 3\pi/4 \le x < \pi \end{cases}$$

Since $\tan x$ and $\cot x$ are periodic functions with period π. So, $f(x)$ is also periodic with period π

It is evident from the graph that $f(x)$ is not continuous at $x = \pi/2$. Since $f(x)$ is periodic with period π. So, it is not continuous at $x = 0, \pm\pi/2, \pm\pi, \neq 3\pi/2$

Also, $f(x)$ is not differentiable $x = \pi/4, 3\pi/4, 5\pi/4$ etc



4 **(c)**

We have,

$f(x) = \{|x| - |x-1\}^2$

$$\Rightarrow f(x) = \begin{cases} (-x+x-1)^2, & \text{if } x<0 \\ (x+x-1)^2, & \text{if } 0 \le x < 1 \\ (x-x+1)^2, & \text{if } x \ge 1 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} 1, & \text{if } x<0 \\ (2\,x-1)^2, & \text{if } 0 < x < 1 \\ 1, & \text{if } x \ge 1 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} 0, & \text{if } x<0 \text{ or if } x>1 \\ 4(2\,x-1), & \text{if } 0<x<1 \end{cases}$$

5 **(b)**

We have,

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$\Rightarrow f'(x_0) = \lim_{x \to x_0} \frac{(x-x_0)\phi(x) - 0}{(x-x_0)}$$

$$\Rightarrow f'(x_0) = \lim_{x \to x_0} \phi(x) = \phi(x_0) \qquad [\because \phi(x) \text{ is continuous at } x = x_0]$$

6 **(b)**

Since, $\lim_{x \to 2^+} f(x) = f(2) = k$

$$\Rightarrow \quad k = \lim_{h \to 0} f(2+h)$$

$$\Rightarrow \quad k = \lim_{h \to 0} \left[(2+h)^2 + e^{\frac{1}{2-(2+h)}}\right]^{-1}$$

$$\Rightarrow \quad \lim_{h \to 0} \quad [4 + h^2 + 4h + e^{-1/h}]^{-1} = \frac{1}{4}$$

7 **(c)**

For $f(x)$ to be continuous at $x = \pi/2$, we must have

$$\lim_{x \to \pi/2} f(x) = f(\pi/2)$$

$$\Rightarrow \lim_{x \to \pi/2} \frac{1 - \sin x}{(\pi - 2x)^2} \cdot \frac{\log \sin x}{\log(1 + \pi^2 - 4\pi x + 4x^2)} = k$$

$$\Rightarrow \lim_{h \to 0} \frac{1 - \cos h}{4h^2} \times \frac{\log \cos h}{\log(1 + 4h^2)} = k$$

$$\Rightarrow \lim_{h \to 0} \frac{1 - \cos h}{4h^2} \times \frac{\log\{1 + \cos h - 1\}}{\cos h - 1} \times \frac{4h^2}{\log(1+4h^2)} \times \frac{\cos h - 1}{4h^2} = k$$

$$\Rightarrow - \lim_{h \to 0} \left(\frac{1 - \cos h}{4h^2}\right)^2 \frac{\log(1 + (\cos h - 1))}{\cos h - 1} \times \frac{4\,h^2}{\log(1 + 4h^2)} = k$$

$$\Rightarrow - \lim_{h \to 0} \left(\frac{\sin^2 h/2}{2\,h^2}\right)^2 \frac{\log(1 + (\cos h - 1))}{\cos h - 1} \times \frac{4h^2}{\log(1 + 4h^2)} = k$$

$$\Rightarrow -\frac{1}{64}\lim_{h\to 0}\left(\frac{\sin h/2}{h/2}\right)^4 \frac{\log(1+(\cos h-1))}{\cos h-1}\times\frac{4h^2}{\log\left(1+4h^2\right)}=k$$

$$\Rightarrow -\frac{1}{64}=k$$

8 **(c)**

$$\text{LHL}=\lim_{h\to 0} f(0-h)=\lim_{h\to 0}\frac{\sin 5(0-h)}{(0-h)^2+2(0-h)}$$

$$=-\lim_{h\to 0}\frac{\frac{\sin 5h}{5h}}{\frac{1}{5}(h-2)}=\frac{5}{2}$$

Since, it is continuous at $x=0$, therefore LHL $=f(0)$

$$\Rightarrow \frac{5}{2}=k+\frac{1}{2} \quad \Rightarrow \quad k=2$$

9 **(a)**

Since $f(x)$ is continuous at $x=0$

$$\therefore \lim_{x\to 0} f(x) = f(0)=0$$

$$\Rightarrow \lim_{x\to 0} x^n \sin\left(\frac{1}{x}\right) = 0 \Rightarrow n>0$$

$f(x)$ is differentiable at $x=0$, if

$\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}$ exists finitely

$\Rightarrow \lim_{x\to 0}\frac{x^n \sin\frac{1}{x}-0}{x}$ exists finitely

$\Rightarrow \lim_{x\to 0} x^{n-1}\sin\left(\frac{1}{x}\right)$ exists finitely

$\Rightarrow n-1>0 \Rightarrow n>1$

If $n\le 1$, then $\lim_{x\to 0} x^{n-1}\sin\left(\frac{1}{x}\right)$ does not exist and hence $f(x)$ is not differentiable at $x=0$

Hence $f(x)$ is continuous but not differentiable at $x=0$ for $0<n\le 1$ i.e. $n\in(0,1]$

10 **(b)**

Clearly, $f(x)$ is not differentiable at $x=3$

Now, $\lim_{h\to 3^-} f(x)=\lim_{h\to 0} f(3-h)$

$=\lim_{h\to 0} |3-h-3|$

$=0$

$\lim_{h\to 3^+} f(x)=\lim_{h\to 0} f(3+h)$

$=\lim_{h\to 0} |3+h-3|=0$

and $f(3)=|3-3|=0$

$\therefore$ $f(x)$ is continuous at $x=3$

11 **(a)**

It can easily be seen from the graphs of $f(x)$ and that both are continuous at $x=0$

Also, $f(x)$ is not differentiable at $x=0$ whereas $g(x)$ is differentiable at $x=0$

12 **(c)**

We have,

$$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{-\sin(a+1)h - \sin h}{-h}$$

$$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\left\{\frac{\sin(a+1)h}{h} + \frac{\sin h}{h}\right\}$$

$$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = (a+1)+1 = a+2$$

and, $\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$

$$\Rightarrow \lim_{x\to 0^+} f(x) = \lim_{h\to 0}\frac{\sqrt{h+bh^2}-\sqrt{h}}{b\,h^{3/2}}$$

$$\Rightarrow \lim_{x\to 0^+} f(x) = \lim_{h\to 0}\frac{h+bh^2-h}{bh^{3/2}(\sqrt{h+bh^2}-\sqrt{h})} = \lim_{h\to 0}\frac{1}{\sqrt{1+bh}+1} = \frac{1}{2}$$

Since, $f(x)$ is continuous at $x=0$. Therefore,

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$$

$$\Rightarrow a+2 = \frac{1}{2} = c \Rightarrow c = \frac{1}{2}, a = -\frac{3}{2} \text{ and } b \in R - \{0\}$$

13 **(c)**

For $f(x)$ to be continuous at $x=0$, we must have

$$\lim_{x\to 0} f(x) = f(0)$$

$$\Rightarrow \lim_{x\to 0}\frac{(9^x-1)(4^x-1)}{\sqrt{2}-\sqrt{2\cos^2 x/2}} = k$$

$$\Rightarrow \lim_{x\to 0}\frac{(9^x-1)(4^x-1)}{\sqrt{2}.2\sin^2 x/4} = k$$

$$\Rightarrow \lim_{x\to 0}\frac{16\times\left(\frac{9^x-1}{x}\right)\left(\frac{4^x-1}{x}\right)}{2\sqrt{2}\left(\frac{\sin x/2}{x/4}\right)^2} = k$$

$$\Rightarrow \frac{16}{2\sqrt{2}}\log 9 \,.\log 4 = k = 4\sqrt{2}\log 9\,.\log 4 = 16\sqrt{2}\log 3\log 2$$

14 **(b)**

Given, $f(x) = [\tan^2 x]$

Now, $\lim_{x\to 0} f(x) = \lim_{x\to 0}[\tan^2 x] = 0$

And $f(0) = [\tan^2 0] = 0$

Hence, $f(x)$ is continuous at $x=0$

15 **(b)**

Let, $f(x) = x$

Which is continuous at $x=0$

Also, $f(x+y) = f(x) + f(y)$

$\Rightarrow f(0+0) = f(0) + f(0)$

$= 0 + 0$

$\Rightarrow f(0)=0$
$f(1+0)=f(1)+f(0)$
$\Rightarrow\ f(1)=1+0$
$\Rightarrow\ f(1)=1$
As, it satisfies it.
Hence, $f(x)$ is continous for every values of x
16 **(c)**
Here, $gof=\begin{cases} e^{\sin x}, & x\geq 0 \\ e^{1-\cos x}, & x\leq 0\end{cases}$
$\therefore\ \text{LHD}=\lim_{h\to 0}\frac{gof(0-h)-gof(h)}{-h}$
$=\lim_{h\to 0}\frac{e^{1-\cos h}-e^{1-\cos h}}{-h}=0$
$\text{RHD}=\lim_{h\to 0}\frac{gof(0+h)-gof(h)}{h}$
$=\lim_{h\to 0}\frac{e^{\sin h}-e^{\sin h}}{h}=0$
Since, RHD=LHD=0
$\therefore\ (gof)'(0)=0$
17 **(b)**
We have,
$$f(x)\begin{cases}(x+1)^{2-\left(\frac{1}{x}+\frac{1}{x}\right)}=(x+1)^2, & x<0\\ 0, & x=0\\ (x+1)^{2-\left(\frac{1}{x}+\frac{1}{x}\right)}=(x+1)^{2-\frac{2}{x}}, & x>0\end{cases}$$
Clearly, $f(x)$ is everywhere continuous except possibly at $x=0$
At $x=0$, we have
$\lim_{x\to 0^-}f(x)=\lim_{x\to 0^-}(x+1)^2=1$
and, $\lim_{x\to 0^+}f(x)=\lim_{x\to 0}(x+1)^{2-\frac{2}{x}}=\lim_{x\to 0}(x+1)^{-2/x}$
$\Rightarrow\lim_{x\to 0^+}f(x)=e^{\lim_{x\to 0}-\frac{2}{x}\log(1+x)}=e^{-2}$
Clearly, $\lim_{x\to 0^-}f(x)=\lim_{x\to 0^+}f(x)$
So, $f(x)$ is not continuous at $x=0$
18 **(b)**
Since $f(x)$ is continuous at $x=0$. Therefore,
$\lim_{x\to 0}f(x)=f(0)$
$\Rightarrow\lim_{x\to 0}f(x)=k$
$\Rightarrow\lim_{x\to 0}\frac{\log(1+ax)-\log(1-bx)}{x}=k$

$$\Rightarrow a\lim_{x\to 0}\frac{\log(1+ax)}{ax}-(-b)\lim_{x\to 0}\frac{\log(1-bx)}{-bx}=k$$

$\Rightarrow a+b=k$

19 **(c)**

Since $f(x)$ is continuous at $x=0$

$\therefore f(0)=\lim_{x\to 0}f(x)$

$$\Rightarrow f(0)=\lim_{x\to 0}\frac{(27-2x)^{1/3}-3}{9-3(243+5x)^{1/5}} \qquad \left[\text{Form }\frac{0}{0}\right]$$

$$\Rightarrow f(0)=\lim_{x\to 0}\frac{\frac{1}{3}(27-2x)^{-\frac{2}{3}}(-2)}{-\frac{3}{5}(243+5x)^{-\frac{4}{5}}(5)}=\left(-\frac{2}{3}\right)\left(-\frac{1}{3}\right)\frac{3^4}{3^2}=2$$

20 **(d)**

$$\lim_{x\to 0}\frac{e^{2x}-1-2x}{x(e^{2x}-1)}$$

$$=\lim_{x\to 0}\frac{2e^{2x}-2}{(e^{2x}-1)+2xe^{2x}} \quad \text{[using L 'Hospital rule]}$$

$$=\lim_{x\to 0}\frac{4e^{2x}}{4e^{2x}+4xe^{2x}}=1 \quad \text{[using L 'Hospital's rule]}$$

Since, $f(x)$ is continuous at $x=0$, then

$\lim_{x\to 0} f(x)=f(0) \quad \Rightarrow \quad 1=f(0)$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	A	C	B	B	C	C	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	C	B	B	C	B	B	C	D